HEALTH & COMMUNITY SERVICES

HEALTH MATHEMATICS
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Chapter 1

A LOOK AT THE "LANGUAGE" OF MATHEMATICS

Mathematics is a continually expanding subject with many branches. Most mathematicians specialize in one branch only. Obviously, this chapter does not seek to cover the terminology applying to all branches, but rather to ensure an understanding of the symbols which are appropriate to the operations used in solving the mathematical problems frequently encountered in nursing.

A SYMBOL is a mark, sign or word which represents an object or idea.

Examples of SYMBOLS which represent ideas are:-

+ represents addition

- represents subtraction

x represents multiplication

÷ represents division.

In mathematics, the processes of addition, subtraction, multiplication and division are called OPERATIONS.

Those interested in nursing tend to relate the term "operation" to surgical operations, but in this area of study we are referring to mathematical operations.

Numbers are represented by symbols known as NUMERALS, because a number is abstract and cannot be written. Down through the ages, different numerical systems have been used to record numbers. We use one system, but the ancient Romans and the ancient Egyptians each used their own. A given finite number is the same to us as it was to them, but the method of representing it is different.

The number five is usually represented by the symbol 5.
15 is a symbol representing the number fifteen. Other symbols or numerals may also be used to represent the number fifteen.

For example, $11 + 4$; $17 - 2$; $5 \times 3$; $60 \div 4$ all represent fifteen.

Generally we use the simplest method of representing a number so, in this case, we would use the symbol or numeral 15.

*NOTE: $60 \div 4$ may also be written as $\frac{60}{4}$

Another well-known symbol = indicates that the numbers represented by the marks on either side of the symbol are the same.

For example:

$2 + 6 = 8$
$9 - 2 = 7$
$14 \div 7 = 2$
$56 - 50 = 2 \times 3$
$7 \times 3 = 19 + 2$
$24 \div 3 = 10 - 2$

Notice that it is the NUMBER represented by the marks which is the same on either side of the = symbol.

NUMERAL SYSTEMS

Throughout the ages, different civilizations have used different systems for representing numbers.

The system we use in everyday life is called the DECIMAL SYSTEM because it is built on groups of ten. (Decimal is derived from the latin word for ten - decem).

Ten symbols called digits are used: 0 1 2 3 4 5 6 7 8 9. Any number can be represented by a numeral consisting of one or more of these digits.

FACE VALUE VERSUS PLACE VALUE

In the decimal system, each of the ten symbols (digits) has both face value and place value.
The face value of a digit never changes, whereas the place value of the digit depends on its position (place) in a numeral.

The numeral 5 contains only one digit. Its face value is five, but its place value is one; that is, it occupies the "ones place".

The numeral 75 contains two digits. The digit 5 occupies the "ones place", whereas the digit 7 occupies the "tens place".

In this numeral, 75, the place value of 5 is one and the place value of 7 is ten.

In the numeral 575 with three digits, the place value of the fives is different.

The digit 5 on the right of the numeral has a place value of one.

The digit 7 has a place value of ten.

The digit 5 on the left of the numeral has a place value of one hundred.

The digit which has a place value of one is said to be in the first place (1st place).

The digit which has a place value of ten is said to be in the second place (2nd place).

The digit which has a place value of one hundred is said to be in the third place (3rd place).

The digit which has a place value of one thousand is said to be in the fourth place (4th place).

Take the numeral 1973 which represents the number one thousand nine hundred and seventy three.

The four digits occupy the following places:

<table>
<thead>
<tr>
<th>4th place</th>
<th>3rd place</th>
<th>2nd place</th>
<th>1st place</th>
</tr>
</thead>
<tbody>
<tr>
<td>(thousand)</td>
<td>(hundred)</td>
<td>(ten)</td>
<td>(one)</td>
</tr>
<tr>
<td>1</td>
<td>9</td>
<td>7</td>
<td>3</td>
</tr>
</tbody>
</table>

Reading and writing of decimal numerals is made easier by grouping of digits. Beginning from the right of the numeral and moving towards the left we separate the digits into groups consisting of three digits. A comma used to be placed between each group, but modern usage leaves a space between each group.

For example:

5943279 is grouped as 5 943 279
379548625 is grouped as 379 548 625
The right hand group is the "ones group" but this is always assumed and is never put into words.

The second group from the right is given the group name of "thousand".

The third group from the right is given the group name of "million".

These are the groupings met with most commonly, however larger groups are often appropriate.

5,943,279 is read as five million nine hundred and forty three thousand two hundred and seventy nine.
When writing four digit numerals, the space between the "one" group and the "thousand" group may be omitted.

One thousand nine hundred and seventy five is more usually written as 1975 rather than 1,975.
Commas may be retained to separate groups of digits when an amount of money is being recorded. One hundred thousand dollars may be written $100,000.

POWERS AND INDICES

If you examine the following true statements, you will discover that in each case the digit 0 is repeated the same number of times on either side of the = symbol.

\[
10 \times 10 = 100 \\
10 \times 10 \times 10 = 1000 \\
10 \times 10 \times 10 \times 10 = 10000 \\
10 \times 10 \times 10 \times 10 \times 10 = 100000
\]

Consequently it was agreed to simplify the writing of expressions and numerals in which the digit 0 is repeated several times by using an index (plural: indices).

10 x 10 can be written as \(10^2\).

The numeral 2 which is written above and to the right of the 10 is called an INDEX and indicates the number of tens which are multiplied together.

10 x 10 x 10 can be written as \(10^3\)

10 x 10 x 10 x 10 can be written as \(10^4\)

10 x 10 x 10 x 10 x 10 can be written as \(10^5\).
$10^2$ is read as "ten to the second power". These are called the powers of ten.

$10^3$ is read as "ten to the third power".

$10^4$ is read as "ten to the fourth power".

$10^5$ is read as "ten to the fifth power".

The first power of ten is usually written as $10^1$. In this context, the 10 is called the base.

The use of powers and indices may be summarised thus:

(a) $10^4$ is read as "ten to the fourth power".
(b) $10^4$ means $10 \times 10 \times 10 \times 10$.
(c) $10^4 = 10000$.
(d) $10^4$ has an index of 4.
(e) $10^4$ has a base of 10.

Any numeral may be written as a base or as an index, for example:

$$3 \times 3 \times 3 \times 3 = 3^4$$
$$4 \times 4 \times 4 = 4^3$$

### PLACE VALUES:

<table>
<thead>
<tr>
<th>PLACE</th>
<th>PLACE VALUE IN WORDS</th>
<th>PLACE VALUE IN ORD. NUMERALS</th>
<th>PLACE VALUE IN POWERS OF TEN</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st place</td>
<td>one</td>
<td>1</td>
<td>$10^0$</td>
</tr>
<tr>
<td>2nd place</td>
<td>ten</td>
<td>10</td>
<td>$10^1$</td>
</tr>
<tr>
<td>3rd place</td>
<td>hundred</td>
<td>100</td>
<td>$10^2$</td>
</tr>
<tr>
<td>4th place</td>
<td>thousand</td>
<td>1000</td>
<td>$10^3$</td>
</tr>
<tr>
<td>5th place</td>
<td>ten thousand</td>
<td>10000</td>
<td>$10^4$</td>
</tr>
<tr>
<td>6th place</td>
<td>hundred thousand</td>
<td>100000</td>
<td>$10^5$</td>
</tr>
</tbody>
</table>

(c) BY
Earlier it was stated "the system we use is known as the decimal system because it is built on groups of ten". Now that we have been reintroduced to the term "base" we may restate this as "the decimal system is based on grouping in tens".

TEN groups of ten make a larger group called one hundred.

TEN groups of one hundred make a larger group called one thousand.

TEN groups of one thousand make a larger group called ten thousand.

Therefore, the decimal system is a place value system with a base of ten.

In a later chapter we will look at other numerical systems with place value, but in which the grouping is not based on ten.

The Metric System of Measurement (see Chapter 5) is described as a decimal system of measures and correlates closely with the decimal system of numerals.

SETS

A set is a collection or a group of things. For example, the SET of digits used in the decimal system. The members of the set are called its ELEMENTS. For example, the elements in the set of digits used in the decimal system are:-

0 1 2 3 4 5 6 7 8 9.

COUNTING NUMBERS

Counting is an attempt to match objects with the numbers in a set on a 'one-to-one' relationship. The set of numbers used in this matching process are called the set of COUNTING NUMBERS.

Zero is not a counting number. The first counting number is one.

WHOLE NUMBERS

A set consisting of zero and all the counting numbers is called the set of WHOLE NUMBERS.
RATIONAL NUMBERS

A rational number is a number which can be obtained by dividing a whole number by a counting number. (NOTE - we never divide by zero.)

For example:

\[
14 \div 2 = 7 \text{ which represents a rational number.}
\]
\[
1 \div 2 = \frac{1}{2} \text{ which represents a rational number.}
\]
\[
53 \div 12 = \frac{53}{12} \text{ which represents a rational number.}
\]

FRACTIONS

A fraction is a numeral which can represent a rational number.

For example, \(\frac{1}{2}\) is a fraction which represents the rational number one-half;

\(\frac{2}{3}\) is a fraction which represents the rational number two-thirds.

EXPANDED NUMERALS

There are times when it is very useful to "take numerals apart" so as to understand the construction of the numeral and perhaps rewrite it in a different way.

Consider the numeral 785 - the digit 7 is in the third place and has a place value of one hundred. It represents 700, i.e., \(7 \times 100\).

The digit 8 is in the second place and has a place value of ten. It represents 80, i.e., \(8 \times 10\).

The digit 5 is in the first place and has a place value of one. It represents 5, i.e., \(5 \times 1\).

The numeral 785 can be written in several other ways which constitute its "expanded form".

For example:-

\[
700 + 80 + 5 = 785
\]
\[
(7 \times 100) + (8 \times 10) + (5 \times 1) = 785
\]
\[
(7 \times 10 \times 10) + (8 \times 10) + (5 \times 1) = 785
\]
\[
(7 \times 10^2) + (8 \times 10^1) + (5 \times 1) = 785
\]
Another example of expanded form:

4 000 + 300 + 0 + 7 = 4307
(4 × 1000) + (3 × 100) + (0 × 10) + (7 × 1) = 4307
(4 × 10 × 10 × 10) + (3 × 10 × 10) + (0 × 10) + (7 × 1) = 4307
(4 × 10³) + (3 × 10²) + (0 × 10¹) + (7 × 1) = 4307

NOTE: In the numeral above there are no tens. This is indicated by a zero in the tens place, and is shown in expanded form as (0 × 10).

In writing these numbers in expanded form, we have demonstrated some SPECIAL PROPERTIES OF ZERO AND ONE.

THE MULTIPLICATION PROPERTY OF ZERO (M.P. ZERO)

This property tells us that the product of any whole number and zero is zero.
In the example above we have 0 × 10 = 0
also 8 × 0 = 0
20 × 0 = 0

THE MULTIPLICATION PROPERTY OF ONE (M.P. ONE)

This property tells us that the product of any whole number and one is the original number.
In the previous examples we have 5 × 1 = 5
7 × 1 = 7
1 × 20 = 20

THE ADDITION PROPERTY OF ZERO (A.P. ZERO)

This property tells us that the sum of any whole number and zero is the original number.
For example:

8 + 0 = 8
20 + 0 = 20
0 + 10 = 10
Chapter 2

A CONTINUING LOOK AT THE LANGUAGE OF MATHEMATICS

SUMS AND PRODUCT

We know that $5 + 5 + 5$ is quite different from $5 \times 5 \times 5$; so also is $3 \times 5$ quite different from $5^3$.

$5 + 5 + 5$ may be written as $3 \times 5$ with the 3 indicating how many fives are added together.

$5 \times 5 \times 5$ may be written as $5^3$ with the 3 as an index, indicating the number of fives which are multiplied together.

The common numeral for $5 + 5 + 5$ or $3 \times 5$ is 15.

The common numeral for $5 \times 5 \times 5$ or $5^3$ is 125.

The operation known as addition leads us to the SUM of the numbers being added.

The operation known as multiplication leads us to the PRODUCT of the numbers being multiplied.

DIFFERENCES AND QUOTIENTS

The operation known as subtraction leads us to the DIFFERENCE between the numbers being subtracted.

The operation known as division leads us to the QUOTIENT of the numbers being divided.

NUMERICAL EXPRESSIONS

The simplest numeral for the number one is 1, but it can be expressed in many different ways.

For example: $1 = \frac{2}{2}; \frac{5}{5}; \frac{10}{5}; \frac{20}{1}; \frac{10}{2}; \frac{20}{9}; 1^2; 1^3; 1 \times 1; 1 - 0; 1 + 0$.

On page 1.2, the number fifteen was represented by the following numerals or numerical expressions:-
15; 11 + 4; 17 - 2; 5 x 3; 60 ÷ 4; \frac{60}{4}

The words "numeral" and "numerical expression" mean exactly the same thing, and either term may be used. However, it is usual to use "numeral" when we want to refer to simpler names for a number, and "numerical expression" when we want to refer to some other name for a number (i.e., a name which involves operations).

In the previous example we usually refer to 15 as the numeral and to 11 + 4; 17 - 2; 5 x 3 etc., as the numerical expression.

The simplest numeral for any number is called its COMMON NUMERAL. 15 is the common numeral for the number fifteen.

Numerical expressions in which two numerals are separated by the symbol for an operation have special names.

For example: 3 + 6 is called an INDICATED SUM, because the operation + indicates that the common numeral for the numerical expression can be obtained by finding the sum of the numbers.

Similarly:-

7 - 2 is called an INDICATED DIFFERENCE;

8 x 5 is called an INDICATED PRODUCT;

15 ÷ 3 is called an INDICATED QUOTIENT.

PUNCTUATED NUMERICAL EXPRESSIONS

We know that a sentence without punctuation may have more than one meaning - i.e., it is ambiguous.

Numerical expressions can also have double meanings. Take as an example the numerical expression

5 + 2 x 3.

Looking at this one way you might think 5 + 2 = 7 and 7 x 3 = 21.

But looking at it another way you could say 2 x 3 = 6 and 5 + 6 = 11.

To prevent confusion, numerical expressions can be punctuated. The symbols used to punctuate numerical expressions are called BRACKETS. They show the order in which the operations should be performed.

The ambiguity can be removed from the expression 5 + 2 x 3 by arrangement of the numerical expression as either:-

[(5 + 2) x 3] or [5 + (2 x 3)]

2.2
(a) \((5 + 2) \times 3\)  \hspace{1cm} \text{or} \hspace{1cm} (b) \(5 + (2 \times 3)\)

First perform the operation indicated by the numerical expression within the brackets.

\[
\begin{align*}
\text{In (a) } & \quad (5 + 2) \times 3 = 7 \times 3 = 21 \\
\text{In (b) } & \quad 5 + (2 \times 3) = 5 + 6 = 11
\end{align*}
\]

There are four different types of brackets used in mathematics to punctuate numerical expressions. They are:

(a) \((\quad)\) called "round brackets" - sometimes shortened to "brackets".

(b) \([\quad]\) called "square brackets".

(c) \{"\quad\} called "curly brackets".

(d) \(\quad\) called "a straight line bracket" or "vinculum".

Brackets, except the vinculum, are always used in pairs, and they show that the numerical expression grouped inside the brackets is to be treated as a single numeral.

In the previous example, the numerical expression \((5 + 2)\) was expressed as the common numeral 7; while the numerical expression \((2 \times 3)\) was expressed as the common numeral 6.

The straight line bracket is seen as a straight line in a fraction \(\frac{1}{2}\) or in the numerical expression \(\frac{3 \times 4}{2 \times 3}\) .

In this case, the numerical expression above the line represents a single numeral, and the numerical expression below the line represents a single numeral.

When working with numerical expressions, the multiplication symbol \(\times\) is sometimes omitted.

\[
\begin{align*}
6(5 + 2) & \text{ means } 6 \times (5 + 2) \\
\left[6 + 7\right]\left[5 - 7\right] & \text{ means } \left[6 + 7\right] \times \left[5 - 7\right]
\end{align*}
\]

A dot on the line has also been used at times as a multiplication symbol, but it has lost favour because of the danger of confusion between it and the decimal point.
The operation of multiplication is intended in each of the following:

\[ 5(7 - 2); \ 6[4]; \ (21)(33). \]

**Simplifying Numerical Expressions**

Simplifying the numerical expression for a number means finding the common numeral for the number. For example, the numerical expression \(3 \times 4 \times 2\) can be simplified to the common numeral 24.

The numerical expression \(\frac{63 \times 2}{9 \times 7}\) can be simplified to its common numeral 2.

Consider the following example where both round and straight line brackets are used:

\[ \frac{5(6 - 2)}{13 - 3} \]

First perform the operation indicated by the numerical expression within the round brackets. Then express as common numerals the numerical expressions above and below the straight line bracket.

\[
\frac{5(6 - 2)}{13 - 3} = \frac{5(4)}{10} = \frac{20}{10} = 2
\]

Note that \(\frac{5(6 - 2)}{13 - 3}\) is the indicated quotient of the number \(5(6 - 2)\) and \(13 - 3\).

If you meet a numerical expression which shows several groupings, one within the other, the procedure is to first simplify the innermost numerical expression and then work outwards, omitting each pair of brackets as the numerical expression grouped inside is simplified.

For example:

\[ [(4 + (2 \times 3)) \times 7] + 8 \]

**STEP 1:** Carry out the operation indicated by the numerical expression within the round brackets - then omit the round brackets.
STEP 2: Carry out the operation indicated by the numerical expression now within the curly brackets - then omit the curly brackets.

STEP 3: Carry out the operation indicated by the numerical expression now within the square brackets - then omit the square brackets.

STEP 4: Carry out the final operation.

This example would be set out as follows:-

\[
\left(\frac{4 + (2 \times 3)}{7}\right) + 8 = \left(\frac{4 + 6}{7}\right) + 8
= \left(\frac{10}{7}\right) + 8
= 70 + 8
= 78
\]

Follow the above steps in simplification of the following expressions.

Simplify \( \left(\frac{6 + 7 (5 - 3)}{10}\right) \div 5 \)

\[
\left(\frac{6 + 7 (5 - 3)}{10}\right) \div 5 = \left(\frac{6 + 7 (2)}{10}\right) \div 5
= \left(\frac{6 + 14}{10}\right) \div 5
= \left(\frac{20}{10}\right) \div 5
= \frac{10}{5}
= 2
\]

Simplify \( \frac{(3 \times 4) + (2 \times 5)}{2} - 7 \)

\[
\frac{(3 \times 4) + (2 \times 5)}{2} - 7 = \frac{12 + 10}{2} - 7
= \frac{22}{2} - 7
= 11 - 7
= 4
\]

2.5
In the examples used so far in this chapter, the operations have involved relatively simple computations. The following notes on multiples and factors can help in simplifying more complex numerical expressions.

**MULTIPLES AND FACTORS**

A multiple of a number is the product of that number and any whole number.

Examples of multiples of 3 are 3, 6, 9, 12, 15, 18, 21, 24 and so on because:

\[ 3 = 3 \times 1 \]
\[ 6 = 3 \times 2 \]
\[ 9 = 3 \times 3 \]
\[ 12 = 3 \times 4 \]

Multiples of 5 are 5, 10, 15, 20, 25, 30 and so on because:

\[ 5 = 5 \times 1 \]
\[ 10 = 5 \times 2 \]
\[ 15 = 5 \times 3 \]
\[ 20 = 5 \times 4 \]

0 is a multiple of every number.

**EVEN NUMBERS** are those which are multiples of 2. For example:

\[ 2 \times 0 = 0 \]
\[ 2 \times 3 = 6 \]
\[ 2 \times 36 = 72 \]
\[ 2 \times 54 = 108 \]

**ODD NUMBERS** are those which are not multiples of 2. For example: 1, 3, 5, 7, 9, 11, 13, etc.
A COMMON MULTIPLE is the product of any two numbers. In the previously used example, i.e., \(5 \times 3 = 15\), 15 is seen to be a common multiple of 3 and 5.

Common multiples can be found by comparing the multiples of several given numbers. For example:

<table>
<thead>
<tr>
<th>Multiples of 4</th>
<th>Multiples of 6</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>6</td>
</tr>
<tr>
<td>8</td>
<td>12</td>
</tr>
<tr>
<td>12</td>
<td>18</td>
</tr>
<tr>
<td>16</td>
<td>24</td>
</tr>
<tr>
<td>20</td>
<td>30</td>
</tr>
<tr>
<td>24</td>
<td>36</td>
</tr>
<tr>
<td>28</td>
<td>42</td>
</tr>
<tr>
<td>32</td>
<td>48</td>
</tr>
<tr>
<td>36</td>
<td>54</td>
</tr>
<tr>
<td>40</td>
<td></td>
</tr>
<tr>
<td>44</td>
<td></td>
</tr>
<tr>
<td>48</td>
<td></td>
</tr>
</tbody>
</table>

From this example we can see that 12, 24, 36 and 48 are common multiples of 4 and 6.

Earlier it was stated that 0 was a common multiple. Excluding 0, we call the next common multiple the LEAST COMMON MULTIPLE or L.C.M. 12 is the least common multiple of 4 and 6.

We know that 20 is a multiple of 1, 2, 4, 5, 10 and 20. Each of these numbers divides evenly into 20 and so may be called a FACTOR of 20.

1 is a factor of 20 (because 1 divides evenly into 20) and 20 is a multiple of 1.

2 is a factor of 20 (because 2 divides evenly into 20).
20) and 20 is a multiple of 2.

4 is a factor of 20 (because 4 divides evenly into 20) and 20 is a multiple of 4.

5 is a factor of 20 (because 5 divides evenly into 20) and 20 is a multiple of 5.

10 is a factor of 20 (because 10 divides evenly into 20) and 20 is a multiple of 10.

20 is a factor of 20 (because 20 divides evenly into 20) and 20 is a multiple of 20.

Numbers can be grouped according to how many factors they have.

1 has only one factor - itself, and so is in a special group on its own. Some numbers have just two factors, the number itself and 1. Other numbers have more than two factors. For example:-

4 has three factors - 1, 2 and 4.

8 has four factors - 1, 2, 4 and 8.

PRIME NUMBERS are those numbers which have only two factors, e.g., 2, 3, 5, 7, 11, 13......are prime numbers.

COMPOSITE NUMBERS are those numbers which have more than two factors, e.g., 4, 6, 8, 9, 10, 12, 14 are composite numbers.

RECOGNISING PRIME NUMBERS

Ask two questions about any number:

1. Does it have factors other than itself and 1?

2. Is it a multiple of any number other than itself and 1?

If the answer to each question is "No" then you have identified a prime number.

If the answer to either question is "Yes" then you have identified a composite number.
WORKING WITH FACTORS

There are some quick ways to recognise factors:

(a) If 2 is a factor of some number, the digit in the first place of that number is even.

28
210
336
74
962

an even digit in first place so 2 is a factor

(b) If 3 is a factor of some number, the sum of the digits of that number must be a multiple of 3, e.g., in 1428 the sum of 1 + 4 + 2 + 8 is 15. 15 is a multiple of 3, so 3 must be a factor of 1428.

In 2573 the sum of 2 + 5 + 7 + 3 is 17. 17 is not a multiple of 3, so 3 is not a factor of 2573.

(c) If 5 is a factor of some number, the digit in the first place of that number must be 0 or 5.

5
25
3695

the digit 5 is in first place so 5 is a factor
the digit 0 is in first place, so 5 is a factor.

Just as we could find the least common multiple by examining the multiples of two or more numbers, we can also find the HIGHEST or GREATEST COMMON FACTOR (H.C.F.) by examining the factors of two or more numbers. The highest or greatest common factor is sometimes referred to as the GREATEST COMMON DIVISOR.

For example:

Factors of 30

1  common factor  1
2  common factor  2
3  common factor  3
5  4
6  common factor  6
10  7
15  12
30  14

Factors of 84

7  11
14  12
21  28
42  42
84  84
The common factors of 30 and 84 are: 1, 2, 3 and 6, but 6 is the greatest common factor.

An easier way of finding the greatest common factor is to write each number as a product of its prime factors.

\[
30 = 2 \times 3 \times 5 \quad (2, 3 \text{ and } 5 \text{ are all prime}) \\
84 = 2 \times 3 \times 7 \times 2 \\n\quad = 2 \times 2 \times 3 \times 7
\]

The common prime factors are 2 and 3, therefore the greatest common factor is a multiple of 2 and a multiple of 3.

The greatest common factor is 6.

Find the greatest common factor of 105 and 75 using the easier method.

\[
105 = 3 \times 5 \times 7 \\n\quad = 3 \times 5 \times 7 \quad (3, 5 \text{ and } 7 \text{ are all prime}) \\
75 = 3 \times 5 \times 5 \\n\quad = 3 \times 5 \times 5 \quad (3 \text{ and } 5 \text{ are all prime})
\]

The common factors are 3 and 5, therefore the greatest common factor is a multiple of 3 and a multiple of 5.

Greatest common factor of 105 and 75 is \(3 \times 5 = 15\).

**NOTE:** It is not essential to place the prime factors in order of ascending face value, but most people find it easier to do so.

The following examples will show two different methods of finding the greatest common factor.

**METHOD A**

1. Find the greatest common factor of 120 and 168.

\[
120 = 2 \times 60 \\n\quad = 2 \times 2 \times 30 \\n\quad = 2 \times 2 \times 2 \times 15 \\n\quad = 2 \times 2 \times 2 \times 3 \times 5 \\
168 = 2 \times 84 \\n\quad = 2 \times 2 \times 42 \\n\quad = 2 \times 2 \times 2 \times 21 \\n\quad = 2 \times 2 \times 2 \times 3 \times 7
\]

Common factors: \(2, 2, 3\)

\[\text{Greatest Common Factor} = 2 \times 2 \times 3 = 2 \times 3 \times 2 = 12\]
Greatest common factor = \( 2 \times 2 \times 2 \times 3 = 24 \).

2. Find the greatest common factor of 180, 18 and 198.

\[
egin{align*}
180 &= 2 \times 90 \\
    &= 2 \times 2 \times 45 \\
    &= 2 \times 2 \times 3 \times 15 \\
    &= 2 \times 2 \times 3 \times 3 \times 5 \\
18 &= 2 \times 9 \\
    &= 2 \times 3 \times 3 \\
198 &= 2 \times 99 \\
    &= 2 \times 3 \times 33 \\
    &= 2 \times 3 \times 3 \times 11
\end{align*}
\]

Common factors: 2, 3, 3.

The greatest common factor = \( 2 \times 3 \times 3 = 18 \).

METHOD B

In this method we divide each number by prime numbers until the final factor is 1. The prime factors of that number can then be seen to be arranged down the left.

We will demonstrate this method in solving the same problems we did by Method A.

1. Find the greatest common factor of 120 and 168.

\[
\begin{array}{c|c}
2 & 120 \\
\hline
2 & 60 \\
\hline
2 & 30 \\
\hline
3 & 15 \\
\hline
5 & 5 \\
\hline
1 & 1 \\
\end{array}
\quad
\begin{array}{c|c}
2 & 168 \\
\hline
2 & 84 \\
\hline
2 & 42 \\
\hline
3 & 21 \\
\hline
7 & 7 \\
\hline
1 & 1 \\
\end{array}
\]

2.12
Select the factors from the left-hand columns which are common to both columns.

2, 2, 2 and 3 are the common factors.
Greatest common factor = 2 x 2 x 2 x 3 = 24.

2. Find the greatest common factor of 180, 18 and 198.

<table>
<thead>
<tr>
<th></th>
<th>180</th>
<th></th>
<th>18</th>
<th></th>
<th>198</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>2</td>
</tr>
<tr>
<td>2</td>
<td>90</td>
<td>3</td>
<td>9</td>
<td>3</td>
<td>99</td>
</tr>
<tr>
<td>3</td>
<td>45</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>33</td>
</tr>
<tr>
<td>3</td>
<td>15</td>
<td>1</td>
<td>11</td>
<td>11</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>5</td>
<td>1</td>
<td></td>
<td>1</td>
<td></td>
</tr>
</tbody>
</table>

There are three columns from which to select the common factors.
Common factors are 2, 3 and 3.
Greatest common factor is 2 x 3 x 3 = 18.
The factors for any number may be written in shorter form by use of indices.

The factors of 120 are $2^3 \times 3 \times 5$.
The factors of 168 are $2^3 \times 3 \times 7$.
The greatest common factor of 120 and 168 is $2^3 \times 3 = 24$.
The factors of 180 are $2^2 \times 3^2 \times 5$.
The factors of 18 are $2 \times 3^2$.
The factors of 198 are $2 \times 3^2 \times 11$.
The greatest common factor of 180, 18 and 198 is $2 \times 3^2 = 18$. 
In the examples used so far in this chapter, the operations have involved relatively simple computations. The following examples show how factors can help in simplifying more complex numerical expressions.

Simplify \(\frac{4 \times 7 \times 8}{112}\) using factors.

Keeping in mind the quick ways of recognising factors shown on pages 2.9 and 2.10 of this chapter, scan the numerals above and below the straight line bracket.

You can immediately recognise that there are common factors.

2 is common to 4 and 112, so divide each number by that common factor.

The numerical expression could now be rewritten as:-

\[
\frac{2 \times 7 \times 8}{56}
\]

2 is also common to 2 and 56 so that the numerical expression could be rewritten again - this time as:-

\[
\frac{1 \times 7 \times 8}{28}
\]

2 is also common to 8 and 28, so the numerical expression could be rewritten again, but now as:-

\[
\frac{1 \times 7 \times 4}{14}
\]

2 is also common to 4 and 14, so the numerical expression can again be rewritten as:-

\[
\frac{1 \times 7 \times 2}{7}
\]

7 is present on both sides of the straight line bracket so the numerical expression can now be written as:-

\[
\frac{1 \times 1 \times 2}{1}
\]

As a final step, the numerical expression can now
be reduced to its common numeral 2.

Such problems are usually set out as:

\[
\frac{1}{2} \times \frac{2}{1} \times \frac{4}{8} = \frac{1 \times 1 \times 2}{1} = 2
\]

The procedure can be reduced even further if the greatest common factor of a numeral above and below the line bracket can be recognised.

For example, if you are asked to simplify \(\frac{4 \times 7 \times 8}{112}\)

4 is the greatest common factor of 4 and 112, so that the numerical expression can be rewritten as:

\[
\frac{1 \times 7 \times 8}{28}
\]

4 is the greatest common factor of 8 and 28, so that the numerical expression can now be written as:

\[
\frac{1 \times 7 \times 2}{7}
\]

7 is again common, so we can write:

\[
\frac{1 \times 1 \times 2}{1}
\]

From this stage we can recognise the common numeral which is 2.

This can be set out as:

\[
\frac{1}{2} \times \frac{7}{1} \times \frac{8}{112} = \frac{1 \times 1 \times 2}{1} = 2
\]
Simplify \( \frac{105 \times 315}{35 \times 7} \) using factors.

35 is the greatest common factor of 35 and 105. The numerical expression can be rewritten as:

\[
\begin{array}{c|c|c|c}
 & 3 & 315 \\
\hline
7 & 7 & 5
\end{array}
\]

7 is the greatest common factor of 7 and 315 so that the numerical expression may be rewritten as:

\[
\begin{array}{c|c|c|c}
 & 3 & 105 \\
\hline
7 & 7 & 5
\end{array}
\]

The common numeral is therefore \( 3 \times 45 = 135 \).

MORE ON H.C.F. AND G.C.D.

In the previous examples used, the greatest common factor is relatively obvious. From time to time the problem may be presented in which the greatest common factor of two numbers may not be immediately apparent. A more direct approach is needed in these cases. As the procedure involves long division we will leave its exploration until the methods of long division have been reviewed in Chapter 3.
Chapter 3

MORE ON SUMS AND PRODUCTS, DIFFERENCES AND QUOTIENTS

In this chapter we will examine some more special properties relating to certain mathematical operations.

COMMUTATIVE PROPERTY OF ADDITION (C.P.A.)

Commut means to interchange.

This property tells us that when two numbers are added it is possible to change the order of the numbers without changing the result.

For example:-

\[ 4 + 2 = 2 + 4 \]

\[ 7 + 6 = 6 + 7 \]

\[ 95 + 35 = 35 + 95 \text{ and so on.} \]

In any numerical expression involving the operation of addition, the numbers to be added are known as ADDENDS.

ASSOCIATIVE PROPERTY OF ADDITION (A.P.A.)

Associate means to group.

This property tells us that when we add three or more numbers, the way in which we group the numbers does not affect the sum.

For example:-

\[ 14 + (16 + 17) = (14 + 16) + 17 \]
Addition may be simplified by one pairing in preference to another.

In the previous numerical expression it can be seen that \((14 + 16) + 17\) is easier to solve than \(14 + (16 + 17)\)

\[
(14 + 16) + 17 = 30 + 17
\]

\[= 47\]

FURTHER EXAMPLES IN ADDITION

Find the sum of 25, 36 and 75. (The indicated sum is \(25 + 36 + 75\).)

By applying the commutative and associative properties of addition we can change the order and the grouping so that the indicated sum is re-expressed as \((25 + 75) + 36\).

\[
(25 + 75) + 36 = 100 + 36
\]

\[= 136\]

This is only one method by which we can find the sum of these three numbers. Let us look at some other methods with which you may be familiar.

Refer back to Chapter 1 page 1.8 where the process of expanding numerals was demonstrated.

Rewrite each numeral in expanded form.

\[25 = 20 + 5 \text{ or } (2 \times 10) + 5\]

\[36 = 30 + 6 \text{ or } (3 \times 10) + 6\]

\[75 = 70 + 5 \text{ or } (7 \times 10) + 5\]

Now, add the numerals in the right-hand column. Use the Associative Property of Addition to arrange a more convenient grouping.

\[
(5 + 5) + 6 = 10 + 6
\]

\[= 16\]

Add the numerals in the left-hand column:-

\[3.2\]
\[(30 + 70) + 20 = 100 + 20\]
\[= 120\]

or
\[[(3 + 7) + 2] \times 10 = (10 + 2) \times 10\]
\[= 12 \times 10\]
\[= 120\]

Now add sixteen to the one hundred and twenty, as below:
\[
\begin{align*}
25 & = 20 + 5 \\
36 & = 30 + 6 \\
+ 75 & = 70 + 5 \\
120 + 16 & = 10 + 6; 120 + 10 = 130 \\
& = 130 + 6 \\
& = 136
\end{align*}
\]

In the next method, the numerals are written down one under the other. The digits in the ones place are added, followed by those in the tens place.

\[
\begin{array}{c}
25 \\
36 \\
+ 75 \\
\hline
16 \text{ i.e., 16 ones} \\
120 \text{ i.e., 12 tens} \\
\hline
136
\end{array}
\]

Our next method expresses the sum of the digits in the ones place as \(10 + 6\). We write the six and carry the ten. This would be set out as:

\[
\begin{array}{c}
25 \\
36 \\
+ 75 \\
\hline
1 \\
\hline
136
\end{array}
\]

In the final method to be demonstrated, we carry the ten in our head and make no note of it on the paper.
25
36
+ 75
\[ \frac{136}{} \]

Note that the digits with the same place value are under one another.

Consider the indicated sum 476 + 345 + 412.

METHOD A:

\[
\begin{align*}
476 & = 400 + 70 + 6 \\
345 & = 300 + 40 + 5 \\
+ 412 & = 400 + 10 + 2 \\
\quad & = 1100 +120 +13 \\
\quad & = 1100 +130 + 3 \\
\quad & = 1200 + 30 + 3 \\
\quad & = 1233
\end{align*}
\]

METHOD B:

\[
\begin{align*}
476 & \\
345 & \\
\text{+ 412} & \quad \text{because } (6 + 5 + 2) \\
\quad & = 13 \\
120 & \quad \text{because } (70 + 40 + 10) \\
1100 & \quad \text{because } (400 + 300 + 400) \\
\quad & = 1233
\end{align*}
\]

METHOD C:

\[
\begin{align*}
476 & \\
345 & \\
412 & \\
11 & \\
\quad & = 1233
\end{align*}
\]
METHOD D:

<table>
<thead>
<tr>
<th>476</th>
</tr>
</thead>
<tbody>
<tr>
<td>345</td>
</tr>
<tr>
<td>412</td>
</tr>
<tr>
<td>1233</td>
</tr>
</tbody>
</table>

Most people tend to use Method D, but some find either Methods A, B or C easier to handle.
Let us look at a further example using Method D.
Consider the indicated sum 19 983 + 309 + 3 950:

| 19 983 |
| 309 |
| 3 950 |
| 24 242 |

Adding the extreme right-hand column (the ones column) gives 12, i.e., 10 + 2 or \((1 \times 10)+(2 \times 1)\), so we write the 2 and carry the 1 to the tens column.
Adding the tens column gives 14 tens, i.e., 140 or \((1 \times 100)+(4 \times 10)\), so we write the 4 and carry the 1 to the hundreds column.
Adding the hundreds column gives 22 hundreds, i.e., 2200 or \((2 \times 1000)+(2 \times 100)\), so we write 2 and carry 2 to the thousands column.
Adding the thousands column gives 14 thousands i.e., 14 000 or \((1 \times 10 000)+(4 \times 1000)\), so we write the 4 and carry 1 to the ten thousands column which then has a sum of 2.
The sum of 19 983 + 309 + 3 950 is 24 242.

THE COMMUTATIVE PROPERTY OF MULTIPLICATION (C.P.M.)

This property tells us that it is possible to change the order when multiplying two whole numbers.
For example:

\[ 5 \times 7 = 7 \times 5 \]
\[ 11 \times 2 = 2 \times 11 \]
\[ 191 \times 7 = 7 \times 191 \] and so on.
THE ASSOCIATIVE PROPERTY OF MULTIPLICATION (A.P.M.)

This property tells us that when multiplying three or more numbers, the way in which we group the numbers does not affect the product.

For example:

\[(17 \times 25) \times 4 = 17 \times (25 \times 4)\]

As in addition, the multiplication may be simplified by one pairing in preference to another.

In the above example, the numerical expression \(17 \times (25 \times 4)\) is easier to solve than the numerical expression \((17 \times 25) \times 4\) or than the other alternative \((17 \times 4) \times 25\).

\[17 \times (25 \times 4) = 17 \times 100\]
\[= 1700\]

Notice how the Commutative and Associative Properties of Multiplication have allowed us to change the order and the grouping.

THE DISTRIBUTIVE PROPERTY OF MULTIPLICATION OVER ADDITION (D.P.M/A)

This property of whole numbers links the operations of multiplication and addition. It shows how complicated multiplication may be reduced to simple multiplication and addition.

Using a simple example this can be shown as:

\[2 \times (3 + 1) = (2 \times 3) + (2 \times 1)\]

Working some slightly more complicated examples shows:

\[3 \times 15 = 3 \times (10 + 5)\]
\[= (3 \times 10) + (3 \times 5)\]
\[= 30 + 15\]
\[= 45\]
23 × 7 = (20 + 3) × 7
     = (20 × 7) + (3 × 7)
     = 140 + 21
     = 161

FURTHER EXAMPLES IN MULTIPLICATION

The problems in multiplication handled so far have all been simple. It is time to look at some other examples including long multiplication.

As with addition, there are various ways of setting out methods of finding the product of two numbers.

Consider the indicated product 24 × 7.

METHOD A:

\[
\begin{align*}
24 &= (20 + 4) \\
\times 7 &= \frac{20 \times 7}{140 + 28} = 168
\end{align*}
\]

METHOD B:

\[
\begin{align*}
24 &= (20 + 4) \\
\times 7 &= \frac{20 \times 7 + 28 \text{ because } (7 \times 4)}{140 \text{ because } (7 \times 20)} = 168
\end{align*}
\]

METHOD C:

\[
\begin{align*}
24 \\
\times 7 \\
28 \\
140 \\
168
\end{align*}
\]

METHOD D:

\[
\begin{align*}
24 \\
\times 7 \\
168
\end{align*}
\]
In Method D, multiplying the four ones by 7 gives 28, i.e., \((20 + 8)\) or \((2 \times 10) + (8 \times 1)\), so we write the 8 and carry the 2 to the tens column. Multiplying two tens by 7 gives 14 tens, i.e., 140 or \((1 \times 100) + (4 \times 10)\). Add to this the two tens carried over and the product is 168.

Consider the indicated product \(1375 \times 9\):

**METHOD A:**

\[
1375 = (1000 + 300 + 70 + 5) \times 9
\]

\[
= 9000 + 2700 + 630 + 45
\]

\[
= 9000 \\
= 2700 \\
= 630 \\
= 45
\]

\[
12375
\]

**METHOD B:**

\[
1375 = (1000 + 300 + 70 + 5) \times 9
\]

\[
= 45 = (5 \times 9) \\
= 630 = (70 \times 9) \\
= 2700 = (300 \times 9) \\
= 9000 = (1000 \times 9)
\]

\[
12375
\]

**METHOD C:**

\[
1375 \times 9
\]

\[
= 45 \\
= 630 \\
= 2700 \\
= 9000
\]

\[
12375
\]

**METHOD D:**

\[
1375 \times 9
\]

\[
12375
\]
In Method D, multiplying the five ones by 9 gives 45, i.e., \((40 + 5)\) or \((4 \times 10) + (5 \times 1)\), so we write the 5 and carry the 4 to the tens column. Multiplying the seven tens by 9 gives 630, i.e., \(600 + 30\) plus the four tens carried over gives 670, i.e., \(600 + 70\) or \((6 \times 100) + (7 \times 10)\). We write down the 7 and carry the 6 to the hundreds column. Multiplying the three hundreds by 9 gives 2700, i.e., \((2000 + 700)\) plus the six hundreds carried over gives 3300. We write down 3 and carry 3 to the thousands column. Multiplying one thousand by 9 gives 9000 plus the three thousands carried over gives 12000.

Consider the indicated product \(67 \times 23\).

In this example both numerals are two digit numerals.

**METHOD A:**

\[
\begin{align*}
67 &= (60 + 7) \\
\times 23 &= x (20 + 3) \\
180 + 21 &= (60 \times 3) + (7 \times 3) \\
+ 1200 + 140 &= (60 \times 20) + (7 \times 20) \\
= 180 + 21 + 1200 + 140 &= 1541
\end{align*}
\]

**METHOD B:**

\[
\begin{align*}
67 &\quad 23 \\
21 &= (3 \times 7) \\
180 &= (3 \times 60) \\
140 &= (20 \times 7) \\
1200 &= (20 \times 60) \\
\hline
1541
\end{align*}
\]
METHOD C:

\[
\begin{array}{c}
67 \\
\times 23 \\
\hline
201 \\
1340 \\
\hline
1541
\end{array}
\]

= (3 \times 67) \\
= (20 \times 67)

METHOD D:

\[
\begin{array}{c}
67 \\
\times 23 \\
\hline
201 \\
1340 \\
\hline
1541
\end{array}
\]

In this last method, multiplying the seven ones by 3 gives 21, i.e., (20 + 1) or (2 \times 10) + (1 \times 1), so we write the 1 and carry the 2 to the tens column. Multiplying six tens by 3 gives 18 tens, i.e., 180 plus the two tens carried over gives 20 tens, i.e., 200.

Multiplying the seven ones by 20 gives 140, i.e., 100 + 40 or (1 \times 100) + (4 \times 10), so we write down the 40 and carry the 1 to the hundreds column. Multiplying six tens by 20 gives 1200, i.e., (1000 + 200) plus the one hundred carried over gives 1300. The sum of 201 + 1340 is 1541 which is the product of 67 \times 23.

Subtraction is the INVERSE of addition, i.e., subtraction is the opposite to addition.

For example:-

\[
\begin{align*}
7 + 4 &= 11 \\
11 - 4 &= 7 \\
11 - 7 &= 4
\end{align*}
\]

Subtraction is not commutative, therefore we cannot change the order of the numbers without changing the result.

7 - 4 is NOT the same as 4 - 7.
Subtraction is not associative.
18 - (5 - 3) is not the same as (18 - 5) - 3.
\[18 - (5 - 3) \quad (18 - 5) - 3\]
\[= 18 - 2 \quad = 13 - 3\]
\[= 16 \quad = 10\]

We use the symbol ≠ to show that two numerical expressions are not the same.

\[\therefore 18 - (5 - 3) \neq (18 - 5) - 3\]

Therefore it does matter in which order we place numbers to be subtracted.

In order to identify the numbers used in subtraction we give them special names.

In the indicated difference \(11 - 4\), eleven is the MINUEND and four is the SUBTRAHEND.

OTHER EXAMPLES IN SUBTRACTION

Consider the indicated difference \(47 - 30\).

\[47 = (40 + 7) = 47\]
\[-30 = (30 + 0) = -30\]
\[10 + 7 = 17\]

In this method, the minuend and subtrahend are rewritten in expanded form before the operation of subtraction is performed.

Consider the indicated difference \(125 - 79\).

\[\text{Step 1} \quad \text{Step 2} \quad \text{Step 3}\]
\[125 = (100+20+5) = (100+10+15) = (110+15) = 125\]
\[-79 = (70+9) = (70+9) = (70+9) = -79\]
\[40 + 6 = 46\]

STEP 1 involves rewriting the numerals in expanded form.

STEP 2 borrows from the tens column, because you cannot subtract 9 from 5.
STEP 3 borrows from the hundreds column because you cannot subtract 70 from 10.

As subtraction is the inverse of addition, the difference may be verified by assuming the difference and subtrahend to be addends. The sum of the addends must then equal the minuend.

Look back to the indicated difference 47 - 30. We found the difference to be 17.
The sum of 17 + 30 is 47.
Look back to the indicated difference 125 - 79. We found the difference to be 46.
The sum of 46 + 79 is 125.

Consider the indicated difference 453 - 178.

<table>
<thead>
<tr>
<th>Step 1</th>
<th>Step 2</th>
<th>Step 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>453 = (400+50+3) = (400+40+13) = (300+140+13) = 453</td>
<td>-178 = (100+70+8) = (100+70+8) = (100+70+8) = -178</td>
<td>200+70+5 = 275</td>
</tr>
</tbody>
</table>

STEP 1 involves rewriting the numerals in expanded form.

STEP 2 borrows from the tens column because you cannot subtract 8 from 3.

STEP 3 borrows from the hundreds column because you cannot subtract 70 from 40.

Consider the indicated difference 800 - 173.

\[
\begin{align*}
800 &= (700 + 90 + 10) = 800 \\
-173 &= (100 + 70 + 3) = -173 \\
600 + 20 + 7 &= 627
\end{align*}
\]

In this example, Steps 1 to 3 have been combined.

Consider the indicated difference 1300 - 876.

Combine Steps 1 to 3.

\[
\begin{align*}
1300 &= (1290 + 10) = 1300 \\
-876 &= (870 + 6) = -876 \\
420 + 4 &= 424
\end{align*}
\]

[3.12]
It is not essential to show these steps on paper. The practice of borrowing from the tens and hundreds columns can be done in your head and the problem set out as:

\[
\begin{array}{c}
1300 \\
- 876 \\
\hline
424
\end{array}
\]

Division is the INVERSE of multiplication, i.e., division is the opposite of multiplication.
For example:–

\[
\begin{align*}
2 \times 5 &= 10 \\
10 \div 2 &= 5 \\
10 \div 5 &= 2
\end{align*}
\]

Division is NOT commutative, therefore we cannot change the order of the numbers without changing the result.

\[
10 \div 5 \text{ is not the same as } 5 \div 10
\]

Division is NOT associative.

\[
(32 \div 4) \div 2 \text{ does not equal } 32 \div (4 \div 2)
\]

Therefore it does matter in which order we place numbers to be divided.
In order to identify the numbers used in division we give them special names.
In the indicated quotient \(10 \div 5\), the 10 is the DIVIDEND and the 5 is the DIVISOR.
A dividend is a number which is to be divided.
A divisor is a number which divides into another number.

FURTHER EXAMPLES IN DIVISION

We have looked at several methods of finding the product of any two numbers, and we will look at more than one method of finding the quotient of any two numbers.

Consider the indicated quotient \(684 \div 2\).
684 = 600 + 80 + 4 = 6 hundreds 8 tens and 4 ones
6 hundreds ÷ 2 = 3 hundreds
8 tens ÷ 2 = 4 tens
4 ones ÷ 2 = 2 ones

The quotient is 3 hundreds 4 tens 2 ones = 300 + 40 + 2 = 342.
This shows that at each step in the division the place value of the quotient is the same as the place value of the dividend.
The division is often set out as:-

\[
\begin{array}{c}
342 \\
2 \overline{684}
\end{array}
\]

Consider the indicated quotient 6240 ÷ 3.
6240 = 6000 + 200 + 40 + 0
= 6 thousands 2 hundreds 4 tens 0 ones
6 thousands ÷ 3 = 2 thousands
2 hundreds ÷ 3 = 0 hundreds and 20 tens over
(20 + 4 tens) 24 tens ÷ 3 = 8 tens
0 tens ÷ 3 = 0 ones
6240 ÷ 3 = 2080
This division is often set out as:-

\[
\begin{array}{c}
2080 \\
3 \overline{6240}
\end{array}
\]

Consider the indicated quotient 165 ÷ 5.
165 = 100 + 60 + 5 = 1 hundreds 6 tens 5 ones
1 hundreds ÷ 5 = 0 hundreds and 10 tens over
(10 + 6 tens) 16 tens ÷ 5 = 3 tens and 10 ones over
(10 + 5 ones) 15 ones ÷ 5 = 3 ones

(\text{3.14})
165 ÷ 5 = 33.

This division is usually set out as:

\[
\begin{array}{c}
5 \longdiv{165} \\
15 \\
\underline{15} \\
0
\end{array}
\]

Alternative methods which may seem easier to some people are shown below.

\[
\begin{array}{c}
5 \longdiv{165} \\
50 \\
115 \\
\underline{115} \\
0
\end{array} \quad \begin{array}{c}
5 \longdiv{165} \\
50 \\
65 \\
50 \\
15 \\
\underline{15} \\
0
\end{array}
\]

\[
\begin{array}{c}
5 \longdiv{165} \\
150 \\
15 \\
\underline{15} \\
0
\end{array} \quad \begin{array}{c}
5 \longdiv{165} \\
150 \\
138 \\
\underline{138} \\
0
\end{array}
\]

In these two methods, the quotient is equal to the sum of the numerals in the right-hand column.

Consider the indicated quotient 828 ÷ 23.

\[
\begin{array}{c}
23 \longdiv{828} \\
69 \\
138 \\
\underline{138} \\
0
\end{array}
\]

\[
3.15
\]
STEP 1:  8 hundreds \div 23 = 0 hundreds and 8 tens over.

STEP 2:  \((80 + 2 \text{ tens}) 82 \div 23 = 3 \text{ tens and}
\quad 130 \text{ ones over.}\n
STEP 3:  \((130 + 8 \text{ ones}) 138 \div 23 = 6 \text{ ones.}\n
828 \div 23 = 30 + 6 = 36

Some may prefer to reason this way:

23 will not go into 8;
23 goes into 82 three times with 13 over;
Bring down the 8 ones to make 138;
23 goes into 138 six times.

We will try this problem using the other two methods shown.

\[
\begin{array}{c|c|c}
\hline
828 & 23 & 828 \\
690 & 30 & \\
138 & 6 & 0 \\
6 & & 36 \\
\hline
\end{array}
\]

When we were considering the indicated difference between two numbers we found we could check the accuracy of our calculations by assuming the difference and subtrahend were addends. The sum of the addends was equal to the minuend.

It is possible to check our calculation of the indicated quotient by finding the product of the quotient and the divisor. If our calculations were correct, the

\[
\text{Product} = \text{Quotient} \times \text{Divisor} = 3 \times 23 = 69
\]

\[
\text{Sum of the addends} = 828 \div 23 = 30 + 6 = 36
\]

\[
\text{Product} - \text{Sum of the addends} = 69 - 36 = 33
\]
product of the quotient and the divisor is the same as the dividend.

For example:

\[ \frac{828}{23} = 36 \]
\[ 36 \times 23 = 828 \]

When using the operations of addition, subtraction, multiplication and division it is possible to work with only two numbers at a time. Any operation which combines only two numbers is called a BINARY OPERATION.

**FINDING THE H.C.F. OR G.C.D. BY LONG DIVISION**

The following method may be used to find the greatest common factor of two numbers when this is not immediately apparent.

**METHOD:**

1. Divide the larger number by the smaller.

2. Divide the divisor, i.e., the smaller number by the remainder and continue this process until there is no remainder.

The last divisor will be the greatest common factor or greatest common divisor.

**Find the H.C.F. of 120 and 168.**

\[
\begin{array}{c|c}
120 & 168 \\
\hline
120 & \\
\hline
48 & 2 \\
\hline
48 & 120 \\
\hline
24 & 96 \\
\hline
24 & 24 \\
\hline
24 & 48 \\
\hline
48 & \\
\hline
0 & \\
\end{array}
\]

The H.C.F. is 24.

Check this with the answer gained on page 2.11 of Chapter 2 by using prime factors.

**Find the H.C.F. of 108 and 744.**
The H.C.F. is 12.

Find the H.C.F. of 9230 and 639.

The H.C.F. is 71.
Chapter 4

FRACTIONS AND DECIMALS

So far, all our examples have dealt with whole numbers. In Chapter 1 we introduced two concepts which now need to be examined in greater detail.

Many of our references to numbers in daily conversation do not refer to whole numbers. For example, we speak about three-quarters of a minute; a quarter of an hour; half a day; half a loaf of bread. We hear someone say, "Please give me a third of that piece" or, "Joe would be about two-thirds of my weight".

Here we see the use of rational numbers. The word "rational" comes from "ratio", which means much the same as "quotient". Rational numbers might have been called "ratio numbers" or "quotient numbers". Remember, a rational number is a number which can be obtained by dividing a whole number by a counting number.

Rational numbers may be represented by fractions. A fraction is a numeral which can represent a rational number. The fractions are the marks on the page, the rational numbers are the things we work with.

For example:

\[
\frac{1}{4} \text{ is the fraction which represents the rational number one-quarter.}
\]

\[
\frac{1}{2} \text{ is the fraction which represents the rational number one-half.}
\]

\[
\frac{2}{3} \text{ is the fraction which represents the rational number two-thirds.}
\]
The word "fraction" comes from a Latin word which means to break into parts. We can use fractions to describe the size of a part by comparing it to the whole quantity.

In the above diagrams, the whole figures have been divided into four equal parts. Of these four parts, three parts have been shaded, so that the shaded area equals three-quarters of the whole and is written as the fraction $\frac{3}{4}$. The unshaded area equals one-quarter of the whole and is written as the fraction $\frac{1}{4}$.

In the fraction $\frac{3}{4}$, the 4 is the DENOMINATOR and indicates the number of parts into which the whole has been divided.

In the fraction $\frac{3}{4}$, the 3 is the NUMERATOR and indicates how many of the equal parts we are considering.

$\frac{3}{4}$ may be described as a PROPER FRACTION, because the numerator is less than the denominator.

We may also have a MIXED NUMERAL which consists of a whole number and a fraction, e.g., $1\frac{3}{4}$.

It is also possible to have an IMPROPER FRACTION where the numerator is greater than the denominator, e.g., $\frac{3}{2}$; $\frac{6}{2}$. Any improper fraction may be converted to a mixed numeral, e.g., $\frac{3}{2} = 1\frac{1}{2}$, or a whole number, e.g., $\frac{6}{2} = 3$.

To change an improper fraction to a mixed numeral or a whole number, you divide the numerator by the denominator.
EQUIVALENT FRACTIONS

Examine the following diagrams. Note that we have four squares of equal size. In (a) we have a line dividing the square into two halves. In (b) the square has been divided into quarters; (c) has been divided into eighths and (d) into sixteenths.

Examine the shaded portions and you will see that in each square half of the area has been shaded. But notice that in (a) the shaded portion represents one-half while in (b) the shaded portion represents two-quarters. In (c) the shaded portion represents four-eighths and in (d) it represents eight sixteenths. We can, therefore say that $\frac{1}{2}$ is the same as $\frac{2}{4}$, $\frac{4}{8}$ or $\frac{8}{16}$.

Using the same diagrams we can also see that $\frac{1}{4}$, $\frac{2}{8}$ and $\frac{4}{16}$ are the same.

These are all examples of equivalent fractions. Equivalent fractions, although expressed in different values are nevertheless equal.

From this we can determine an important principle. The VALUE of a fraction remains unaltered when the numerator and denominator are multiplied or divided by the same number.

Sometimes we meet fractions with such large denominators that we find interpretation difficult. Frequently such fractions may be reduced to a more manageable size. Take, for example, the fraction $\frac{50}{1000}$. If we divide the numerator and denominator by two, the fraction is reduced to $\frac{25}{500}$. We can now divide both the numerator and denominator by five, reducing
the fraction to \( \frac{5}{10} \). Further division by five results in a fraction of \( \frac{1}{20} \) which is more meaningful than \( \frac{50}{1000} \).

This process of reducing a fraction to its lowest terms is done by "cancellation" or "canceling out". It is frequently presented as follows:

\[
\begin{array}{c}
1 \\
25 \\
50 \\
1000 \\
\hline
560 \\
100 \\
\hline
20
\end{array}
\]

The progressive divisions are done mentally and the results of each division are entered to the right and slightly above or below the previous quotient. (A quotient is the result of the division.)

Notice that at each stage, the numerator and denominator are both divided by the same number. To avoid confusion the succeeding quotients must be entered neatly and clearly.

This method of reducing a fraction to its lowest terms works well when the prime factors are easily recognised. An alternative method may be more suitable when the prime factors are not so readily recognised. In the alternative method we find the greatest common factor or greatest common divisor of the numerator and denominator of the fraction, using the procedure shown on page 3.17 of Chapter 3.

We will demonstrate this method firstly by using the previous example.

Reduce \( \frac{50}{1000} \) to its lowest terms.

\[
\begin{array}{c}
20 \\
50 \\
1000 \\
1000 \\
\hline
0
\end{array}
\]

H.C.F. = 50

By dividing the numerator and denominator by 50, the fraction is reduced to its lowest terms \( \frac{1}{20} \).
Reduce $\frac{625}{1000}$ to its lowest terms by finding the H.C.F. of the numerator and denominator.

\[
\begin{array}{ccc}
625 & 1 \\
375 & 250 & 1 \\
125 & 250 & 2 \\
0 & & \\
\end{array}
\]

H.C.F. = 125

By dividing the numerator and denominator by 125 the fraction is reduced to its lowest terms: $\frac{5}{8}$.

Reduce $\frac{472}{708}$ to its lowest terms by finding the H.C.F. of the numerator and denominator.

\[
\begin{array}{ccc}
472 & 1 \\
236 & 2 \\
472 & 0 \\
\end{array}
\]

H.C.F. = 236.

By dividing the numerator and denominator by 236 the fraction is reduced to $\frac{2}{3}$.

Cancellation of fractions appears to be a more common procedure than expansion of fractions. To expand a fraction both the numerator and the denominator must be multiplied by the same number.

We can expand fractions by applying some principles we have already explored.

We know that $\frac{1}{2} = \frac{1}{2} \times 1$ (see Multiplication Property of One)

\[
\begin{align*}
&= \frac{1}{2} \times \frac{2}{2} \\
&= \frac{2}{4}
\end{align*}
\]
Also \( \frac{1}{2} = \frac{1}{2} \times 1 \) (M.P. One)
\[ = \frac{1}{2} \times \frac{5}{5} \quad (\frac{2}{5} \text{ is a numeral for number one}) \]
\[ = \frac{5}{10} \]

Equivalent fractions all represent the same rational number. Refer back to those four equal squares on page 45. Examine the various squares and notice that:

\[ \frac{1}{4} \text{ is less than } \frac{1}{2} \]
\[ \frac{1}{4} \text{ is greater than } \frac{1}{8} \]
\[ \frac{1}{8} \text{ is greater than } \frac{1}{16} \]

This should clear up a common misconception. Many people look at two fractions such as \( \frac{1}{4} \) and \( \frac{1}{8} \) and say "as eight is greater than four, one-eighth must be greater than one-quarter". This is not so!

The denominator indicates the number of equal parts into which a whole number has been divided. The greater the number of parts which have to be obtained from the whole, the smaller will be each section (fraction).

This can be verified by expressing the two fractions with a suitable common denominator. Any common denominator would do, but it is usual to choose the least common denominator.

If we take \( \frac{1}{8} \) and \( \frac{1}{16} \), both fractions can be expressed with the common denominator of sixteen because \( \frac{1}{8} = \frac{2}{16} \).

It is now easy to see that \( \frac{1}{8} \) or \( \frac{2}{16} \) is greater than \( \frac{1}{16} \).

**USES OF EQUIVALENT FRACTIONS**

When it is necessary to add, subtract or determine the larger of two or more fractions, these are best considered as equivalent fractions.
EXAMPLES

Identify the larger of the following fractions:

\[ \frac{2}{8} \text{ and } \frac{1}{6} \]

METHOD:

1. Find the least common denominator of the two denominators.
   
   L.C.D. of 8 and 6 is 24.

2. Express both fractions with the common denominator of 24.
   
   \[ \frac{2}{8} = \frac{2}{8} \times \frac{3}{3} = \frac{6}{24} \]
   
   \[ \frac{1}{6} = \frac{1}{6} \times \frac{4}{4} = \frac{4}{24} \]

\( \frac{2}{8} \) is greater than \( \frac{1}{6} \) by \( \frac{2}{24} \) or \( \frac{1}{12} \).

Identify the greatest of the following fractions:

\[ \frac{1}{2}, \frac{5}{6}, \frac{2}{3}, \text{ and } \frac{2}{5} \]

METHOD:

1. Find the L.C.D.

   L.C.D. of 2, 6, 3 and 5 is 30.

2. Express each fraction with the common denominator of 30.

   \[ \frac{1}{2} = \frac{1}{2} \times \frac{15}{15} = \frac{15}{30} \]

   \[ \frac{5}{6} = \frac{5}{6} \times \frac{5}{5} = \frac{25}{30} \]
\[
\frac{2}{3} = \frac{2}{3} \times \frac{10}{10} = \frac{20}{30}
\]
\[
\frac{2}{5} = \frac{2}{5} \times \frac{6}{6} = \frac{12}{30}
\]

\[
\frac{5}{6} \text{ or } \frac{25}{30} \text{ is the greatest of the four fractions.}
\]

**Addition of Fractions**

To find the indicated sum of two or more fractions, first express them as equivalent fractions with a common denominator.

**Example**

Find the indicated sum of \(\frac{2}{8}\) and \(\frac{1}{6}\)

**Method:**

Express \(\frac{2}{8}\) and \(\frac{1}{6}\) as equivalent fractions with the common denominator of 24.

\[
\frac{2}{8} + \frac{1}{6} = \frac{6}{24} + \frac{4}{24}
\]

\[
= \frac{10}{24} \text{ or } \frac{5}{12}
\]

Find the indicated sum of \(\frac{1}{2}\), \(\frac{5}{6}\), \(\frac{2}{3}\), and \(\frac{2}{5}\)

\[
\frac{1}{2} + \frac{5}{6} + \frac{2}{3} + \frac{2}{5} = \frac{15}{30} + \frac{25}{30} + \frac{20}{30} + \frac{12}{30}
\]

\[
= \frac{72}{30}
\]

\[
= \frac{24}{10} \text{ or } \frac{2}{5}
\]
SUBTRACTION OF FRACTIONS

To find the indicated difference between two fractions, first express them as equivalent fractions with a common denominator.

EXAMPLE

Find the indicated difference between $\frac{2}{8}$ and $\frac{1}{6}$

\[
\frac{2}{8} - \frac{1}{6} = \frac{6}{24} - \frac{4}{24}
\]
\[
= \frac{2}{24} \text{ or } \frac{1}{12}
\]

When whole numbers are involved it is usually easier if the mixed numeral is expressed as an improper fraction.

EXAMPLE

Find the indicated difference between $3\frac{1}{5}$ and $1\frac{2}{15}$

\[
3\frac{1}{5} - 1\frac{2}{15} = \frac{16}{5} - \frac{17}{15}
\]
\[
= \frac{48}{15} - \frac{17}{15}
\]
\[
= \frac{31}{15}
\]
\[
= 2\frac{1}{15}
\]

L.C.D. = 15

\[
\therefore \frac{16}{5} = \frac{16}{5} \times \frac{3}{3} = \frac{48}{15}
\]

4.9
RECIPROCALS

Consider the following numerical expressions:

\[ 5 \times \frac{1}{5} ; \quad \frac{7}{3} \times \frac{3}{7} ; \quad \frac{8}{19} \times \frac{19}{8} \quad \text{or} \quad \frac{8}{19} \times \frac{23}{8}. \]

These all simplify to 1, and so we say that each number is the RECIPROCAL of the other.

That is:

- 5 is the reciprocal of \( \frac{1}{5} \) (because \( 5 \times \frac{1}{5} = 1 \))
- \( \frac{1}{5} \) is the reciprocal of 5 (because \( \frac{1}{5} \times 5 = 1 \))
- \( \frac{7}{3} \) is the reciprocal of \( \frac{3}{7} \) (because \( \frac{7}{3} \times \frac{3}{7} = 1 \))
- \( \frac{3}{7} \) is the reciprocal of \( \frac{7}{3} \) (because \( \frac{3}{7} \times \frac{7}{3} = 1 \))
- \( \frac{8}{19} \) is the reciprocal of \( \frac{19}{8} \) (because \( \frac{8}{19} \times \frac{19}{8} = 1 \))
- \( \frac{23}{8} \) or \( \frac{19}{8} \) is the reciprocal of \( \frac{8}{19} \) because \( \frac{19}{8} \times \frac{8}{19} = 1 \)

If we wish to divide by a rational number, we need only multiply by its reciprocal (i.e., the reciprocal of the divisor). For example: to divide by 3, you multiply by \( \frac{1}{3} \), because \( \frac{1}{3} \) is the reciprocal of 3.

While we use some fractions almost without thinking, many fractions are unwieldly and so if there is a more meaningful way of saying the same thing without using fractions it seems sense to use it.

In Chapter 1 we looked at our decimal system of numeration which we saw had both face value and place value.
<table>
<thead>
<tr>
<th>Place</th>
<th>5th Place</th>
<th>4th Place</th>
<th>3rd Place</th>
<th>2nd Place</th>
<th>1st Place</th>
</tr>
</thead>
<tbody>
<tr>
<td>ten thousand</td>
<td>one thousand</td>
<td>one hundred</td>
<td>ten</td>
<td>one</td>
<td>Place values in words.</td>
</tr>
<tr>
<td>10 000</td>
<td>1000</td>
<td>100</td>
<td>10</td>
<td>1</td>
<td>Place values in ordinary numerals.</td>
</tr>
<tr>
<td>$10^4$</td>
<td>$10^3$</td>
<td>$10^2$</td>
<td>$10^1$</td>
<td>$10^0$</td>
<td>Place values in powers of ten.</td>
</tr>
</tbody>
</table>

In the above table you will notice that each place has a place value which is ten times the place value of the place on its immediate right. In order to represent larger numbers we simply use more places to the left.

The table also shows that each place has a place value which is one-tenth of the place value of the place on its immediate left. This suggests we may be able to represent smaller numbers by using more places to the right.

In the place on the immediate right of the first place we would expect to have a place value of $\frac{1}{10}$ (i.e., $\frac{1}{10}$ of 1). This is often shown $10^{-1}$. The place to the right of this would have a place value of $\frac{1}{100}$ (i.e., $\frac{1}{10}$ of $\frac{1}{10}$). This is often shown as $10^{-2}$.

We call the places to the right of the ones place, DECIMAL PLACES. This can be shown in a table. (See top of page 4.12.)

When we write a numeral consisting of a series of digits we need something to show us which digit occupies the first place. For this purpose we use a dot (known as the DECIMAL POINT) placed between the lines and to the right of the digit in the first place, e.g., 5.5.

For ease of typing, the decimal points in these notes are often shown on the line. When writing decimal numerals please place the decimal point above the line.

The place value of the left-hand 5 is five ones.
The place value of the right-hand 5 is five tenths.

In Chapter 1 we found it helpful to take numerals apart, i.e., to expand them. We will now expand a numeral which contains decimal places.

4.11
<table>
<thead>
<tr>
<th>Place values in words</th>
<th>3rd Place</th>
<th>2nd Place</th>
<th>1st Place</th>
<th>1st Decimal Place</th>
<th>2nd Decimal Place</th>
<th>3rd Decimal Place</th>
</tr>
</thead>
<tbody>
<tr>
<td>one hundred</td>
<td></td>
<td></td>
<td></td>
<td>one tenth</td>
<td>one hundredth</td>
<td>one thousandth</td>
</tr>
<tr>
<td>ten</td>
<td>100</td>
<td>10</td>
<td>1</td>
<td>$\frac{1}{10}$</td>
<td>$\frac{1}{100}$</td>
<td>$\frac{1}{1000}$</td>
</tr>
<tr>
<td>one</td>
<td>$10^2$</td>
<td>$10^1$</td>
<td>$10^0$</td>
<td>$10^{-1}$</td>
<td>$10^{-2}$</td>
<td>$10^{-3}$</td>
</tr>
</tbody>
</table>

729.3564

$$= (7 \times 100) + (2 \times 10) + (9 \times 1) + (3 \times \frac{1}{10}) + (5 \times \frac{1}{100}) + (6 \times \frac{1}{1000}) + (4 \times \frac{1}{10000})$$

$$= (7 \times 10^2) + (2 \times 10^1) + (9 \times 10^0) + (3 \times 10^{-1}) + (5 \times 10^{-2}) + (6 \times 10^{-3}) + (4 \times 10^{-4})$$

$$= 700 + 20 + 9 + \frac{3}{10} + \frac{5}{100} + \frac{6}{1000} + \frac{4}{10000}$$

The numeral 729.3564 is read as "seven hundred and twenty nine point three, five, six, four".

In 729.3564:-

the 3 is in the first decimal place and represents three tenths;

the 5 is in the second decimal place and represents five hundredths;

the 6 is in the third decimal place and represents six thousandths;
the 4 is in the fourth decimal place and represents four ten thousandths.

Numerals in which a decimal point is shown will from now on be referred to as 'decimals' to distinguish them from numerals for whole numbers. Whenever we write the decimal equivalent of a numeral of less than one we always place a zero in front of the decimal point, so that the presence of the decimal point is emphasised.

For example:-

0.5; 0.125; 0.075

MULTIPLYING BY POWERS OF TEN

We know that \(10 \times 5 = 50\)

\[10 \times 9 = 90\]

\[10 \times 17 = 170\]

These examples show that when we multiply a whole number by ten, the product shows a single zero to the right of the numeral for that number. They also show that the place value of the digit (or digits) representing that whole number has changed, or increased, by one place.

We also know that \(100 \times 5 = 500\)

\[100 \times 9 = 900\]

\[100 \times 17 = 1700\]

These examples show that when we multiply a whole number by one hundred \((10^2)\) the product shows two zeros on the right of the numeral for that number. This means the place value of the digit (or digits) representing that whole number has changed, or increased, by two places.

We also know that \(1000 \times 5 = 5000\)

\[1000 \times 9 = 9000\]

\[1000 \times 17 = 17000\]
These examples show that when we multiply a whole number by one thousand \((10^3)\) the product shows three zeros on the right of the numeral for that number. The place value of each digit (or digits) has changed, or increased, by three places.

Therefore, we can say - if we multiply any whole number by any power of ten, a number of zeros corresponding to the power of ten are added to the right of the numeral for the number.

Expressing this in a different way gives - when multiplying a whole number by a power of ten, the place value of each digit changes (increases) by the same number of places as the power of ten by which the number is multiplied.

The same rule follows if we multiply a rational number by a power of ten.

For example:

\[
\frac{1}{100} \times \frac{1}{3} = \frac{1}{300} \quad \text{or} \quad 10^{-2} \times \frac{1}{3} = \frac{1}{300}
\]

\[
\frac{1}{10} \times \frac{1}{1000} = \frac{1}{10000} \quad \text{or} \quad 10^{-1} \times 10^{-3} = \frac{1}{10000} \quad \text{or} \quad 10^{-4}
\]

\[
\frac{1}{100} \times \frac{1}{100000} = \frac{1}{10000000} \quad \text{or} \quad 10^{-2} \times 10^{-5} = 10^{-7}
\]

If multiplying by a power of ten involves a change in the place value of each digit, let us look at what happens when we multiply a decimal by a power of ten.

\[
1.5 \times 10 = 15
\]

\[
37.2 \times 10 = 372
\]

\[
1.25 \times 10 = 12.5
\]

\[
5.875 \times 10 = 58.75
\]

In each of the above examples, the place value of each digit has increased by ten (i.e., one place) and the decimal point appears to have moved one place towards the right of the numeral.
1.5 \times 100 = 150
37.2 \times 100 = 3720
12.5 \times 100 = 125
5.875 \times 100 = 587.5

Because one hundred is the second power of ten, we find the place value of each digit in the above examples has increased by two places, and the decimal point appears to have moved two places towards the right of the numeral.

1.5 \times 1000 = 1500
37.2 \times 1000 = 37200
1.25 \times 1000 = 1250
5.875 \times 1000 = 5875

Because one thousand is the third power of ten, we find the place value of each digit in the above examples has increased by three places, and the decimal point appears to have moved three places to the right of the numeral.

DIVIDING BY POWERS OF TEN

We know that 80 \div 10 = 8
200 \div 10 = 20
350 \div 10 = 35

These examples show that when we divide a multiple of ten by ten, we remove one zero from the right of the numeral for that multiple of ten. Again there is a change (a reduction) in the place value of each digit by one place.

800 \div 100 = 8
2000 \div 100 = 20
3500 \div 100 = 35

4.15
The above examples show that when we divide a multiple of ten containing more than one zero by one hundred \((10^2)\) we remove two zeros from the right of the numeral for that multiple. This means there is a change in the place value of each digit by two places.

We also know that:

\[
\begin{align*}
8000 \div 1000 &= 8 \\
20000 \div 1000 &= 20 \\
350000 \div 1000 &= 350
\end{align*}
\]

These examples show that when we divide any multiple of ten containing three or more zeros by one thousand \((10^3)\) we remove three zeros from the right of the numeral for that multiple. In other words, the place value of each digit decreases by three places.

Let us now look what happens when we divide decimals by a power of ten:

\[
\begin{align*}
1.5 \div 10 &= 0.15 \\
37.2 \div 10 &= 3.72 \\
1.25 \div 10 &= 0.125 \\
5.875 \div 10 &= 0.5875
\end{align*}
\]

In these examples, the place value of each digit has decreased by one place, and the decimal point appears to have moved one place towards the left of the numeral.

\[
\begin{align*}
1.5 \div 100 &= 0.015 \\
37.2 \div 100 &= 0.372 \\
1.25 \div 100 &= 0.0125 \\
5.875 \div 100 &= 0.05875
\end{align*}
\]

In these examples, the place value of each digit has decreased by two places, and the decimal point appears to have moved two places towards the left of the numeral. Notice that in three examples it was necessary to
insert a zero into the tenths place in order to obtain the correct placement of the other digits.

We can now see that to divide by one thousand, the place value of each digit will decrease by three places and the decimal point will appear to move three places to the left of the numeral.

CHANGING FRACTIONS AND MIXED NUMERALS TO DECIMALS

If the fraction has a denominator which is a power of ten, the fraction can easily be converted to a decimal.

For example:

\[
\frac{27}{100} = 27 \div 10^2 = 0.27
\]

This may be shown in the following steps:

\[
\frac{27}{100} = \frac{20 + 7}{100}
\]

\[
= \frac{20}{100} + \frac{7}{100}
\]

\[
= \frac{2}{10} + \frac{7}{100}
\]

\[
= 0.27
\]

Now consider the fraction \( \frac{379}{10,000} \)

\[
\frac{379}{10,000} = 379 \div 10^4 = 0.0379
\]

Notice that in order to place the nine in the fourth decimal place it was necessary to place a zero in the first decimal place.

When a fraction does not have a denominator of a power of ten, see whether it has an equivalent fraction with such a denominator.
Consider the fraction \( \frac{3}{5} \).

\[
\frac{3}{5} = \frac{3}{5} \times 1 \text{ (M.P. One)}
\]
\[
= \frac{3}{5} \times \frac{2}{2} \text{ (} \frac{2}{2} \text{ is a numeral for one)}
\]
\[
= \frac{6}{10}
\]
\[
\therefore \frac{3}{5} = 0.6
\]

Consider the fraction \( \frac{3}{8} \).

\[
\frac{3}{8} = \frac{3}{8} \times 1 \text{ (M.P. One)}
\]
\[
= \frac{3}{8} \times \frac{125}{125} \text{ (} \frac{125}{125} \text{ is a numeral for one)}
\]
\[
= \frac{375}{1000}
\]
\[
\therefore \frac{3}{8} = 0.375
\]

An alternative method is to divide the numerator by the denominator.

For example:

\[
\frac{3}{8} = 8 \div 3.000
\]
\[
\begin{array}{c|c}
2 & 4 \\
60 & 56 \\
40 & 40 \\
0 & 0
\end{array}
\]

Any number of zeros may be placed to the right of the decimal point.

The stages in which this division is performed are important to ensure that digits in the quotient appear in the correct places.

We will set it out again using an arrow and circled number to show the order of working.
\[
\frac{3}{8} = \overline{8.3\,0\,0\,0}
\]

STEP 1: 8 will not go into 3, so a zero is placed in the quotient.

STEP 2: The decimal point is placed in the quotient immediately above the decimal point in the dividend.

STEP 3: 8 goes into 30 tenths, three times with 60 hundredths over.

STEP 4: 8 goes into 60 hundredths seven times with 40 thousandths over.

STEP 5: 8 goes into 40 thousandths five times.

When you wish to convert a mixed numeral to decimals, it is only the part representing the fraction which is expressed in a different way.

For example:

\[
1\frac{3}{5} = 1 + \frac{3}{5}
\]

\[
\frac{3}{5} = \frac{6}{10} = 0.6
\]

\[
= 1 + 0.6
\]

\[
= 1.6
\]

If \(1\frac{3}{5}\) was expressed as the improper fraction \(\frac{8}{5}\), it could be expressed immediately as a mixed numeral or left as the improper fraction, in which case the procedure would be to divide the numerator by the denominator.

\[
\frac{8}{5} = 5 \overline{1.6}
\]

\[
\frac{5}{5}
\]

\[
\frac{30}{30}
\]

\[
\frac{0}{0}
\]

\[
4.19
\]
ADDITION OF DECIMALS

Consider the indicated sum 0.274 + 1.325.
One way to simplify this is to expand each decimal.

\[
0.274 + 1.325 = \left( \frac{2}{10} + \frac{7}{100} + \frac{4}{1000} \right) + \left( 1 + \frac{3}{10} + \frac{2}{100} + \frac{5}{1000} \right)
\]

\[
= 1 + \left( \frac{2}{10} + \frac{3}{10} \right) + \left( \frac{7}{100} + \frac{2}{100} \right) + \left( \frac{4}{1000} + \frac{5}{1000} \right)
\]

\[
= 1 + \frac{5}{10} + \frac{9}{100} + \frac{9}{1000}
\]

\[
= 1.599
\]

This could also be set out as:

\[
0.274 = 0 + \frac{2}{10} + \frac{7}{100} + \frac{4}{1000}
\]

\[
+ 1.325 = 1 + \frac{3}{10} + \frac{2}{100} + \frac{5}{1000}
\]

\[
= 1 + \frac{5}{10} + \frac{9}{100} + \frac{9}{1000}
\]

\[
= 1.599
\]

Both methods show that tenths are added to tenths; hundredths to hundredths and thousandths to thousandths.

When we found the sum of various whole numbers we made sure that digits with the same place values were placed beneath each other. The same procedure works when finding the sum of various decimals. Provided the decimal points are kept in line, the other digits will automatically be in their correct position.

For example:

\[
0.274
\]

\[
+ 1.325
\]

\[
= 1.599
\]

Consider the indicated sum 37.39 + 9.984 + 0.069.
Write the numerals in columns keeping the decimal points one under the other.
37.39
9.984
+ 0.069

When you come to add the thousandths column, there are no thousandths in the numeral 37.39. For clarity this may be expressed as 37.390 and the indicated sum set out as:

\[
\begin{array}{c}
37.390 \\
9.984 \\
+ 0.069 \\
\hline
47.443
\end{array}
\]

**SUBTRACTION OF DECIMALS**

When we subtracted whole numbers we made sure that digits with the same place value were placed beneath each other. Subtraction of decimals is just as easy if we ensure that all digits with the same place value are written beneath each other, so that thousandths are subtracted from thousandths; hundredths are subtracted from hundredths; tenths subtracted from tenths and so on.

Consider the indicated difference 9.895 - 7.563.

\[
9.895 = 9 + \frac{8}{10} + \frac{9}{100} + \frac{5}{1000}
\]

\[
-7.563 = 7 + \frac{5}{10} + \frac{6}{100} + \frac{3}{1000}
\]

\[
= (9-7) + \frac{8-5}{10} + \frac{9-6}{100} + \frac{5-3}{1000}
\]

\[
= 2 + \frac{3}{10} + \frac{3}{100} + \frac{2}{1000}
\]

\[
= 2.332
\]

Correct placement of all digits will occur if the decimal points are kept one under the other.

For example:

\[
\begin{array}{c}
9.895 \\
- 7.563 \\
\hline
2.332
\end{array}
\]

This result can be checked by finding the sum of 2.332 + 7.563.
Now consider the indicated difference 2.3 - 1.956.

The initial step should be to express 2.3 as 2.300 seeing as the subtrahend is a three decimal numeral.

Simplifying this by expanding each numeral leads to quite a lengthy series of steps, but it is well worth following these through.

STEP 1

\[ 2.300 = 2 + \frac{3}{10} + \frac{0}{100} + \frac{0}{1000} = 1 + \frac{13}{10} + \frac{0}{100} + \frac{0}{1000} \]

\[ -1.956 = 1 + \frac{9}{10} + \frac{5}{100} + \frac{6}{1000} = 1 + \frac{9}{10} + \frac{5}{100} + \frac{6}{1000} \]

STEP 2

\[ = 1 + \frac{12}{10} + \frac{10}{100} + \frac{0}{1000} = 1 + \frac{12}{10} + \frac{9}{100} + \frac{10}{1000} \]

\[ = 1 + \frac{9}{10} + \frac{5}{100} + \frac{6}{1000} \]

\[ = \frac{1}{10} + \frac{5}{100} + \frac{6}{1000} \]

\[ = (1-1)+\left(\frac{12}{10} - \frac{9}{10}\right)+\left(\frac{9}{100} - \frac{5}{100}\right)+\left(\frac{10}{1000} - \frac{6}{1000}\right) \]

STEPS 6 - 9

\[ = 0 + \frac{3}{10} + \frac{4}{100} + \frac{4}{1000} \]

\[ = 0.344 \]

STEP 1: As the subtrahend is a three decimal numeral, express the minuend as a three decimal numeral.

STEP 2: Expand both numerals, noting that you cannot subtract \( \frac{6}{1000} \) from \( \frac{0}{1000} \) nor \( \frac{5}{100} \) from \( \frac{0}{100} \).

STEP 3: Borrow ten tenths from the ones column so that you have 10 + 3 tens, i.e., \( \frac{13}{10} \).

STEP 4: Borrow ten hundredths from the tenths column.
STEP 5: Borrow ten thousandths from the hundredths column.

STEP 6: Subtract \( \frac{6}{1000} \) from \( \frac{10}{1000} \).

STEP 7: Subtract \( \frac{5}{100} \) from \( \frac{9}{100} \).

STEP 8: Subtract \( \frac{9}{10} \) from \( \frac{12}{10} \).

STEP 9: Subtract 1 from 1.

Most people would do the borrowing mentally and set this out as:

\[
\begin{align*}
2.300 \\
-1.956 \\
\hline
0.344
\end{align*}
\]

The result is checked by finding the sum of 0.344 and 1.956.

MULTIPLICATION OF DECIMALS

When we want to find the product of decimals we may use any of the methods of multiplication shown in Chapter 3. That is, we commence the multiplication as though we were finding the product of whole numbers. When the product is found, it is then adjusted to show the appropriate number of decimal places.

Consider the indicated product \( 0.7 \times 1.2 \).

\[
\begin{align*}
0.7 & \times 1.2 = 12 \\
x \frac{7}{0.7} & \times \frac{12}{7} \\
84 & = 0.84
\end{align*}
\]

When multiplying decimals, the number of decimal places in the product is the sum of the number of decimal places in the decimals of the indicated product.

In the example shown above there are a total of two decimal places in the indicated product and so there are two decimal places in the product.

\( 1.2 \times 0.7 \) may also be simplified by the following method:-
\[ 1.2 \times 0.7 = \frac{12}{10} \times \frac{7}{10} = \frac{84}{100} = 0.84 \]

Consider the indicated product 7 \times 1.35.
To simplify 7 \times 1.35 we must first multiply 135 by 7 to obtain 945. Then we must write 945 as a two decimal numeral - i.e., a numeral with two decimal places - 9.45, so:
\[ 7 \times 1.35 = 7 \times 1.35 = \frac{135}{7} = \frac{945}{7} = 9.45 \]

Consider the indicated product 0.3 \times 1.312.
To simplify 0.3 \times 1.312 we first multiply 1312 by 3 to obtain the product 3936. As there is a total of four decimal places in the indicated product we must write 3936 as a numeral with four decimal places, i.e., 0.3936, so:
\[ 0.3 \times 1.312 = 0.3 \times 1.312 = \frac{1312}{3} = \frac{3936}{3} = 0.3936 \]

Consider the indicated product 1.72 \times 2.3.
\[ 1.72 \times 2.3 = 172 \times 23 = 516 \]
\[ 3440 \]
\[ 3956 = 3.956 \]

STEP 1: Find the product of 172 and 23.

STEP 2: Adjust the product so that it becomes a numeral with three decimal places (because 1.72 contains two decimal places plus one decimal place in 2.3).
MULTIPLICATION BY MULTIPLES OF TEN

Earlier we looked at examples involving multiplying whole numbers by powers of ten and in multiplying decimals by powers of ten.

Consider the indicated product $1.739 \times 300$.
This may be considered as:-

$$(1.739 \times 100) \times 3$$

$$= 173.9 \times 3$$

$$= 521.7$$

STEP 1: Expand 300 to $(3 \times 100)$.

STEP 2: Rearrange the problem so that you can multiply 1.739 by a power of ten $(10^2)$, thus adjusting the place value of each digit by two places.

STEP 3: Multiply the intermediate product 173.9 by 3 to find the final product of 521.7.

Consider the indicated product $0.315 \times 40$.
This can be re-expressed as:-

$$(0.315 \times 10) \times 4$$

$$= 3.15 \times 4$$

$$= 12.60$$

$$= 12.6$$

DIVISION BY DECIMALS

The method used for division of decimals is similar to that shown in Chapter 3 for division of whole numbers and that shown earlier in this chapter where fractions were converted to decimals.

Consider the indicated quotient $6.248 \div 2$. 

4.25
3.124

(The numerals in circles show the order in which the division is done.)

2 / 6.248

1. 2 into 6 is 3.
2. The decimal point comes after the 6, so we place the decimal point after the 3 in the quotient. (This shows that the digits 6 and 3 both have a place value of one.)
3. 2 into 2 is 1.
4. 2 into 4 is 2.
5. 2 into 8 is 4.

Consider the indicated quotient 0.0212 ÷ 4.

\[
\begin{array}{c}
0.0053 \\
4 \overline{)0.0212}
\end{array}
\]

Note the order in which the division proceeds.
1. 4 into 0 is 0.
2. Put the decimal point into the quotient.
3. 4 into 0 is 0.
4. 4 into 2 is 0 and 2 over.
5. 4 into 21 is 5 and 1 over.
6. 4 into 12 is 3.
There are several ways of solving this type of problem. The method presented here is preferred. However, if you are familiar and competent with another method, keep to it.

**Consider the indicated quotient 6.51 ÷ 0.7.**

The division must not be attempted while the divisor remains a decimal. (In this case the divisor is 0.7). It must be changed to a whole number. We know that multiplying by ten will convert 0.7 to the whole number 7.

To maintain the correct proportions, whatever we do to the divisor we must also do to the dividend.

So, if we multiply 0.7 by ten we must also multiply 6.51 by ten.

\[
6.51 \div 0.7 = (6.51 \times 10) \div (0.7 \times 10) = \frac{65.1}{7}
\]

The problem now becomes \( \frac{65.1}{7} \) and you can proceed just as you did for the previous examples on page 4.26.

\[
\begin{array}{c}
9.3 \\
7 \overline{)65.1} \\
63 \\
21 \\
\end{array}
\]

It is possible to check the accuracy of this by multiplying 9.3 by 0.7, which equals 6.51

As another example, **consider the indicated quotient 12.038 ÷ 0.26.**

The divisor must be converted to a whole number, so multiply 0.26 by one hundred, i.e. shift the decimal point two places to the right (0.26 \times 100 = 26).

Do the same with the dividend (12.038 \times 100 = 1203.8)
This may be set out as -

\[ 12.038 \div 0.26 \]

\[ = (12.038 \times 100) \div (0.26 \times 100) \]

\[ = 1203.8 \div 26 \]

\[ \begin{array}{c}
46.3 \\
26)1203.8 \\
104 \\
163 \\
156 \\
78 \\
78 \\
\vdots \\
\end{array} \]

The accuracy of this can be checked by multiplication

\[ 46.3 \times 0.26 = 12.038 \]

Sometimes an extra zero may need to be placed in the dividend.

Consider the indicated quotient 537.6 \div 0.32

Both divisor and dividend are multiplied by one hundred.

so 537.6 \div 0.32

becomes (537.6 \times 100) \div (0.32 \times 100)

or 53760 \div 32

\[ \begin{array}{c}
1680. \\
32)53760. \\
32 \\
217 \\
192 \\
256 \\
256 \\
\vdots \\
\end{array} \]

so 537.6 \div 0.32 = 1680
Consider the indicated quotient $7.95 \div 2.5$

Both divisor and dividend are multiplied by ten so

$$7.95 \div 2.5$$

becomes $(7.95 \times 10) \div (2.5 \times 10)$

$$= 79.5 \div 25$$

\[
\begin{array}{c}
25)79.50 \\
75 \\
\underline{45} \\
25 \\
\underline{200} \\
200 \\
\ldots
\end{array}
\]

Note that an extra zero was placed in the dividend to enable the division to continue.

**DIVISION BY A MULTIPLE OF TEN**

Consider the indicated quotient $93.6 \div 30$.

This is the same as:-

$$93.6 \div (10 \times 3) = (93.6 \div 10) \div 3$$

$$= 9.36 \div 3$$

$$= 3.12$$

**STEP 1:** Express the divisor in expanded forms.

**STEP 2:** Divide by ten, i.e., adjust the place value of each digit by one place.

**STEP 3:** Divide by three.
Consider the indicated quotient $812.2 \div 400$.

\[
812.2 \div 400 = 812.2 \div (100 \times 4) \\
= (812.2 \div 100) \div 4 \\
= 8.122 \div 4 \\
= 2.0305
\]

**RECURRING DECIMALS**

All the problems involving division presented so far have worked out exactly, so that at some stage in the division there is no remainder. These are called terminating decimals. However, divisions do not always work out exactly.

Consider the indicated quotient $2 \div 3$.

\[
\begin{array}{c}
3 \\ 0.666 \\
2.000 \\
\hline
18 \\
18 \\
20 \\
18 \\
2 \\
\end{array}
\]

No matter how far we continue this division we still get a remainder of 2 and the 6 in the quotient keeps on repeating or recurring.

We can abbreviate this decimal by writing the recurring digit only once and placing a dot above it to show that it recurs.

That is, $0.6\overline{6}$ is written as $0.6$.

This is read as "zero point six recurring".
Consider the indicated quotient $4 \div 11$.

\[
\begin{array}{c}
\frac{0.3636}{11} \quad \text{4,0000} \\
3 \quad \text{3} \\
\frac{70}{66} \\
\frac{40}{33} \\
\frac{70}{66} \\
\frac{4}{1}
\end{array}
\]

This is another division which cannot be completed. No matter how far we go we find the remainders of 7 and 4 recurring.

This quotient is written as 0.36.

In this case a dot is placed above both digits because both recur.

The quotient is read "zero point three six recurring, the three six recurring".

Consider the indicated quotient $4 \div 37$.

\[
\begin{array}{c}
\frac{0.108108}{37} \quad \text{4,000000} \\
3 \quad \text{7} \\
\frac{300}{296} \\
\frac{40}{37} \\
\frac{300}{296} \\
\frac{4}{1}
\end{array}
\]

Although three digits recur in this quotient we only place a dot above the first and last digit in the series, i.e., 0.108.

The quotient is read "zero point one zero eight recurring, the one zero eight recurring".

If a series of digits recurs, a dot is placed above the first and last digits in the series, showing that the same sequence of digits will keep repeating itself.

ROUNDING NUMBERS

You will notice that some quotients show a large number of decimal places - for example, four, five, six or more decimal places. For most purposes it is not

4.31
essential to be accurate to that degree and an approximate answer is acceptable.

Depending on the degree of accuracy required you may be asked to express an answer "correct to the first decimal place" or "correct to the second decimal place" and so on.

If you are asked to express an answer correct to the first decimal place you must take your division to the second decimal place.

If the digit in the second decimal place is 1, 2, 3 or 4, the answer is left as the digit in the first decimal place.

For example: 0.21 ) would all be expressed as
0.22 ) 0.2 (correct to the first
0.23 ) decimal place)
0.24 )

If the digit in the second decimal place is 6, 7, 8 or 9, the digit in the first decimal place is increased by one.

For example: 0.26 ) would all be expressed as
0.27 ) 0.3 (correct to the first
0.28 ) decimal place)
0.29 )

When the digit in the second decimal place is five, mathematicians have agreed to round it to the nearest even number.

This means 0.25 would be expressed as 0.2 (correct to the first decimal place), while 0.35 would be expressed as 0.4 (correct to the first decimal place).

If you are asked for an answer "correct to the second decimal place" you must continue your calculations to the third decimal place.

If the digit in the third decimal place is 1, 2, 3 or 4, the answer is rounded to the digit in the second decimal place. But, if the digit in the third decimal place is 6, 7, 8 or 9, the digit in the second decimal place is increased by one.

For example: 1.121 ) would be rounded to 1.12
1.122 ) (correct to the second
1.123 ) decimal place)
1.124 )
1.126) would be rounded to 1.13
1.127) (correct to the second
decimal place)
1.128)
1.129)

Because of the agreement to round to the nearest even number, 1.125 would be expressed as 1.12 (correct to the second decimal place).

Now consider the number 0.635.

Because the digit in the third decimal place is 5, the result of rounding 0.635 to the second decimal place depends on the digit in the second decimal place. The digit in the second decimal place is 3 which represents an odd number, so that 0.635 is rounded to 0.64 (correct to the second decimal place) rather than 0.63.

It should be obvious by now that when rounding decimals, the division must be continued one place beyond the place to which you are asked to give the answer.

**Simplifying Numerical Expressions**

In Chapter 2 we examined the use of common factors to help simplify numerical expressions involving whole numbers. We will now look at similar problems involving decimals.

Simplify \( \frac{2 \times 1.5}{1 \times 0.5} \)

N.B. Numbers on the lower line act as divisors. Numbers on the upper line act as dividends.

When demonstrating division of decimals we did not perform the division until the divisor was changed to a whole number. Also, in order to maintain the correct proportion, what we did to the divisor we also did to the dividend.

We can apply the same principles here.

\[
\frac{2 \times 1.5}{1 \times 0.5} = \frac{2 \times 1.5 \times 10}{1 \times 0.5 \times 10}
\]

\[
= \frac{2 \times (1.5 \times 10)}{1 \times (0.5 \times 10)}
\]

\[
= \frac{2 \times 15}{1 \times 5}
\]

\[
= \frac{2 \times 3}{1} = 6
\]

4.33
An alternative method is:-

\[
\frac{2 \times 1.5}{1 \times 0.5} = \frac{2 \times 1.5 \times 1}{1 \times 0.5 \times 1}
\]

\[
= \frac{2 \times 3 \times 1}{1 \times 1 \times 1}
\]

\[
= 6
\]

**STEP 1:** Express 1.5 as the improper fraction \(\frac{15}{10}\) and 0.5 as the proper fraction \(\frac{5}{10}\).

**STEP 2:** To divide by \(\frac{5}{10}\), multiply by its reciprocal \(\frac{10}{5}\).

**STEP 3:** Simplify as far as possible by dividing by common factors.

**STEP 4:** When no further common factors can be identified, find the product of the remaining numbers.

Simplify \(\frac{1.4 \times 0.22}{1.1 \times 2.8}\)

In this problem we have a divisor formed from the indicated product 1.1 x 2.8. We saw earlier that the product will have a total of two decimal places because there is one decimal place in each number of the indicated product.

When we have a divisor with two decimal places we need to use the equivalent numeral:

\[
1 = \frac{100}{100} \text{ or } 1 = \frac{10 \times 10}{10 \times 10}
\]

so

\[
\frac{1.4 \times 0.22}{1.1 \times 2.8} = \frac{1.4 \times 0.22 \times 100}{1.1 \times 2.8 \times 100}
\]

\[
= \frac{1.4 \times (0.22 \times 100)}{(1.1 \times 10) \times (2.8 \times 10)}
\]

\[
= \frac{140}{280} = \frac{1}{2} = 0.5
\]

\[
4.34
\]
\[ \frac{1.4 \times 22^1}{4.1 \times 28^1} = \frac{1.4 \times 1}{1 \times 14} = 0.1 \]

Some students prefer to work these problems without any decimal places remaining.

If choosing this method, scan both upper and lower lines and decide which line has the greater number of decimal places. In the above example, there are three decimal places in the upper line and two decimal places in the lower line.

All decimal places will be removed if both lines are adjusted by three lots of ten (ten to the third power).

So \( \frac{1.4 \times 0.22}{1.1 \times 2.8} \) can be written as \( \frac{1.4 \times 0.22 \times 1000}{1.1 \times 2.8 \times 1000} \)

or \( \frac{1.4 \times 0.22 \times (10 \times 100)}{1.1 \times 2.8 \times (10 \times 10 \times 10)} \)

\[ = \frac{(1.4 \times 10) \times (0.22 \times 100)}{(1.1 \times 10) \times (2.8 \times 10) \times 10} \]

\[ = \frac{14 \times 22 \times 1}{11 \times 28 \times 10} \]

\[ = \frac{14^1 \times 22^1 \times 1}{11^1 \times 28^2 \times 10} \]

\[ = \frac{1}{10} \]

\[ = 0.1 \]
Using the alternative method, express each decimal as a fraction. For example: \(1.4 = \frac{14}{10}; 0.22 = \frac{22}{100};\)
\(1.1 = \frac{11}{10}\) and \(2.8 = \frac{28}{10}.

To divide by \(\frac{11}{10}\) and \(\frac{28}{10}\), multiply by their respective reciprocals, \(\frac{10}{11}\) and \(\frac{10}{28}\).

So \(\frac{1.4 \times 0.22}{1.1 \times 2.8} = \frac{1 \times 2 \times 1 \times 1}{10 \times 10 \times 1 \times 1}\)

\(= \frac{1 \times 1 \times 1 \times 1}{1 \times 10 \times 1 \times 1}\)

\(= \frac{1}{10}\)

\(= 0.1\)

Simplify \(\frac{1.05 \times 3.15}{3.5 \times 0.007}\)

\(\frac{1.05 \times 3.15}{3.5 \times 0.007} = \frac{1.05 \times 3.15}{3.5 \times 0.007} \times \frac{10000}{10000}\)

\(= \frac{1.05 \times 3.15}{3.5 \times 0.007} \times \frac{(100 \times 100)}{(10 \times 1000)}\)

\(= \frac{(1.05 \times 100) \times (3.15 \times 100)}{(3.5 \times 10) \times (0.007 \times 1000)}\)

\(= \frac{3 \times 45}{1 \times 1} = 135.\)

\(= 4.36\)
Using the alternative method, express all decimals as fractions. For example: $1.05 = \frac{105}{100}$; $3.15 = \frac{315}{100}$; 
$3.5 = \frac{35}{10}$ and $0.007 = \frac{7}{1000}$.

To divide by $\frac{35}{10}$ and $\frac{7}{1000}$, multiply by their respective reciprocals $\frac{10}{35}$ and $\frac{1000}{7}$.

$$
\begin{align*}
1.05 \times 3.15 &= \frac{3 \times 45 \times 1}{100 \times 35^2 \times 7^2}, \\
3.5 \times 0.007 &= \frac{40 \times 1 \times 1}{7 \times 35 \times 7}, \\
&= \frac{3 \times 45 \times 1 \times 1}{1 \times 1 \times 1 \times 1}, \\
&= 135.
\end{align*}
$$
SOME APPLICATIONS OF MEASUREMENTS IN NURSING

LINEAR

MASS

VOLUME

Injections
Intra-venous

Drainage

Medicines and Lotions
Chapter 5

MEASUREMENT

By measurement we can determine the size of anything in terms of a suitable unit of measurement.

In the previous chapter we used some expressions such as "three-quarters of a minute" and "a quarter of an hour". These were expressions of quantities and their measures.

In the examples above, the number part of the quantity is its measure, therefore in the statement "three-quarters of a minute", the "three-quarters" is the measure and the "minute" is the unit of measurement. In the statement "a quarter of an hour", "one quarter" is the measure and "hour" is the unit of measurement.

Measurement of various quantities occupies a great part of our lives. In this, and following chapters, we will be considering mainly those forms of measurement which directly concern a nurse in carrying out her duties, or in understanding particular measurements used by other members of the health team. However, from time to time, it will be easier to take examples from everyday life.

In this chapter we will be concentrating on the units of measurement.

Some form of measurement has been used by people since earliest times. Primitive man gathered together his objects and counted them. When he wanted to represent the size of an object, he related it to some other object. Often he used parts of his body - for example, the width of his thumb, the width of his hand, the length of his arm. Larger distances were represented in terms such as a "stone's throw", an "arrow's flight" or "a day's walk".

We can see that such measurements would vary from person to person. Gradually it was seen that objects
could be represented by numbers and those numbers represented by symbols. Also it was seen that if measurement was not to vary from person to person, some standard units must be accepted. Throughout the centuries various standards have been accepted in different countries.

The idea of adopting a universal system of measurement was suggested as early as the seventeenth century, however, it obtained little support. After France adopted the metric system of measurement there was a gradual acceptance of this system in scientific circles throughout the world. This meant that many countries utilized a dual system of measurement - one system for everyday measurement, business and commerce, and another for the sciences and technology. The changeover to metric measurement has gradually occurred in more and more countries.

There have been some slight changes in the metric system of measurement over the years, and the system adopted by Australia is known as the Systeme International d'Unités or S.I. System. This is the modern form of the metric system approved by the Conférence Générale des Poids et Mesures in 1960 and has been described as an extension and refinement of the traditional metric system.

When developing systems of measurement, there has been an attempt to find some naturally occurring fixed measure. When the metric system was being developed it was decided to relate the fixed measure to the earth itself. The measure chosen was the metre (named from the Greek word - to measure) and was calculated to be one ten millionth part of the quadrant of the terrestrial meridian. A terrestrial meridian is an imaginary circle drawn across the earth's surface passing through the Poles. A quadrant of a meridian is the distance on that circle between the Equator and a Pole. Because the work was being done in France, the quadrant chosen was on the meridian which passes through Paris and Barcelona. A metal bar was made from an alloy of platinum and iridium and two fine lines were engraved upon it a metre apart. This was the standard metre and various copies of it were made for other countries wishing to use it. Later it was found that the measurement of the meridian had not been accurate and so the metal bar was accepted as the standard. In recent years, the standard has been related to an atomic phenomenon.
The metric system is entirely decimal and so is related to our place value system of numeration. Special Greek and Latin prefixes are used to name the decimal steps, and these remain the same no matter which units are being used.

Those prefixes and their values which are likely to be met in nursing practice are shown in the table below.

<table>
<thead>
<tr>
<th>Factor By Which the Unit is Multiplied</th>
<th>Prefix</th>
<th>Symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^6$ (one million)</td>
<td>mega</td>
<td>M</td>
</tr>
<tr>
<td>$10^3$ (one thousand)</td>
<td>kilo</td>
<td>k</td>
</tr>
<tr>
<td>$10^2$ (one hundred)</td>
<td>hecto</td>
<td>h</td>
</tr>
<tr>
<td>$10^1$ (ten)</td>
<td>deka or deca</td>
<td>da</td>
</tr>
<tr>
<td>$10^{-1}$ (one tenth)</td>
<td>deci</td>
<td>d</td>
</tr>
<tr>
<td>$10^{-2}$ (one hundredth)</td>
<td>centi</td>
<td>c</td>
</tr>
<tr>
<td>$10^{-3}$ (one thousandth)</td>
<td>milli</td>
<td>m</td>
</tr>
<tr>
<td>$10^{-6}$ (one millionth)</td>
<td>micro</td>
<td>μ</td>
</tr>
</tbody>
</table>

In the S.I. system, preference is given to the use of those prefixes related to the S.I. unit by powers of $10^3$.

For example: micro ($10^{-6}$), milli ($10^{-3}$), kilo ($10^3$) and mega ($10^6$).

**MEASUREMENT OF LENGTH**

The base unit for measurement of length is the METRE. The Americanised spelling "meter" is not acceptable.

The most frequently used measures are:
The kilometre (km) pronounced "kill-o-metre", with the accent on the first syllable and the "o" as in "oh". It does not rhyme with speedometer.

The metre (m);

The millimetre (mm);

1 kilometre (km) = 1000 metres (m)
1 metre (m) = 1000 millimetres (mm)

These measures conform to the preferred practice of using multiples and submultiples related by the third power of ten.

1 kilometre = 10³ metres.
1 millimetre = 10⁻³ metres.

The centimetre (cm) (10⁻² metres), although a non-preferred unit is often used to express dimensions of the human body such as: the length of a baby at birth and during various developmental stages, and the height of a person, as well as the linear dimensions of larger body organs. The dimensions of the cellular components of the body are often recorded in micrometres (microns) (10⁻⁶ m (metres)). Abbreviation for micrometre (micron) is µm.

When recording linear measure, preferably choose a unit for which the value to be expressed will be between 0.1 and 1000. For example, 45 m rather than 45 000 mm or 0.045 km.

Conversion of millimetres to metres, or metres to kilometres, and vice versa, is performed by adjusting the place value of the digits either by multiplying or dividing by the appropriate power of ten.

For example, 2500 mm would be expressed as 2.5 metres.

1000 mm = 1 metre
1 mm = \frac{1}{1000} metre

\[ \therefore \ 2500 \text{ mm} = (2500 \div 10^3) \text{ metres.} \]

= 2.5 metres.
Express 5 500 metres in kilometres.

1000 metres = 1 km

1 metre = \( \frac{1}{1000} \) km

\((5500 \div 10^3)\) metre = 5.5 km

Express 0.75 km in metres.

1000 metres = 1 km

1 metre = \( \frac{1}{1000} \) km

\((0.75 \times 10^3)\) metre = 0.75 km

750 metres = 0.75 km

Express 65 centimetres in millimetres.

1 cm = 10 mm

\(\frac{1}{10}\) cm = 1 mm

65 cm = \((65 \times 10)\) mm

65 cm = 650 mm

Express 250 centimetres in metres.

100 cm = 1 metre

1 cm = \( \frac{1}{100} \) metre

250 cm = \((250 \div 10^2)\) metres

250 cm = 2.5 metres

\[ 5.5 \]
If you are not already familiar with these units of length, make sure you gain experience by examining rulers or tape measures calibrated in millimetres and centimetres. Take a long stride - the distance covered is approximately a metre. If you have not a metric tape measure, measure a piece of string one metre in length and use this to estimate the linear dimensions of various objects. Portion of a metric linear scale is shown below.

![Metric Linear Scale](image)

**FIG. 5.1** Portion of a 30 cm rule

Units of area and volume are derived from the unit of length. The unit of area is the SQUARE METRE (m²) - that is, the area enclosed by a square each side of which is a metre in length.

The most common units of area met with in nursing will be square metre (m²), square millimetre (mm²) and sometimes square centimetre (cm²).

The relationship between each decimal unit, for example, square millimetre and square centimetre is $10^2$, so $100 \text{ mm}^2 = 1 \text{ cm}^2$.

The volume of an object is the space it occupies. The unit of volume is the CUBIC METRE (m³), i.e., the volume of a cube with all sides measuring 1 metre. The other common units of volume you will meet are cubic millimetre (mm³) and cubic centimetre (cm³). (Some old measuring equipment may still have the abbreviation "cc" on it. This was once the accepted abbreviation for cubic centimetre, but is no longer to be used.)

As will be seen later, there is another set of units which may be applied to the volume or capacity of liquids and gases.
The relationship between each decimal unit, for example, cubic centimetre to cubic millimetre is $10^3$, so $1000 \text{ mm}^3 = 1 \text{ cm}^3$.

MEASUREMENT OF FLUIDS AND GASES

The capacity of a cube each side of which measures 10 centimetres or 1 decimetre is a cubic decimetre or one LITRE. (The Americanised spelling of "liter" is not acceptable.)

The interchangeable relationship between units of volume and capacity is shown in the following table, however, in nursing the "litre" derivatives are used more frequently.

- $1000$ cubic millimetres ($\text{mm}^3$) $= 1$ cubic centimetre ($\text{cm}^3$) $= 1$ millilitre ($\text{mL}$).
- $1000$ cubic centimetres ($\text{cm}^3$) OR $1000$ millilitres ($\text{mL}$) $= 1$ cubic decimetre ($\text{dm}^3$) $= 1$ litre ($\text{L}$).
- $1000$ cubic decimetres ($\text{dm}^3$) OR $1000$ litres ($\text{L}$) $= 1$ cubic metre ($\text{m}^3$) $= 1$ kilolitre ($\text{kL}$).

In nursing, the litre and millilitre ($10^{-3}$ litres) will be the most frequently used units.

If you are not familiar with these units, obtain some measuring equipment; for example, a syringe, medicine measure, measuring cylinder, measuring jug, which are graduated in millilitres. A medicine measure can be bought for a few cents at most large supermarkets and at all pharmacies.

Conversion of litres to millilitres or vice versa can again be performed by adjusting the place value of the digits. See examples below Fig. 5.3.

N.B. Because of the danger of confusing a lower case 'l' with numeral 1, a capital "L" is used as the abbreviation for litre.
FIG. 5.2 Diagram representing a cube, each side of which measures 10 cm (1 dm). The volume of this cube is 1 dm$^3$ and its capacity may be stated as 1000 cm$^3$, 1000 mL or 1 litre. The cube is subdivided to show that 1 dm$^3$ = 1000 cm$^3$. 
FIG. 5.3 Diagram representing:-
(a) one thousandth of the
volume of the cube; or
(b) one of the subdivisions
of the cube shown in
Fig. 5.2.
Each side of this cube
measures 1 cm and its volume
may be expressed as
1 cm\(^3\) or 1 mL.

Examples:-

Express 0.75 litres in millilitres.

\[
\frac{1}{1000} \text{ litre} = 1 \text{ mL}
\]

\[
0.75 \text{ litre} = (0.75 \times 10^3) \text{ mL}
\]

\[
\therefore 0.75 \text{ litre} = 750 \text{ mL}
\]

Express 3400 millilitres in litres.

\[
\frac{1}{1000} \text{ litre} = 1 \text{ mL}
\]

\[
(3400 \div 10^3) \text{ litres} = 3400 \text{ mL}
\]

\[
\therefore 3.4 \text{ litres} = 3400 \text{ mL}
\]
MEASUREMENT OF MASS

In the past there has been a tendency to use the term 'weight' when referring to the quantity of matter in an object. Strictly speaking, the weight of an object is determined by the gravitational pull on it and may therefore vary. We demonstrate this when we refer to the 'weightlessness' of objects in space technology. The amount of matter in an object is still the same whether or not it is under the influence of gravitational pull. It is, therefore, desirable that the term MASS be used whenever we refer to the quantity of matter in an object.

The SI base unit for mass is the KILOGRAM, and the standard is a metal cylinder of platinum - iridium alloy which is held in the custody of the Bureau International des Poids et Mesures at Sèvres, a suburb of Paris.

The kilogram used to have a connection with the litre, because 1 litre of water was said to weigh 1 kilogram. This is no longer exact under all conditions, however, the relationship is very useful, and for practical purposes we can say '1 litre of water has a mass of 1 kilogram'; '1 milliliter of water has a mass of 1 gram'; and therefore, '1 cubic centimetre of water has a mass of 1 gram'. This demonstrates a very interesting relationship between length, mass and volume.

'Gramme' was sometimes used as an alternative spelling, but the shorter version 'gram' is preferred.

The most frequently used units are the kilogram (kg), gram (g), milligram (mg) and microgram (μg). These conform to the preferred relationship of the third power of ten.

\[
\begin{align*}
1 \text{ kilogram} &= 10^3 \text{ grams} \\
1 \text{ milligram} &= 10^{-3} \text{ grams} \\
1 \text{ microgram} &= 10^{-6} \text{ grams}.
\end{align*}
\]

A mass of 1 gram is a very small quantity when applied to commercial or industrial articles, so this is the reason why the kilogram is more often used.

The mass of an adult should be recorded in kilograms, but the mass of a newborn baby may be more conveniently recorded in grams.
Drugs may be measured in gram, milligram and microgram doses. Extremely potent drugs are presented in microgram doses, and it is vital that the units "microgram" and "milligram" are not confused, otherwise fatal overdosages could be given.

Conversion of kilograms to grams; grams to milligrams; milligrams to micrograms and vice versa is again accomplished by adjustment of the place value of each digit.

Examples:

Express 1342 grams in kilograms.

\[
1000 \text{ grams} = 1 \text{ kilogram} \\
1 \text{ gram} = \frac{1}{1000} \text{ kilogram} \\
1342 \text{ grams} = (1342 \div 10^3) \text{ kg} \\
\therefore 1342 \text{ grams} = 1.342 \text{ kg}
\]

Express 0.92 kg in grams.

\[
1000 \text{ grams} = 1 \text{ kg} \\
1 \text{ gram} = \frac{1}{1000} \text{ kg} \\
(0.92 \times 10^3) \text{ gram} = 0.92 \text{ kg} \\
\therefore 920 \text{ grams} = 0.92 \text{ kilogram}
\]

Express 0.12 gram in milligrams.

\[
1000 \text{ mg} = 1 \text{ gram} \\
1 \text{ mg} = \frac{1}{1000} \text{ gram} \\
(0.12 \times 10^3) \text{ mg} = 0.12 \text{ gram} \\
\therefore 120 \text{ mg} = 0.12 \text{ gram}
\]
Express 957 milligrams in grams.

\[1000 \text{ mg} = 1 \text{ gram}\]
\[1 \text{ mg} = \frac{1}{1000} \text{ gram}\]
\[957 \text{ mg} = (957 \div 10^3) \text{ g}\]
\[\therefore 957 \text{ mg} = 0.957 \text{ gram}\]

Express 0.015 milligrams in micrograms.

\[1000 \mu\text{g} = 1 \text{ mg}\]
\[1 \mu\text{g} = \frac{1}{1000} \text{ mg}\]
\[(0.015 \times 10^3) \mu\text{g} = 0.015 \text{ mg}\]
\[\therefore 15 \mu\text{g} = 0.015 \text{ mg}\]

Express 1500 micrograms in milligrams.

\[1000 \mu\text{g} = 1 \text{ mg}\]
\[1 \mu\text{g} = \frac{1}{1000} \text{ mg}\]
\[1500 \mu\text{g} = (1500 \div 10^3) \text{ mg}\]
\[\therefore 1500 \mu\text{g} = 1.5 \text{ mg}\]

Always endeavour to record mass in the most suitable unit. For example - 11 725 micrograms would be better recorded as 11.725 milligrams and 0.001 25 gram would be better recorded as 1.25 milligrams.

If you have been observant, you will have noticed the following facts during this discussion of the SI system.

(a) The prefixes indicating the multiples and submultiples of the various units do not begin with capitals, unless the word comes at the start of a sentence.
(b) The abbreviations for the prefixes representing multiples and submultiples as high as $10^3$ (i.e., kilo) do not have capitals.

(c) A space is left between the number and the unit symbol, e.g., 15 mm, 250 mL.

(d) No full stop follows the abbreviations for the units, unless a unit comes at the end of a sentence.

(e) The abbreviations are the same in the singular and the plural.

(f) There is no space between the prefix symbol and the unit symbol.

(g) Indices are used to show the decimal relationships between the multiples and submultiples.

(h) The indices two and three are used to represent square and cubic measures respectively, and these indices are placed to the right of the unit symbol. For example, km$^2$ means square kilometre, m$^2$ means square metre and mm$^3$ means cubic millimetre.

(i) Care must be taken that the abbreviations for litre cannot be mistaken for the digit one, therefore a capital L should be used.

(j) As always, the decimal point is shown at the mid-height of the digits.

(k) When the numeral represents a number of less than one, the usual practice is followed of placing a zero before the decimal point.

(l) When expressing a quantity we use only one unit. For example, 375 millimetres may be expressed as 375 millimetres, 37.5 centimetres or 0.375 metres, but NOT 37 cm 5 mm or 3 dm 7 cm 5 mm.

Other points which have not been demonstrated but are accepted practice in the SI system are shown below.

1. As the metric system is a decimal system, it is

5.13
preferable always to use decimals rather than equivalent fractions - e.g., 0.5 instead of $\frac{1}{2}$; 0.25 instead of $\frac{1}{4}$; 0.75 instead of $\frac{3}{4}$.

2. When there are more than four digits on either side of the decimal point, a space is left between each group of three digits to show the breakdown into groups of thousands or thousandths - e.g., 15125.5; 0.176 54. In Chapter 1 we saw that it was not essential to leave this space between the third and fourth digit if there were only four digits in the numeral.
SOME APPLICATIONS OF MEASUREMENTS
IN NURSING

Temperature  Pulse

Blood Pressure  O₂ Therapy

Autoclave (Sterilization under pressure) Relative Density (Specific gravity)
Chapter 6

MEASUREMENT CONTINUED

In Chapter 5 we discussed the units by which we could measure length, volume and capacity and mass. A nurse is often called upon to undertake other types of measurement. In addition, she needs an understanding of other measurements made by various members of the health team. This chapter will consider a few of these measurements.

MEASUREMENT OF TEMPERATURE

One of the important duties of a nurse is the measurement of temperature. What is temperature? It may be simply defined as the "hotness" or "coldness" of an object. An instrument which measures the degree of hotness or coldness is called a thermometer. Thermometers vary slightly according to the function for which they were designed. Most thermometers used by nurses have a small bulb or reservoir connected to a fine bore tube which bears numerous markings on its outer surface. In the bulb is either mercury or coloured alcohol. These have been found to be the most suitable substances because of their high degree of expansion when heat is applied. The expansion of the liquid in the bulb causes the level in the fine bore tube to rise. The extent of this rise can be determined by the graduations on the stem. When heat is withdrawn the substance contracts, causing the level in the bore tube to fall.

Scientists have established what are known as temperature scales. The main scale by which we not only record atmospheric temperature, but also the temperature of lotions, baths and a patient's body
temperature is called the CELSIUS scale.

To make a temperature scale it is necessary to find some basis against which all other temperatures can be compared. The basic points chosen for graduating thermometers have been:-

(a) The freezing point of pure water; and

(b) The boiling point of pure water measured at sea level.

It is necessary to make the stipulation "at sea level" because the boiling point of water is raised if atmospheric pressure is increased and lowered if the pressure is decreased. So, at the top of a high mountain water will boil at a lower temperature than it will at sea level.

The Celsius scale sets the freezing point of pure water at sea level at 0°C and the boiling point at 100°C. These fixed points are not the highest or lowest temperatures which can be recorded. Some substances have a lower freezing point than pure water, and others have a higher boiling point than pure water. Thermometers with extended scales are used to measure such temperatures.

When measuring the temperature of a substance you should select the thermometer which is most appropriate for the job required.

Lotion thermometers are used to measure the temperature of the lotions and solutions used in carrying out nursing procedures. Sometimes only a section of the scale appears on the stem of the thermometer.

Clinical thermometers are used to measure body temperature. Body temperature does not vary greatly even in sickness, so only a limited section of the scale is shown. Try to examine a clinical thermometer, and compare it with Figure 6.1. Usually an arrow is engraved on the stem to indicate normal body temperature.

When a thermometer is removed from its warm surroundings the mercury contracts. Because of this, a thermometer is always read while the bulb is surrounded by the substance of which the temperature is being measured. Imagine the difficulty of trying to read a thermometer while it is still in the patient's mouth! To overcome this problem the clinical thermometer has a special constriction in the stem which breaks the column of mercury and prevents it from contracting back into
FIG. 6.1 Clinical Thermometer

the bulb. This means the temperature can be read after the thermometer is removed from the patient's mouth. Vigorous shaking will force the mercury back past the constriction so that another reading may be taken.

The base unit for temperature difference in the SI system is the KELVIN. On the Kelvin scale the freezing point of pure water is 273 K and the boiling point of water is 373 K—a difference of one hundred kelvin. Absolute zero at 0 K (−273°C) is the lowest temperature which can be recorded.

MEASUREMENT OF FORCE

A force may be defined as that which tends to cause or alter the motion of matter.

In the SI system the NEWTON is the unit of force. The components from which the unit of force is derived involve distance (measured in metres), mass of the object (measured in kilograms) and time (measured in seconds).

A newton (N) is the force which when applied to a body having a mass of 1 kilogram causes an acceleration of 1 metre per second in the direction of the application of the force.

Higher multiples of the newton may be encountered, e.g., the kilonewton (kN) and meganewton (MN).

MEASUREMENT OF ENERGY

For a force to be applied to matter, energy must be expended. The SI unit for the measurement of energy, work or quantity of heat is the JOULE (J).

A joule is defined as the work done, or the energy expended when a force of one newton moves the point of application a distance of one metre in the direction of
that force.

We have different forms of energy - e.g., chemical energy, mechanical energy, heat energy. Because energy may be converted from one form to another we measure different kinds of energy in the same unit.

The calorie and kilocalorie (Calorie) are metric units which were used to measure heat energy, but with the change to the SI system, all energy is measured in the same units, viz., the joule or kilojoule (kJ). The energy needs of man and the energy provided by the food he eats are expressed in kilojoules:

\[
\begin{align*}
1 \text{ kilocalorie} & = 4200 \text{ joules} = 4.2 \text{ kilojoules} \\
1 \text{ Calorie} & 
\end{align*}
\]

MEASUREMENT OF ELECTRIC CURRENT

The basic unit of electric current, i.e., the rate at which current flows through a conductor, in the SI system is the ampere (amp).

Electricity appears to be extremely important in the normal functioning of the human body. The muscles, heart and nervous system all depend on it and can generate it.

Physiological studies of the body will be found to refer to electric current within the body in terms of milliamperes (mA).

The unit of electrical potential, i.e., the concentration of electrical charge at any point, is the volt. Electrical potentials within the human body may be expressed in terms of millivolts (mV) or microvolts (μV).

The product of amps and volts gives the unit of power known as the watt (W). In the SI system, the watt is equal to one joule per second. This means that all forms of power whether produced by heat, electricity or mechanical movement can be related to the same unit - the watt and its frequently used multiple, the kilowatt (kW).

MEASUREMENT OF PRESSURE

Many nursing procedures involve the measurement or monitoring of pressures.

Pressure is defined as the force exerted divided by the area over which it acts. If a force of 1 newton is applied to a square metre (N/m²) the pressure exerted is called a pascal (Pa). The pascal is too small a unit for
convenient use in many areas, including most areas of health care, so the unit of 1000 pascal or 1 kilopascal (kPa) is more appropriate.

The term atmospheric or barometric pressure is a familiar one. By this we understand that the gases which make up the air or atmosphere around us exert a pressure. It is still quite common to hear that pressure expressed in different ways.

The meteorologist, in preparing weather forecasts, talks about barometric (atmospheric) pressure in terms of millibars (1 millibar (mb) = 0.1 kPa). Others prefer to express the pressure in kilopascals.

You will recall that air is a mixture of gases, especially nitrogen, oxygen, carbon dioxide and water vapour, present in reasonably constant proportions. Air may be said to consist of approximately:

- 78% nitrogen
- 20% oxygen
- 0.4% carbon dioxide
- 1.6% water vapour and other gases

The barometric pressure at sea level is very close to 100 kPa, which means that the partial pressures (in kilopascals) of each gas comprising the air around us will be almost the same as its percentage composition.

So, the partial pressure of the nitrogen would be approximately 78 kPa, while the partial pressure of the oxygen would be approximately 20 kPa.

Pressures can be measured with instruments known as manometers.

Many pressures significant to human physiology are still expressed in millimetres of mercury (mm Hg). This implies that a mercury manometer was used to make the measurement.

(At sea level, the atmosphere will support a column of mercury 760 mm high.)

Most of the manometers which are used in nursing practice are graduated so that zero on the manometer scale represents atmospheric pressure (i.e. 760 mm Hg). The pressure recorded on such a scale is known as a gauge pressure.

If you wish to correlate pressures in kilopascals with those in millimetres of mercury (1 kilopascal = 7.5 mm Hg).

A frequent nursing procedure is the measurement of blood pressure using a piece of apparatus known as a
SPHYGMOMANOMETER. One type of sphygmomanometer is shown in Figure 6.2.

FIG. 6.2 Sphygmomanometer for measuring blood pressure.

You should note the inflatable cuff which is wound around the upper arm; the rubber bulb-pump or "bellows" with which the cuff is inflated; and the tubing leading to the reservoir of mercury at the base of the calibrated tube. The calibrated tube and reservoir comprise the mercury pressure gauge or manometer.

To record the blood pressure using this apparatus, sufficient air is pumped into the cuff to compress the blood vessels in the arm. (If the fingers of one hand are held on the radial pulse i.e., the pulse taken at the wrist, while the other hand pumps the bulb to inflate the cuff, the operator can tell when blood flow ceases because the radial pulse disappears). The cuff is inflated further. The column of mercury in the manometer will have risen to equal the pressure inside the cuff. A stethoscope is then placed over the brachial artery, just in front of the elbow. Using the release valve, the pressure in the cuff is gradually decreased while the operator waits to hear the first tapping sounds through the stethoscope. The point at which these sounds first appear is called the
systolic blood pressure and the level of the mercury in the
gauge at that point is noted. Systolic pressure is the
pressure exerted by the forceful ejection of blood when the
ventricles of the heart contract. When the pressure in the
sphygmomanometer cuff equals the systolic pressure, blood
will begin to flow through the arteries beneath the cuff.
As the cuff pressure is further reduced, the intensity of
the sounds heard through the stethoscope increases until
they suddenly become muffled and finally disappear. The
point at which the sounds become muffled is taken as the
diastolic blood pressure, and the level of the mercury at
that point is noted. Diastolic pressure is the residual
pressure in the vessels when the heart is not contracting.

The blood pressure determined by such apparatus is
recorded in millimetres of mercury but is generally written
as a fraction, e.g. $\frac{120}{80}$. In such records, the upper number
represents the systolic pressure (in mm Hg) and the lower
number represents the diastolic pressure (in mm Hg).
Pressures considerably above these readings may be recorded
in a hypertensive patient i.e. a person with abnormally
high blood pressure.

Pressure of blood is highest in the arteries, but
falls gradually as the blood flows through the arterioles,
capillaries and veins. By the time blood enters the large
veins draining into the right side of the heart, the
pressure has normally reduced to zero gauge pressure (i.e.
atmospheric pressure).

Unless otherwise stated, blood pressure is always
recorded with the sphygmomanometer cuff around the upper
arm, thus permitting meaningful comparisons to be made.

Respiration occurs because there are alternate in-
creases and decreases of pressure within the thorax. When
the diaphragm contracts and the dome part descends, it
increases the volume of the thoracic cavity and thus
decreases the pressure. As a result, air rushes in through
the respiratory passages (inspiration) into the lungs in
an effort to equalize the pressure. When the intra-
thoracic pressure becomes greater than atmospheric pressure
due to recoil of the lung tissue and relaxation of the
diaphragm, expiration occurs.

These fluctuations in pressure are sometimes referred
to as 'positive pressure' or 'negative pressure'. We need
to remind ourselves that they imply pressures slightly
above (+ve pressure) or slightly below (-ve pressure) the
atmospheric pressure.

6.7
Destruction of disease causing organisms is frequently performed by applying heat under pressure in an autoclave or pressure sterilizer. It is a physical law that so long as the volume remains constant, the temperature of a gas increases as the pressure increases. The autoclave is a chamber with a fixed volume. Air is removed from the chamber before the steam is introduced. As the pressure of the steam increases so does the temperature and this ensures a temperature high enough throughout the materials being sterilized to ensure destruction of all microorganisms and spores.

Oxygen and other mixtures of gases are supplied in cylinders or bulk storage tanks in which the gas is held under high pressure. Before oxygen can be administered to a patient it must pass through a pressure reducing valve. Another valve regulates the flow of oxygen, so that it can be adjusted to provide a flow of a given number of litres per minute.

Some of the masks used in oxygen therapy enable a specified percentage concentration to be administered e.g. oxygen 28% or oxygen 35%. These masks are designed so that the oxygen entering from the tubing connected to the cylinder or central supply also draws in air which dilutes the oxygen.

Because of the earlier relationship described between percentage concentrations of the gases in air and the partial pressure of a gas in kilopascals, it is obvious that a mask delivering 28% oxygen is delivering oxygen at approximately 28 kPa.

The rate of infusion of fluids into the body, either in intravenous feeding, in gastric lavage or enemas, depends to a large extent on the pressure of the fluid, and this depends on the height above the patient that the reservoir is positioned. The higher the reservoir, the greater will be the pressure and therefore the faster will be the flow.

Cerebro spinal fluid forms a water cushion around the brain and spinal cord. In certain conditions such as head injury, tumor of the brain, meningitis and hydro-cephalus the pressure of the C.S.F. rises. Measurement of the pressure exerted by the C.S.F. is an aid to diagnosis. A water manometer is used. The normal pressure in a person lying in the horizontal position averages 130 mm of water, although pressures varying between 70 mm and 180 mm of water may be found in a normal person. The following equivalents help relate the units we have been discussing:

6.8
1 kPa = 100 mm water (H₂O)

1 mm mercury (Hg) = 13.6 mm water (H₂O)

The kilopascal is now the preferred unit of measurement for pressures, but other pressure units are still being widely used in special situations, and will probably remain acceptable for a considerable time. Unless other units are specifically mentioned, one would expect the kilopascal (10³ Pa) to be the unit of choice.

**MEASUREMENT OF RELATIVE DENSITY**

Nurses carry out a number of routine tests on urine - the fluid excretion of the kidneys. One of these tests is to determine the relative density. Relative density is the ratio of the density of any substance to that of pure water. Density is a relationship between the mass and volume of a substance. The density of water is 1 gram per cubic centimetre (or 1 gram per millilitre). Because relative density is a ratio it is expressed as a number. In other words it signifies how many times lighter or heavier a substance is than an equal volume of water. The relative density of water is 1.000. Some texts ignore the decimal point and express it as 1000, relating it to the density of 1 litre of water.

As a short cut to finding the relative density of liquids we use an instrument called a hydrometer. A hydrometer is an instrument which will float in liquid. It consists of a weighted bulb and a stem with a graduated scale. When placed in a liquid it floats upright. A dense liquid buoys up the hydrometer, while a less dense liquid allows it to sink more deeply into the liquid. To measure the relative density of urine we use a special type of hydrometer called a urinometer. Urine always contains some dissolved solids and therefore its relative density is always greater than that of an equal volume of water. The range of the scale on the stem of the urinometer is 1.000 to 1.060, usually represented as 0-60. The relative density of urine usually lies between 1.010 and 1.030. This means that urine is 1.010 to 1.030 times denser than water.

The urinometer illustrated in Figure 6.3 gives a reading of 20, which means the relative density is 1.020.
FIG. 6.3 Urinometer

In order to obtain an accurate reading from the urinometer it must float freely in the urine - not touching the glass of the cylinder at any place.

If the volume of the specimen is so small that the urinometer touches the bottom of the cylinder and will not float, an equal volume of water may be added to the specimen. Measure accurately the volume of the urine specimen and then add an exactly equal volume of water. Because the volume has been doubled the relative density reading will be halved. To gain the correct relative density the urinometer reading must be doubled.

If the volume of the specimen is 60 mL and a further 60 mL of water is added to give a urinometer reading of 06, the correct relative density of the specimen would be 1.012, i.e., twice the urinometer reading.
In some cases it may be necessary to double the volume of water added. If the volume of the specimen is increased from 60 mL to 180 mL (120 mL of water added) with a reading of 04 on the urinometer, the correct relative density of the specimen would be 1.012, i.e., three times the urinometer reading.

The relative density of blood varies between 1.041 and 1.067. It increases as the volume of water in the blood decreases and as the substances dissolved in it or floating in it increase.

Remember, relative density is a ratio comparing the density of one liquid with the density of water. It has no unit of measurement, since it is simply a comparison between two substances.

MEASUREMENT OF HYDROGEN ION CONCENTRATION

This is not a procedure carried out by nurses, but it is a common enough measurement in text books and on laboratory reports.

No doubt you are familiar with the words "acid", "neutral" and "alkaline". You may even be able to quote examples of acids and alkalis. The acidity of a substance is determined by the concentration in it of free hydrogen ions. Scientists have developed what is known as a pH scale by which to measure hydrogen ion concentration. Water is said to be neutral and it is placed at pH 7 on the scale. Acid solutions have a pH value of less than 7, and the smaller the pH number, the greater the degree of acidity, which means there are more free hydrogen ions present in the solution. Alkaline solutions have a pH value greater than 7, and the greater the pH value, the smaller number of hydrogen ions are present. This may seem paradoxical. Interested students can consult reference books to find why the scale was devised in this way.

The concentration of hydrogen ions has a marked effect on many of the chemical reactions which take place in the body. Body fluids cannot tolerate a great degree of variation from their normal hydrogen ion concentration. Blood and the cerebro spinal fluid which bathes the brain and spinal cord show the least variation within normal limits, rarely fluctuating beyond the range of 7.3 to 7.5. The highly acid gastric juice may vary from 2 to 4. The pH of urine varies with the needs of the body. If there is too much alkali in the blood, the kidneys
FIG. 6.4 pH SCALE.
excrete alkali and the pH of urine increases. If there is too much acid in the blood, the kidneys excrete acid and the pH of urine decreases. The normal range of urine, blood and gastric juice are shown on the pH scale in Figure 6.4.

MEASUREMENT OF THE CHEMICAL CONSTITUENTS OF THE BODY

As seen earlier, the mass of the body as a whole and its individual organs can be recorded in units of mass such as the kilogram or gram. The volume of the fluids it contains, or the secretions it produces, can be recorded in units of volume (fluid capacity) such as the litre and millilitre. There are many substances in the body for which the estimation of the total mass of the substance present is of little relevance, but the amount present in a stated volume is of greater significance. Various methods have been used to record this vital information. Sometimes the values were recorded in mass per unit volume e.g. grams or milligrams per hundred millilitres; or grams or milligrams per cent; or grams or milligrams per litre.

Physiology books have recorded that every 100 mL of blood contains approximately 15 grams of haemoglobin in its red corpuscles; or, that there is normally about 90 mg of glucose in every 100 mL of blood. Haemoglobin levels have also been recorded as 15 g per cent or as 150 g per litre and blood sugar levels as 90 mg per cent or 900 mg per litre.

It is now known that the biological activity of a substance in the body depends upon the number of molecules of the substance per unit volume of body fluid rather than on the mass of the substance per unit volume. Expressing a concentration in "grams or milligrams per litre" does not give any indication of the number of molecules present, because some molecules are heavier than others. So, physiologists now consider the molar concentrations (i.e. the number of molecules present in 1 litre of fluid) instead of the weight concentrations.

You will recall that

all matter is composed of minute particles called atoms which consist of even smaller fundamental particles called protons, neutrons and electrons.
Atoms consist of:

(i) a "nucleus" containing varying numbers of protons and neutrons;

(ii) one or more "shells" containing varying numbers of orbiting electrons.

The protons in the nucleus carry a positive electrical charge.

The electrons carry a negative electrical charge.

A molecule is the smallest particle of an element or compound which exists in a free state.

One atom of hydrogen cannot exist by itself. It always combines with another atom of hydrogen to form a molecule of hydrogen which can exist alone.

A molecule of a compound must contain at least two different atoms. For example:
A molecule of carbon monoxide consists of one carbon atom and one oxygen atom.
A molecule of carbon dioxide consists of one carbon atom and two oxygen atoms.
A molecule of water consists of two atoms of hydrogen and one atom of oxygen.

Each atom has a property called "atomic mass" or "atomic weight". This may be considered as a ratio between the mass or weight of the atom to that of an atom of hydrogen which is considered to be one (1).

Carbon is considered to be twelve times heavier than hydrogen so its atomic mass or weight is given as twelve (12). Oxygen is considered to be sixteen times as heavy as hydrogen and its atomic mass or weight is given as sixteen (16).

Molecular weight is the sum of the atomic weights of the constituent atoms. We could say the molecular weight of water equals:

\[
(2 \times 1) + (1 \times 16) = 18.
\]

\[
H_2O
\]

A glucose molecule consists of six atoms of carbon plus twelve atoms of hydrogen plus six atoms of oxygen. So the molecular weight of glucose is:

6.14
\[ (6 \times 12) + (12 \times 1) + (6 \times 16) = 180 \]

\[ C_6 \quad H_{12} \quad O_6 \]

When we know the molecular weight of a substance important to the body, we can gain a greater appreciation of its biological activity.

A unit which can reflect the biological activity is called a \textit{mole} and it represents the weight of a chemical in units of mass (usually grams) numerically equal to its molecular weight.

A molecule of water has a molecular weight of 18 so we can say "1 mole of water contains 18 grams of water". So, the number of grams equal to the molecular weight of the molecule represents 1 mole of that molecule. The molecular weight of glucose is 180 so we can say "1 mole of glucose contains 180 grams of glucose".

The mole is rather a large unit for use in human physiology, so the millimole is more frequently used.

The molecular weight in milligrams of the substance constitutes a \textit{millimole} (mmol).

\[
1 \text{ millimole} = \frac{1}{1000} \text{ of a mole.}
\]

When certain blood constituents are expressed in mass per unit volume it would appear that some are present in greater amounts, but further investigation has revealed that the variation is due to differences in molecular weight rather than to the numbers of molecules present. A sample of blood may contain 30 mg/100 mL urea; 90 mg/100 mL glucose and 200 mg/100 mL cholesterol. When these values are considered in terms of the molecules present we find that there are approximately the same number of molecules of urea, glucose and cholesterol present in the blood sample. This means that no matter what substance is being considered, one mole always contains the same number of molecules.

IONS AND ELECTROLYTES

In any atom, the number of protons in the nucleus is equal to the number of electrons surrounding the nucleus. Atoms as a whole are electrically neutral because the positive and negative electrical charges cancel out.

Some atoms can readily lose one or more of their electrons. Other types of atoms can readily pick up one or more electrons. When an atom has picked up or lost electrons it is called an \textit{ion}.
Hydrogen (H), Sodium (Na) and Potassium (K) are elements important in the human body which can each lose an electron. (We could call them "electron donors").

When an electron is lost, or donated, the number of protons in the nucleus does not change. There is therefore one positive charge on a proton which is not balanced by a corresponding negative charge on an electron.

When atoms of hydrogen, sodium and potassium have lost (donated) an electron they are no longer electrically neutral and are called POSITIVE IONS and are sometimes represented by the symbols H⁺; Na⁺; K⁺.

When an atom picks up, receives or gains extra electrons it has additional negative charges which are not balanced by positive charges in the nucleus, so it becomes a negatively charged ion or NEGATIVE ION.

Chlorine (Cl) is an element important in the human body and it readily picks up (receives) an additional electron. The chloride ion which results is often represented by the symbol Cl⁻.

All atoms of elements we call metals tend to lose electrons and become positive ions or cations.

Some positive ions derived from metals which are important in human physiology are sodium, potassium and iron.

Once an atom becomes an ion, its chemical properties change.

Atoms of non-metals have a tendency to gain electrons and become negatively charged for example, chlorine.

The splitting off of electrons from substances in solution can be referred to as DISSOCIATION. Substances which dissociate in this way are generally called ELECTROLYTES because they carry an electrical charge.

Electrolytes are found in all body fluids and are essential to life. Much of the study of physiology relates to the role and balance of the various electrolytes in the body.

Taken as a whole, the healthy body is electrically neutral, i.e. the number of positive ions (cations) and the number of negative ions (anions) are equal. Local imbalances are bound to occur, but they are soon corrected by various physiological processes which act to stabilize the situation. (This tendency of the body to retain a steady state or dynamic equilibrium is called HOMEOSTASIS.)

Electrolyte balance is probably affected in most cases of illness, but only in some conditions is the variation serious enough to drastically upset the normal activities.
of cells or lead to death. Any condition which causes unnatural losses of body fluids will result in loss of electrolytes and therefore upset the delicate balance. The primary aim of treatment of severe vomiting and diarrhoea, burns, haemorrhage and shock is to correct the gross imbalance of electrolytes which occurs due to fluid loss. A patient admitted to hospital with any of these conditions is likely to be placed on a "Fluid and Electrolyte Balance Sheet" like the one shown as Figure 12.5.

Because of the need of cations to balance anions, the concentrations of electrolytes in the body have been expressed in terms of a unit designed to reflect chemical equivalence. If the blood plasma contained only sodium cations and bicarbonate anions, there would need to be enough sodium to combine with the bicarbonate without leaving any sodium or bicarbonate over. To maintain equivalence, we could say "that 1 equivalent of sodium would need to combine with 1 equivalent of bicarbonate". (One equivalent was considered to be the equivalent weight in grams of any element or ion). As it is a large unit and therefore not so appropriate for use in human physiology, the smaller unit of milli-equivalent (mEq) has been preferred. Electrolyte concentrations within the body have often been expressed in milli-equivalents per litre (mEq/L).

Many text books and laboratory reports record electrolyte concentrations in milli-equivalents per litre, but the millimole per litre (mmol/L) has certain advantages and will probably be used more frequently in future.

NOTE: At the close of Chapter 5 we summarised some important facts about the SI system. In this chapter we have noted the exceptions to the rule that the abbreviations for the name of units are not written with capitals unless they begin a sentence. The exception applies to all those units which have been named in honour of a person, e.g., Celsius, Kelvin, Joule, Ampere, Volt, Watt, Newton and Pascal. When we write the unit name in full, no initial capital is used unless the word happens to begin a sentence; but when we use its abbreviation a capital letter is used.
MEASUREMENT OF TIME

In everyday life we are used to our 24 hour day being divided into two twelve hour periods. To distinguish between the two periods we use the abbreviations a.m. and p.m.

Some hospitals have adopted another method of recording time. This method is used in a number of professions including the armed services. Starting from midnight the twenty-four hourly segments are named using the numbers one to twenty-four. Fractions of an hour are still recorded on the basis of 60 minutes to the hour.

The spoken form of these times includes the word "hundred" after the appropriate numeral - for example 1 a.m. is spoken of as "one hundred hours".

The written form of these times may show a dot following whichever numeral between one and twenty-four is appropriate. For example, twelve midday is written as 12.00 hours or 1200 hours.

The written form of the hours one to nine shows a zero in front of the appropriate digit. This means that the written form of these times always shows a total of four digits, two representing the hours and two representing the minutes.

For example:

1 a.m. is written as 01.00 or 0100 hours.
2 a.m. is written as 02.00 or 0200 hours.
6 a.m. is written as 06.00 or 0600 hours.
12 noon is written as 12.00 or 1200 hours.
2 p.m. is written as 14.00 or 1400 hours.
6 p.m. is written as 18.00 or 1800 hours.
10 p.m. is written as 22.00 or 2200 hours.
12 midnight is written as 24.00 or 2400 hours.

When fractions of an hour need to be recorded, the numeral indicating the number of minutes is written to the right of the dot or the digits representing the hour. For example:

5 minutes past one (p.m.) is written as 13.05 or 1305 hours.
20 minutes past one (p.m.) is written as 13.20 or 1320 hours.
A quarter to two (p.m.) is written as 13.45 or 1345 hours.
Between (midnight) 24.00 or 2400 hours and
(1 a.m.) 01.00 or 0100 hours the hour component
is represented by two zeros.

So, a quarter past midnight would be recorded
as 00.15 or 0015 hours.

This method of recording time is favoured in some
hospitals for recording the intervals at which drugs should
be given. It removes any doubt as to whether the dose is
to be given in the morning, afternoon or evening.
Chapter 7

QUANTITIES AND THEIR MEASURES

At the start of Chapter 5 we defined the "number part of a quantity" as its "measure", and then proceeded to look at various metric units of measurement. The examples given in the introductory paragraph demonstrated that we do not have to work with the actual objects, but can use numbers to represent measures and can, therefore, make use of all the properties of numbers which we have examined in earlier chapters.

From what has gone before, it should be obvious that to have meaning, any measurement must possess two characteristics - a numerical value and a unit. The numerical value 25 standing alone has little significance as a measurement, nor does the unit "gram" have much significance alone, but when the two concepts are combined, that is, 25 grams, the statement takes on specific meaning. Expressing this another way we can say that "to measure" is to "assign a number". But before we can assign a number we need to have a "unit of measurement".

RELATED QUANTITIES

Some quantities may be said to be directly related. If, for example, double one quantity corresponds to double the other quantity they are said to be directly related.

Other quantities are said to be inversely related. If, for example, double one quantity corresponds to half the other, you have an inverse relationship.

Consider the following directly related quantities:
(a) 1 tin has a mass of 2.5 kg
2 tins have a mass of 5.0 kg
5 tins have a mass of 12.5 kg
10 tins have a mass of 25 kg
100 tins have a mass of 250 kg

(b) 1 orange costs 5 cents
2 oranges cost 10 cents
5 oranges cost 25 cents
10 oranges cost 50 cents

(c) If a man walks 1 kilometre in half an hour, assuming he continues to walk at the same pace, he will cover 2 kilometres in 1 hour; 4 kilometres in 2 hours; 10 kilometres in 5 hours.

(d) A drug is ordered to be given in the dosage of 0.5 milligram per kilogram of body weight.

A baby weighing 5 kilograms would be given 2.5 mg of drug.

A child weighing 10 kilograms would be given 5 mg of the drug.

Consider the following examples of inversely related quantities.

(a) One man would take 2 days to complete a job. Assuming they worked at the same rate, 2 men would complete the same job in 1 day.

(b) If it takes one nurse 2 hours to make 20 beds, two nurses working together should be able to make the same number of beds in 1 hour. Four nurses should be able to make the 20 beds in half an hour.

(c) With 25 patients in the ward, it takes 2 nurses 20 minutes to take the T.P.R's (temperature, pulse and respiration rates). One nurse would take 40 minutes (assuming that she worked at the same rate).
Consider each of the latter three examples. If there are more people to do a job, the job will be completed in less time, so there is an inverse relationship between the number of persons involved on the job and the time taken to complete the job.

UNITARY METHOD

Problems involving quantities which are either directly or inversely related can be solved by what is known as UNITARY METHOD.

If we are presented with a problem such as:
5 tins have a mass of 12.5 kg, what will be the mass of 25 similar tins?

STEP 1: State in full the information which is given, i.e., the mass of 5 tins is 12.5 kg.

STEP 2: Survey the problem and note that you are dealing with directly related quantities.

STEP 3: Find the mass of one tin, i.e.,
the mass of 1 tin = \( \frac{12.5}{5} \) kg

STEP 4: Having found the mass of 1 tin and noting the direct relationship, find the mass of 25 tins.

This is set out as:
the mass of 5 tins = 12.5 kg
\[ . \text{. the mass of 1 tin} = \frac{12.5}{5} \text{ kg} \]
= 2.5 kg
\[ . \text{. the mass of 25 tins} = (25 \times 2.5) \text{ kg} \]
= 62.5 kg
Consider this problem:
A baby weighing 5 kg is given 2.5 mg of a certain drug. What would be the weight of a baby given 4 mg of the same drug assuming that the dose rate is the same as for the 5 kg baby?

Drug dosage of 2.5 mg = 5 kg total body weight

Drug dosage of 1 mg = \( \frac{5.0}{2.5} \) kg total body weight

= 2 kg

Drug dosage of 4 mg = (4 \times 2) kg

= 8 kg total body weight

Therefore the baby would weigh 8 kg.

Consider this problem:
Five wardsmails can serve the meals to 40 patients in 10 minutes. How long will it take 3 wardsmails?

Note that you have an inverse relationship presented.

Time taken by 5 wardsmails = 10 minutes

\[ \therefore \text{Time taken by 1 wardsmail} = (10 \times 5) \text{ minutes} \]

= 50 minutes

\[ \therefore \text{Time taken by 3 wardsmails} = \frac{50}{3} \text{ minutes} \]

= 16 \( \frac{2}{3} \) minutes

= 17 minutes approx.

Check that the answer is reasonable. Three wardsmails will take longer than 5 wardsmails to complete the same job. Seventeen minutes is longer than 10 minutes, so the answer is reasonable.

These problems have demonstrated the use of unitary method because we have taken a given set of information and used it to find the relationship to one. The relationship to one has then allowed us to find the unknown quantity.
RATIOS

Nurse A receives $120 in her pay packet. Nurse B receives $60 so we can say Nurse A has twice as much as Nurse B; or, Nurse B has half as much as Nurse A.

If baby M weighs 5 kg and baby N weighs 6 kg, then baby M's weight is \( \frac{5}{6} \) of baby N's weight; or, baby N's weight is \( \frac{6}{5} \) of baby M's weight.

When we compare the measures of two quantities by division in this way we form a RATIO.

In the first example we can say: the number 2 is the ratio of the number 120 to the number 60.

This is sometimes shown as 2:1.

The number \( \frac{1}{2} \) is the ratio of the number 60 to the number 120.

This is sometimes shown as 1:2.

In the second example we can say: the number \( \frac{5}{6} \) is the ratio of the number 5 to the number 6.

This is sometimes shown as 5:6.

The number \( \frac{6}{5} \) is the ratio of the number 6 to the number 5.

This is sometimes shown as 6:5.

A ratio is a number. It is the number obtained by dividing one number by another number (not zero).

In the previous chapter we spoke about the relative density being a ratio comparing the density of a liquid against the density of water. As density is a relationship between the mass of a body and the volume of a body we can express the density of water as 1 gram per cubic centimetre.

If we have a solution A, the density of which is 2 grams per cubic centimetre, the ratio between solution A and water is 2:1. The relative density of solution A would be 2.

PROPORTIONS

A proportion is equality of ratios between two pairs of quantities.

Earlier we looked at various quantities which are directly related. We could transpose the words "directly related" for "directly proportional".

7.5
Similarly the terms "inversely related" may be replaced by "inversely proportional".

DIRECT PROPORTION

Looking back to the examples we used earlier we had a series of statements relating to the mass of a number of tins. By taking any two entries from that series of statements we can show the direct relationship.

In one statement, the mass 2.5 kg corresponds to 1 tin.
In another statement, the mass 5.0 kg corresponds to 2 tins.

Ratio of number of tins = \( \frac{1}{2} \)

Ratio of corresponding masses = \( \frac{2.5}{5.0} = \frac{1}{2} \)

\[ \therefore \frac{\text{the ratio of the number of tins}}{\text{the ratio of the corresponding masses}} \]

Take two more entries from the same series of statements.
For example:

The mass 12.5 kg corresponds to 5 tins.
The mass 25 kg corresponds to 10 tins.

Ratio of numbers of tins = \( \frac{5}{10} = \frac{1}{2} \)

Ratio of corresponding masses = \( \frac{12.5}{25} = \frac{1}{2} \)

\[ \therefore \text{ratio of number of tins = ratio of corresponding masses} \]

So we can say the number of tins is directly proportional to mass, and mass is directly proportional to the number of tins.

Take the example of the drug dosages.

The dose of 0.5 mg corresponds to 1 kg body weight.
The dose of 5 mg corresponds to 10 kg body weight.
Ratio of drug dosages \(= \frac{0.5}{5.0} = \frac{1}{10}\)

Ratio of corresponding mass \(= \frac{1}{10}\)

\[
\therefore \text{ratio of drug dosages = ratio of corresponding masses}
\]

So we can say drug dosage is directly proportional to mass of body, and mass of body is directly proportional to drug dosage. 

Direct proportion helps us solve certain problems where we have an unknown quantity.

For example:

I need 7 metres of curtaining material and the cost of 7 metres is $5.60, however I find I only have $4. How much material can I purchase for that amount?

After studying the question, decide whether the quantities are directly proportional. As they are we can say the ratio of lengths equals the ratio of corresponding costs.

Now choose an alphabetical letter to represent the unknown length.

Let \(x = \) unknown length.

(express costs in cents for ease of working)

Ratio of lengths \(= \frac{x}{7}\)  
(\(\frac{400}{560}\)) these ratios are equal 

so \(\frac{x}{7} = \frac{\frac{400}{560}}{\frac{400}{560} = (5 \times 80)}\)

or \(\frac{x}{7} = \frac{\frac{5 \times 80}{7 \times 80}}{\frac{5 \times 80}{7 \times 80}}\)

i.e., \(\frac{x}{7} = \frac{5}{7} \times \frac{1}{1}\) (because M.P. One says \(\frac{80}{80} = 1\))

\[
\therefore x = 5
\]

So $4 will buy 5 metres of material.
An alternative method of working from the stage

\[ \frac{x}{7} = \frac{400}{560} \]

\[ = 7 \times \frac{x}{7} = 7 \times \frac{400}{560} \quad \text{[Multiplying by 7 on either side of the equals sign, gives } x \text{ a denominator of 1.]} \]

\[ = x \times \frac{7}{7} \times \frac{1}{80} \]

\[ = x \times \frac{5}{1} \]

\[ x = 5 \]

\[ \therefore \text{ $4 \text{ will buy 5 metres of material.}} \]

Disposable syringes cost 92 cents per dozen. What will be the cost of 20 dozen?

Choose a variable and summarise the information given in similar terms.

Let 20 dozen (240) syringes cost \( x \) cents

12 syringes cost 92 cents

240 syringes cost \( x \) cents

As the quantities are directly proportional - ratio of costs = ratio of number of syringes

so

\[ \frac{x}{92} = \frac{240}{12} \]

\[ \therefore \frac{x}{92} = \frac{60 \times 4}{3 \times 4} \quad \text{[240 = } 60 \times 4, \quad 12 = 3 \times 4]} \]

or

\[ \frac{x}{92} = \frac{60}{3} \quad \text{(because M.P. One says } \frac{4}{4} = 1) \]

so

\[ 92 \times \frac{x}{92} = \frac{92 \times 60}{3} \quad \text{[Multiplying by 92 on either side of the equals sign gives } x \text{ a denominator of one]} \]

7.8
\[
\frac{1}{92} \times x \cdot \frac{20}{92} = \frac{92 \times 60}{3} \\
\]
so \[
x = 92 \times 20 \\
x = 1840 \\
x = 1840 \text{ cents} \\
\therefore x = \$18.40
\]

Most people would look at this problem and make some quick mental calculations and give the answer. It has been set out in detail as an example of the method which can be used to simplify more difficult problems.

The mental arithmetic would probably be done in the following stages.

20 dozen = \((2 \times 10)\) dozen

10 dozen at 92 cents per dozen = 920 cents or \$9.20

\$9.20 \times 2 = \$18.40

Some people prefer to tackle these problems a different way when direct proportions are involved. They reason from two known statements such as those with which we commenced this chapter.

For example if 1 orange costs 5 cents and 2 oranges cost 10 cents it can be seen that the product of the diagonally opposite numbers are equal, i.e.,

\[2 \times 5 = 1 \times 10.\]

When given any three of these values, the fourth can be found. Again, the unknown quantity is represented by an alphabetical letter.

Suppose we need to find the cost of two oranges. The information will be set out as:-

1 orange costs 5 cents
2 oranges cost \(x\) cents (\(x\) represents the unknown quantity.)
If the products of the diagonally opposite numbers
are equal:

\[ 1 \times x = 2 \times 5. \]

To find the value of \( x \), multiply the right hand side
of the equation by the reciprocal of the number in the
left hand equation - in this case, one.

\[ \therefore x = 2 \times 5 \times 1 \]

\[ \therefore x = 10 \text{ cents} \]

Working another example by this method:
If two tins have a mass of 5 kg, how many similar tins
have a mass of 25 kg?

Let \( x \) equal the unknown number;
then 2 tins have a mass of 5 kg
so \( x \) tins have a mass of 25 kg

\[ x \times 5 = 2 \times 25 \]

\[ x = 2 \times 25 \times \frac{1}{5} \] (multiply by
the reciprocal
of 5)

\[ = 2 \times 5 \times \frac{5}{5} \times \frac{1}{5} \]

\[ = 2 \times 5 \times \frac{1}{1} \]

\[ = 10 \]

Therefore 10 tins have a mass of 25 kg.

This method may only be used when the quantities
are directly related.

By gathering together information from the preceding
examples we can summarise some facts about ratios and
proportion.

A convenient method of comparing the size or
magnitude of like quantities or values is to state the
ratio of one to the other.

The ratio of one number to another is the comparison
of one quantity or value to another quantity or value.
From the definition of ratio shown on page 7.5 we can see that a ratio may be considered as a fraction.

A ratio may be expressed in different ways. For example, "the ratio of 2 to 6" may be expressed as:-

\[
\frac{2}{6}; \quad 2:6; \quad 2-6
\]

In the latter case, the context must make it clear that the dash is not to be considered as a minus sign.

The finding of an equivalent fraction may be considered as a problem in proportion.

If we refer back to the diagrams on page 4.3 of Chapter 4 we can see again that:

\[
\frac{1}{2} = \frac{2}{4}
\]

\[
\frac{2}{4} = \frac{4}{8}
\]

\[
\frac{4}{8} = \frac{8}{16}
\]

This may be expressed in other ways. We can say "one is to two as two is to four". This can be written as:

\[
1:2 = 2:4
\]

or \[
1:2 :: 2:4
\]

again \[
2:4 = 4:8
\]

or \[
2:4 :: 4:8
\]

When a proportion is written as above, the terms may be named:

The first and fourth terms are called the "extremes".

The second and third terms are called the "means".

To solve problems in proportion we may use the basic rule - "The product of the 'means' equals the product of the 'extremes'".

\[
7.11
\]
In the latter examples, 2 and 8 are the 'extremes' while 4 and 4 are the 'means'.
We can see that the indicated product of 2 and 8 equals the indicated product of 4 and 4.
Turn back to the examples in direct proportion in this chapter and we will restate the examples given.
The following statements can be summarised:-

"the mass of 2.5 kg corresponds to 1 tin"
"the mass of 5.0 kg corresponds to 2 tins"
so 2.5 is to 1 as 5 is to 2
or 2.5:1 :: 5:2.

Note that the product of means (1 x 5) equals the product of the extremes (2.5 x 2).

"the mass of 12.5 kg corresponds to 5 tins"
"the mass of 25 kg corresponds to 10 tins"
so 12.5 is to 5 as 25 is to 10
or 12.5:5 :: 25:10.

Turning to the examples of drug dosages in relation to mass, the information may be summarised as:-

0.5:1 :: 5:10

Let us assume now that one of the essential terms of the ratio is unknown. The problem could be posed: 'If a dose of 0.5 mg is given for every kilogram of body weight, what would be the mass of a child given 5 mg of drug?'

(a) Let $x$ represent the unknown mass.

(b) Summarise the information:

0.5:1 :: 5:$x$

(c) If the product of the means equals the product of the extremes we can say:-

7.12
\[ 0.5 \times x = 1 \times 5 \]
\[ x = \frac{1 \times 5}{0.5} \]
\[ x = \frac{(1 \times 5 \times 10)}{(0.5 \times 10)} \]
\[ = \frac{50}{5} \]
\[ = 10 \text{ kg} \]

Just for practice we will find the unknown terms in the following proportions.

Find the value of \( x \) given the following values.
\[ \frac{1}{60} : \frac{1}{20} :: 30 : x \]

Using the rule "the product of the means equals the product of the extremes" we can say:-
\[ \frac{1}{60} \times x = \frac{1}{20} \times 30 \]
\[ x = \frac{1 \times 30 \times 60}{20} \]
\[ = 90 \]
\[ \text{so} \frac{1}{60} : \frac{1}{20} :: 30 : 90 \]

Check to see that the answer gained is reasonable. As \( \frac{1}{20} \) is greater than \( \frac{1}{60} \), so 90 is greater than 30, showing that the answer is reasonable.

If desirable, the above ratios may be simplified.

\[ \frac{1}{60} : \frac{1}{20} \text{ can be re-expressed as } \frac{1}{60} : \frac{3}{60} \text{ (because } \frac{1}{20} = \frac{3}{60} \)
\[\frac{1}{60} : \frac{3}{60}\] can be re-expressed as \(1 : 3\)

\[30 : 90\] can be re-expressed as \(1 : 3\)

Find the value of \(x\) given \(0.19 : x :: 15.2 : 8.8\)

\[
\begin{align*}
0.19 \times 8.8 &= x \times 15.2 \\
1.672 &= x \times 15.2 \\
\frac{1.672}{15.2} &= x \\
0.11 &= x
\end{align*}
\]

\[0.19 : 0.11 :: 15.2 : 8.8\]

Check again that the answer is reasonable.

**Inverse Proportion**

Consider the problem of the time taken to make beds. The number of nurses and the time taken are inversely related. If the time is halved, the number of nurses must be doubled, or if the number of persons is doubled, the time taken is halved.

The information given is that 2 nurses take 1 hour and 4 nurses take half an hour.

The working may be easier to follow if we express the time in minutes, i.e., 1 hour = 60 minutes; \(\frac{1}{2}\) hour = 30 minutes.

2 nurses correspond to 60 minutes
4 nurses correspond to 30 minutes

Ratio of nurses = \(\frac{2}{4} = \frac{1}{2}\)

Ratio of correspondence times = \(\frac{60}{30} = 2\)

Note that \(\frac{1}{2}\) is the reciprocal of 2 and 2 is the reciprocal of \(\frac{1}{2}\).

\[
\text{the ratio of the number of nurses} = \text{reciprocal of the ratio of the corresponding times.}
\]

\(7.14\)
Consider the inverse proportion demonstrated by comparing the time taken by one nurse and that taken by 6 nurses to complete the same task.

1 nurse corresponds to 2 hours (120 mins)
6 nurses correspond to $\frac{1}{3}$ hours (20 mins)

Ratio of nurses = $\frac{1}{6}$

Ratio of time = $\frac{120}{20} = 6$

Note that $\frac{1}{6}$ is the reciprocal of 6 and 6 is the reciprocal of $\frac{1}{6}$.

\[
\begin{align*}
\text{the ratio of the} & \quad = \quad \text{reciprocal of the ratio} \\
\text{number of nurses} & \quad = \quad \text{of corresponding times}
\end{align*}
\]

We can summarize this by saying that:

(a) The time taken to complete a given task is inversely proportional to the number of people involved in the task.

(b) The number of people involved with a particular task is inversely proportional to the time needed to complete that task.

Now we will see how inverse proportions can be used to solve problems.

A team of painters plan to paint the inside of a ward. It is estimated that the usual team of 4 painters would take 10 days to complete the job. It is considered that 4 days is the maximum period for the ward to be closed. How many painters would need to be put onto the job to complete it in 4 days?

Choose a letter of the alphabet to represent the unknown quantity. For example: Let $x$ equal the unknown number of painters.

If 4 painters take 10 days

$x$ painters take 4 days.
After careful consideration we can see that the quantities are inversely related, so we write the inverse proportion.

\[
\text{ratio of painters} = \text{reciprocal of ratio of corresponding times}
\]

\[
\therefore \quad \frac{x}{4} = \text{reciprocal of} \quad \frac{4}{10}
\]

i.e. \[
\frac{x}{4} = \frac{10}{4} \quad \left[\begin{array}{l}
10 = 5 \times 2 \\
4 = 2 \times 2
\end{array}\right]
\]

so \[
\frac{x}{4} = \frac{5 \times 2}{2 \times 2}
\]

\[
\therefore \quad \frac{x}{4} = \frac{5}{2} \quad (\text{because M.P. One says} \quad \frac{2}{2} = 1)
\]

\[
\frac{4 \times x}{4} = \frac{4 \times 5}{2} \quad \text{[multiplying by 4 on either side of the equation gives x]} \\
\text{a denominator of one and retains the correct proportion]}
\]

\[
\frac{1}{K} \times x = \frac{2}{K_1} \times 5
\]

\[
x = 2 \times 5
\]

\[
x = 10
\]

Therefore, 10 painters will be needed to complete the job in 4 days.

Check this problem by working with the unitary method.

4 painters take 10 days

1 painter takes (10 x 4) days - (inverse relationship)

10 painters take 4 days
The relationship between pressure and volume of gases is an inverse one. This can be demonstrated by a simple experiment using a syringe.

If we take a 10 mL syringe and withdraw the plunger to the 10 mL marking we can say the syringe contains 10 mL of air. The pressure of that air will be the same as the pressure of the atmosphere outside the syringe. If the end of the syringe is sealed by placing a finger over it, the plunger can be pushed in to the 5 mL mark and held there, thus compressing the gas. The increase in pressure can be felt by the finger covering the end of the syringe.

FIG. 7.1(a) Syringe with plunger withdrawn to 10 mL. Air inside at atmospheric pressure.

FIG. 7.1(b) Syringe with end sealed and plunger pushed into and held at the 5 mL mark. Volume halved, pressure doubled.

If the finger is released, air rushes out from the syringe until the air inside is once again at atmospheric pressure.

Next, with the plunger of the syringe at the 5 mL mark cover the end of the syringe and withdraw the plunger to the 10 mL mark and hold it there. The air inside the syringe now has to occupy twice the space and so its pressure is reduced to half. The effect of this
FIG. 7.2(a) Syringe containing 5 mL of air at atmospheric pressure.

FIG. 7.2(b) Syringe sealed with plunger withdrawn from 5 mL to 10 mL. Volume doubled, pressure halved.

reduction in pressure can be felt by the sucking sensation on the finger sealing the end of the syringe. When the finger is removed this time, air will rush into the syringe until the pressures inside and outside are again equal.

These experiments have demonstrated some very important facts about the behaviour of gases.

1. Gases can be compressed, and when this happens their pressure rises.

2. If a given quantity of gas has to occupy a larger volume its pressure falls.

3. Gases flow from regions where the pressure is high to regions where the pressure is lower. The flow continues until the two pressures are equal.

From these experiments we can say "for any given mass of air, the volume is inversely proportional to the pressure".
If a mass of air at a pressure of 102 kPa has a volume of 340 litres, what volume will it have when the pressure is increased to 124 kPa?

The problem is dealing with quantities which are inversely proportional. Select an alphabetical letter to represent the unknown quantity.

Let 'a' = the unknown volume of gas.

At 102 kPa the volume of air is 340 litres.

\[ a \]  

... At 124 kPa the volume of air is 'a' litres.

**Ratio of volumes** = **reciprocal of ratios of corresponding pressures**

\[
\frac{a}{340} = \text{reciprocal of } \frac{124}{102}
\]

\[ \therefore \frac{a}{340} = \frac{102}{124} \]

\[ \frac{340 \times a}{340} = \frac{340 \times 102}{124} \]

\[ \frac{1}{340 \times a} = \frac{170}{124} \]

\[ \frac{31}{340 \times a} = 62 \]

\[ a = \frac{170 \times 51}{31} \]

\[ a = \frac{8670}{31} \]

\[ a = 279.68 \text{ litres} \]

(correct to two decimal places)

**NOTE:** This answer is reasonable in the light of the properties of gases we discovered by our previous experiments. An increase in pressure results in a decrease in volume.

(cont'd on next page)
124 kPa is greater than 102 kPa.

279.68 litres is less than 340 litres.
Chapter 8

PERCENTAGES AND FURTHER APPLICATIONS OF PROPORTIONS

In previous chapters we have seen that a rational number can be represented as a fraction or as a decimal. It is also possible to represent a rational number as a percentage. The words "per cent" (%) stand for "per hundred", "in every hundred", "divided by 100" or "hundredth".

So \( \frac{7}{100} \) is another way of writing 7 in every hundred; \( \frac{7}{100} \); or 0.07. That means \( \frac{7}{100} \) and 0.07 are three numerals for the same number.

\( 3 \frac{1}{2} \% \) is another way of writing \( 3 \frac{1}{2} \) or 3.5 in every hundred. So \( 3 \frac{1}{2} \%, \frac{3}{2} \frac{}{100} \; \frac{3.5}{100} \) and 0.035 are five numerals for the same number.

125% is another way of writing 125 in every 100.

So 125%, \( \frac{125}{100} \) and 1.25 are three numerals for the same number.

100% is another way of writing 100 in every 100.

So 100%, \( \frac{100}{100} \) and 1 are three numerals for the same number.

To change a percentage to a decimal, divide the number by the second power of ten, i.e., 100.

For example: Express 126% as a decimal:

\[ 126\% = \frac{126}{100} = 126 : 10^2 = 1.26. \]

Express \( 8 \frac{1}{2} \% \) as a decimal.

\[ \frac{8 \frac{1}{2}}{3} = \frac{8.3}{100} = 8.3 : 10^2 = 0.083. \]
Express $12\frac{1}{2}$% as a decimal.

\[
12\frac{1}{2}\% = \frac{12\frac{1}{2}}{100} = \frac{12.5}{100} = 12.5 \div 10^2 = 0.125
\]

Express $\frac{1}{4}$% as a decimal.

\[
\frac{1}{4}\% = \frac{\frac{1}{4}}{100} = \frac{0.25}{100} = 0.25 \div 10^2 = 0.0025
\]

If \( r \) is a rational number;

then \( r\% = \frac{r}{100} \) or \( r \times \frac{1}{100} \)

so $25\% = \frac{25}{100}$ or $25 \times \frac{1}{100}$

Change $\frac{1}{5}$ to a percentage.

\[
\frac{1}{5} = \frac{1}{5} \times \frac{100}{100} \quad \text{(M.P. One says $\frac{100}{100} = 1$)}
\]

\[
= \frac{100}{5} \times \frac{1}{100}
\]

\[
= 20 \times \frac{1}{100} \quad \text{(The 100 in the denominator is not simplified because it is replaced by the % symbol.)}
\]

\[
\frac{1}{5} = 20\%
\]

Change 1.75 to a percentage.

\[
1.75 = 1.75 \times \frac{100}{100}
\]

\[
= (1.75 \times 100) \times \frac{1}{100}
\]
\[ = 175 \times \frac{1}{100} \]

\[ 1.75 = 175\% \]

**Change 0.16 to a percentage.**

\[ 0.16 = 0.16 \times \frac{100}{100} \]

\[ = (0.16 \times 100) \times \frac{1}{100} \]

\[ = 16.6 \times \frac{1}{100} \]

\[ 0.16 = 16.6\% \]

**MULTIPLICATION AND PERCENTAGES**

There are two usual ways of approaching problems involving multiplication by percentage or finding the percentage of a given number.

In the first method, the percentage is expressed as a fraction, while in the second method, the decimal equivalent of the percentage is used.

**EXAMPLES:**

1. **Find 2\% of 50.**

   (a) \[ 2\% = \frac{2}{100} \text{ or } \frac{1}{50} \]

   \[ \frac{2}{100} \times 50 = 1 \text{ OR } \frac{1}{50} \times 50 = 1 \]

8.3
(b) \( 2\% = 0.02 \)

\[
0.02 \times 50 = (0.02 \times 10) \times 5 \\
= 0.2 \times 5 \\
= 1.0 \\
= 1
\]


(a) \( 10\% = \frac{10}{100} \) or \( \frac{1}{10} \)

\[
\frac{1}{10} \times 2000 = 200
\]

(b) \( 10\% = 0.1 \)

\[
0.1 \times 2000 = (0.1 \times 1000) \times 2 \\
= 100 \times 2 \\
= 200
\]

3. Find 2\(\frac{1}{2}\)\% of 1000.

(a) \( 2\frac{1}{2}\% = \frac{2.5}{100} \) or \( \frac{5}{200} \) or \( \frac{1}{40} \)

\[
\frac{1}{40} \times 1000 \quad \text{OR} \quad \frac{5}{200} \times 1000 \\
\frac{1}{40} \times 1000 = 25 \quad \frac{5}{200} \times 1000 = 25
\]

\[
\frac{25}{1} \quad \frac{5}{1}
\]
(b) \( \frac{1}{4}\% = 2.5\% = 0.025 \)
\[ 0.025 \times 1000 = 25 \]

4. Find 75\% of 624.

(a) \[ 75\% = \frac{75}{100} \text{ or } \frac{3}{4} \]
\[ \frac{75}{100} \times 624 = 468 \quad \text{OR} \quad \frac{3}{4} \times 624 = 468 \]

(b) \[ 75\% = 0.75 \]
\[ 0.75 \times 624 = 468 \]

<table>
<thead>
<tr>
<th>624</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.75</td>
</tr>
<tr>
<td>436.8</td>
</tr>
<tr>
<td>468.00</td>
</tr>
</tbody>
</table>

EQUIVALENT RATIO AND PERCENTAGE STRENGTHS

At the beginning of this chapter we saw that a percentage was a way of writing the information "per hundred", "in every hundred", "divided by one hundred" or "hundredth".

Perhaps you realized at the time that the percentage could also be written as a ratio. For example, we wrote seven per cent as \( \frac{7}{100} \) and 0.07.

7\% can also be written as 7:100 or 7-100.
In addition to the numerals shown on page 1 of the chapter 3\%\% can be written as 3.5:100 or 3.5-100 or even as 35:1000 or 35-1000.
2.5\% can be written as 2.5:100 (2.5-100), 25:1000 (25-1000) or 1:40 (1-40). The latter ratio is the simplest and so is the preferable alternative to 2.5\%.
Note the ratio strengths equivalent to the following

8.5
percentage strengths:

\[50\% = \frac{50}{100} = \frac{1}{2} \quad (50-100) = 1:2 \quad (1-2)\]

\[5\% = \frac{5}{100} = \frac{1}{20} \quad (5-100) = 1:20 \quad (1-20)\]

\[0.5\% = \frac{0.5}{100} = \frac{1}{200} \quad (0.5-100) = 1:200 \quad (1-200)\]

\[0.05\% = \frac{0.05}{100} = \frac{1}{2000} \quad (0.05-100) = 1:2000 \quad (1-2000)\]

Check these by applying the rule "the product of the means equals the product of the extremes".

\[
50:100 :: 1:2
\]

so \[50 \times 2 = 100 \times 1\]

\[100 = 100\]

\[
5:100 :: 1:20
\]

so \[5 \times 20 = 100 \times 1\]

\[100 = 100\]
0.5:100 :: 1:200
so 0.5 \times 200 = 100 \times 1
\therefore 2(0.5 \times 100) = 100 \times 1
2 \times 50 = 100 \times 1
100 = 100

0.05:100 :: 1:2000
so 0.05 \times 2000 = 100 \times 1
\therefore 2(0.05 \times 1000) = 100 \times 1
2 \times 50 = 100 \times 1
100 = 100

Express the percentage strengths equivalent to the following ratio strengths.

1:10 = \frac{1}{10} \times \frac{100}{100} \quad (M.P. One says \frac{100}{100} = 1)
= \frac{100}{10} \times \frac{1}{100}
= 10 \times \frac{1}{100} \quad (The 100 in the denominator is not simplified because it is replaced by the % symbol.)
1:10 = 10$

1:200 = \frac{1}{200} \times \frac{100}{100}
= \frac{100}{200} \times \frac{1}{100}
= \frac{1}{2} \times \frac{1}{100}
1:200 = \frac{1}{2} \% \text{ OR } 1:200 = 0.5\%
1:1000 = \frac{1}{1000} \times \frac{100}{100} = \frac{100}{1000} \times \frac{1}{100} = \frac{1}{10} \times \frac{1}{100} \Rightarrow 1:1000 = \frac{1}{10\%} = \text{OR} \ 1:1000 = 0.1\% \\
1:5000 = \frac{1}{5000} \times \frac{100}{100} = \frac{100}{5000} \times \frac{1}{100} = \frac{1}{50} \times \frac{1}{100} \Rightarrow 1:5000 = \frac{1}{50\%} = \text{OR} \ 1:5000 = 0.02\% \\
1:12 = \frac{1}{12} \times \frac{100}{100} = \frac{100}{12} \times \frac{1}{100} = \frac{25}{3} \times \frac{1}{100} \Rightarrow 1:12 = 8\frac{1}{3}\% = \text{OR} \ 1:12 = 8.3\%

Perhaps you would rather tackle these a different way. Because the percentage symbol means "so many in one hundred" we can express the unknown percentage as the ratio x:100.
so \(1:10 :: x:100\)

i.e., \(1 \times 100 = 10 \times x\) (Because the product of the means equals the product of the extremes.)

\[
\frac{100}{10} = x
\]

\[
\therefore 10 = x
\]

\[
\therefore 1:10 = 10:100
\]

so \(1:10 = 10\%\)

\[1:200 :: x:100\]

\[200 \times x = 1 \times 100\]

\[
\frac{200 \times x}{200} = \frac{1 \times 100}{200}
\]

\[
x = \frac{1 \times 100}{200}
\]

\[
x = \frac{1}{2} \text{ OR } 0.5
\]

\[
\therefore 1:200 = 0.5:100
\]

so \(1:200 = 0.5\%\)

\[1:1000 :: x:100\]

\[1000 \times x = 1 \times 100\]

\[
\frac{1000 \times x}{1000} = \frac{1 \times 100}{1000}
\]

\[
x = \frac{1 \times 100}{1000}
\]

\[
x = \frac{1}{10} \text{ OR } 0.1
\]

\[
\therefore 1:1000 = 0.1:100 \text{ SO } 1:1000 = 0.1\%
\]

8.9

\[\text{© BY} \]
\[ \frac{1:5000}{5000} \times x = \frac{1 \times 100}{5000} \]

\[ x = \frac{1 \times 100}{5000} \]

\[ x = \frac{1}{50} \text{ OR } 0.02 \]

\[ \therefore \frac{1:5000}{5000} = 0.02:100 \]

so \[ \frac{1:5000}{5000} = 0.02\% \]

\[ \frac{1:12}{12} :: x:100 \]

\[ 12 \times x = 1 \times 100 \]

\[ \frac{12 \times x}{12} = \frac{1 \times 100}{12} \]

\[ x = \frac{1 \times 100}{12} \]

\[ x = 8\frac{1}{3} \text{ OR } 8.3 \]

\[ \therefore \frac{1:12}{12} = 8.3:100 \]

so \[ \frac{1:12}{12} = 8.3\% \]

**APPLICATION OF PERCENTAGE**

In an examination, 65% of the students passed, but 14 students failed. How many students passed?

Let \( x \) students = the number of students who passed.

Then \( x \) students = 65% of the group.

14 students = \((100-65)=35\)% of the group.

Ratio of students = ratio of corresponding percentages.
\[
\frac{x}{14} = \frac{65}{35}
\]

\[
= \frac{14 \times x}{14} = \frac{14 \times 65}{35}
\]

\[
= \frac{1}{\frac{14 \times x}{14}} = \frac{13}{\frac{14 \times 65}{35}}
\]

\[
= \frac{2}{1} = \frac{13}{\frac{7}{1}}
\]

\[x = 26\]

\[\therefore \text{No. of students who passed} = 26\]

Approximately 70% of the human body is water. If a man weighs 70 kg, how many kilograms is water?

Let \[x = \text{the unknown weight in kg}.
\]

Then \[x \times 70 = 70\%
\]

so \[70 \text{ kg} = 100\%.
\]

Ratio of weights = ratio of corresponding percentages.

\[
\frac{x}{70} = \frac{70}{100}
\]

\[
\frac{1}{\frac{70 \times x}{70}} = \frac{70 \times 7}{100}
\]

\[
= \frac{7}{1} = \frac{70 \times 7}{100}
\]

\[x = 7 \times 7
\]

\[x = 49 \text{ kg}
\]

Express the 49 kg of water in terms of litres.

Refer back to Chapter 5 where we found 1 kg to be the weight of 1 litre of water.

\[\therefore 49 \text{ kg is the weight of 49 litres of water.}\]
So a man weighing approximately 70 kg contains approximately 49 litres of water.

Some students will probably see a shorter way of solving this problem.

If 70% of the body is water that means \(\frac{70}{100}\) of the body is water.

If the total weight is 70 kg,

Then \(\frac{70}{100}\) of 70 kg \(\left(\frac{\text{of}}{100}\right)\) is another way of saying "find the product of".

\[
\frac{70 \times 70}{100} = 49 \text{ kg}
\]

In Chapter 6 we discussed how electrolytes within the body fluids are usually described in values of millimoles or (milliequivalents per litre). Other constituents are expressed either in percentages of total volume or in grams or milligrams per 100 mL (frequently abbreviated to "grams \%" or "mg \%".)

Blood is composed of plasma and blood corpuscles.
The plasma constitutes approximately 55% of the total volume of the blood.

In 500 mL of blood what would be the volume of blood cells (corpuscles)?

Let \(c\) mL represent the volume of blood corpuscles.
The percentage of corpuscles in 500 mL = (100-55) 45%.
Then \(c\) mL = 45%

so 500 mL = 100%.

Ratio of millilitres = corresponding ratio of percentages.

\[
\frac{c}{500} = \frac{45}{100}
\]

\[
\therefore \frac{500 \times c}{500} = \frac{500 \times 45}{100}
\]

\[
c = 5 \times 45
\]

\[
c = 225 \text{ mL}
\]

8.12
... volume of corpuscles in 500 mL of blood
= 225 mL.

For those who wish to reason the shorter way:

If whole blood = plasma + corpuscles

then 100% = 55 + (100 - 55)

= 55 + 45

... % of corpuscles = 45%

45% of 500 mL = \(\frac{45 \times 500}{100}\)

= 225 mL

The normal level for haemoglobin in the male is 15 grams per 100 mL. What percentage below normal would be a reading of 12 grams per 100 mL?

Let \(x\) represent the unknown percentage.

Let \(x\% = 12\) g per 100 mL

and 100% = 15 g per 100 mL

Ratio of percentages = ratio of corresponding grams per 100 mL.

\[
\frac{x}{100} = \frac{12}{15}
\]

\[
1 \times \frac{x}{100} = \frac{100}{1} \times \frac{12}{15}
\]

\[
x = 20 \times \frac{4}{5}
\]

\[
x = 80\%
\]

... 12 grams per 100 mL is equivalent to a haemoglobin level in a male of 80%.

8.13
Normal life processes require approximately 1 square metre of alveolar surface in the lungs per kilogram of body weight.

A person weighing 60 kilograms is estimated to have only 75% healthy lung tissue. What area of alveolar surface must be incapable of carrying out its function?

From the information given, a person weighing 60 kg should have an effective area of 60 square metres.

Let $x$ represent the area of ineffective tissue.

then $x \text{ m}^2 = (100 - 75) = 25\%$ of the total area

$60 \text{ m}^2 = 100\%$

Ratio of square metres = ratio of corresponding percentages

\[
\frac{x}{60} = \frac{25}{100}
\]

\[
\frac{1}{60} \times x = \frac{3}{60} \times \frac{5}{100}
\]

\[
\frac{1}{60} \times x = \frac{3 \times 5}{60 \times 100}
\]

\[
x = 3 \times 5
\]

\[
x = 15 \text{ m}^2
\]

\[\therefore \text{ 15 square metres of alveolar surface must be incapable of carrying out its functions.}\]

There are other ways in which this problem might be solved.

A person may choose to determine the area of healthy tissue (i.e., 75% of the lung tissue) and then subtract this area from the total area of 60 square metres.

For example:
75% of 60 square metres = \( \frac{75}{100} \) of 60 m\(^2\)

\[
\frac{3}{100} \times 60 \equiv \frac{25}{100} \times \frac{60}{4} = 45 \text{ m}^2
\]

If 45 m\(^2\) = effective area of lung tissue
and 60 m\(^2\) = total area of lung tissue
then (60-45)m\(^2\), i.e., 15 m\(^2\) = the ineffective area of lung tissue.

In a mixture of two or more gases, each component in the mixture acts as though it was the only gas present. The total pressure of the mixture is the sum of all the partial pressures of each of the gases.

The air we breathe is a good example of a mixture of gases. The composition of air is often quoted as approximately 79% nitrogen, 20% oxygen, 0.04% carbon dioxide, plus traces of other gases.

If the pressure of air is given as 760 mm mercury (mm Hg), what is the partial pressure of oxygen?

Let \( x \) represent the unknown pressure of oxygen
then \( \frac{x}{760} \) mm Hg represents the partial pressure of 20% oxygen
and 760 mm Hg represents the total pressure of 100% air.

Ratio of pressures = ratio of corresponding percentages

\[
\frac{x}{760} = \frac{20}{100} \Rightarrow \frac{1}{760} \times x = \frac{152}{760} \times \frac{1}{100} \Rightarrow \frac{x}{760} = \frac{152}{100} \times \frac{1}{5} \Rightarrow 8.15
\]
\[ x = 152 \text{ mm Hg}. \]

The partial pressure of oxygen in air at 760 mm Hg pressure is 152 mm Hg.

Solutions for use as disinfectants and antiseptics are sometimes expressed in percentages.

A 50% solution is a means of stating that in every 100 parts of solution there would be 50 parts of pure solution - the other 50 parts would be a suitable solvent - for example, water. This might also be expressed as "for every 100 mL of solution, there will be 50 mL of pure solution".

A 5% solution is a means of stating that in every 100 parts there would be 5 parts of pure solution. This might also be expressed as "for every 100 mL of solution there will be 5 mL of pure solution".

An 0.5% solution is a means of stating that in every 100 parts of solution there would be only 0.5 parts of pure solution. This could also be stated as "for every 100 mL of solution there will be 0.5 mL of pure solution" or "for every 200 mL of solution there will be 1 mL of pure solution", or "for every 1000 mL of solution there will be 5 mL of pure solution".

FURTHER EXAMPLES OF PROPORTIONS

Throughout the study of nursing, both in theory and practice, we meet a number of situations which are examples of direct or indirect proportions.

Let us look at some practical examples of directly related factors.

1. OSMOTIC PRESSURE OR OSMOTIC PULL

This relationship is best demonstrated by experiment.

Take four identical containers and place in each a cylinder, the lower end of which is covered by a membrane which allows only water molecules to cross it - this is known as a semi-permeable membrane.

Into two of the containers place an equal volume of water.

Into the other two containers place an equal volume of protein solution which contains 5 grams of protein per litre.

Into the cylinders in containers A and C we place
some protein solution in the strength of 10 grams of protein per litre.

All four experiments are left for 24 hours and then the results are noted.

See Figure 8.1 for the situation at the beginning of the experiments, and 24 hours later.

In container A, the level of fluid in the cylinder containing the 5 g/L protein solution has risen by x mm.

In container B, the level of the fluid in the cylinder containing the 10 g/L protein solution has risen to twice the height of that in the cylinder in container A — i.e., 2x mm.

There is no change in the level of the fluid in the cylinder in container C.

In the cylinder in container D there is again a rise in the level of fluid of the same magnitude as in container A.

The force which has moved water from the containers A, B and D into the cylinders within these containers is called osmotic pressure or osmotic pull.

In container B, where the strength of the protein solution in the cylinder was doubled, the osmotic pressure was doubled.

The osmotic pressure depends on the concentration difference between the two solutions on either side of the semi-permeable membrane.

When concentrations are equal as in experiment C, no osmotic pressure develops.

When concentrations differ, the greater the concentration difference, the higher is the osmotic pressure. In container D, the concentration difference is the same as in container A.

2. TRANSFER OF HEAT

When two objects are in actual contact, heat flows from the hotter object to the colder one. For example, if you place your hand into cold water heat will pass from your hand to the water. The greater the temperature difference, the more rapid is the heat flow.

If the temperature of your hand is 37°C and the temperature of the water is 37°C no heat will be transferred from your body to the water or from the water to your body.
FIG. 8.1 Series of Experiments Demonstrating Osmotic Pressure
If your body temperature remains at 37°C and your hand is placed into water at 36°C there will be a slight loss of heat from your hand to the water.

If the water is cooled to 27°C, i.e., a difference of 10 degrees, heat will flow away from the body ten times more rapidly and the water will appear much colder.

3. RELATIONSHIP OF HEAT LOSS TO VOLUME AND SURFACE AREA

Heat is produced by the tissues throughout the whole volume of the body, but a considerable amount is lost across the body surface.

Babies, particularly premature babies, have difficulty in maintaining their body temperature compared to an adult.

A simple demonstration may help you to understand this. Examine these two cubes and the details given.

![Two Cubes Diagram]

**FIG. 8.2** Two Cubes

The small cube with its sides of 1 cm has a volume of 1 cubic centimetre. The larger cube with its sides of 4 cm has a volume of 64 cubic centimetres. In other words, this large cube contains 64 times as much material as the small cube. This is similar to the ratio of the mass of a grown man to the mass of a premature baby.

If we imagine heat being produced within each cube, we can compare the heat loss through the surface area to the heat produced within the cube.
Each cube has 6 sides. Each side of the small cube has an area of 1 square centimetre while each side of the large cube has an area of 16 square centimetres.

The total surface area of the small cube is $(1 \times 6) \times 6 \, \text{cm}^2$, while the total surface area of the large cube is $(16 \times 6) \times 96 \, \text{cm}^2$. For every cubic centimetre in the large cube there is $1.5 \, \text{cm}^2$ of surface area, whereas for every cubic centimetre in the small cube there is $6 \, \text{cm}^2$ of surface area.

Thus, in relation to volume, there is four times the surface area in the small cube as in the larger one, therefore heat loss from the smaller cube would be greater than heat loss from the larger cube.

The following is an example of an inversely related factor. In health, the relative density of urine is inversely proportional to the volume secreted. The greater the volume of urine secreted, the lower its relative density. A symptom of the disease, diabetes mellitus (sugar diabetes), is a greater volume of urine with a high relative density.

This chapter is not attempting to highlight all the examples of proportion which a nurse may meet throughout her study. It is designed rather to stimulate a sense of enquiry so that you will see how a knowledge of mathematical principles can help in understanding other areas of study and will be constantly on the watch for similar examples.
Chapter 9

Calculation of Drug Dosages

One of many of a nurse's important duties is to see that patients receive the medications ordered by their doctors.

The range of drugs prescribed increases almost daily and this places heavy responsibilities on nurses to see that the right drug is given in the right amount to the right patient at the right time.

This chapter will deal with some medicines which are designed to enter the body through the gastro-intestinal tract. These are taken orally - i.e., by mouth, and may be in the form of liquids, tablets, capsules or powders. Also included will be those drugs which are injected into the body tissues. These are almost always presented in liquid form, although rarely, a powder may have to be dissolved in a suitable sterile fluid medium.

All drugs are potentially dangerous, therefore drug administration calls for a high degree of accuracy. The label must be read with care and the volume and strength of the drug noted. The measuring apparatus used must be suitable for the amount required and the actual measurement made accurately.

You should already be familiar with the 30 to 40 mL medicine measure which can be purchased from your local pharmacy or supermarket. When volumes of less than 5 mL are being given a special 5 mL measure should be used - see Figure 9.1.

When reading the level of a liquid in a medicine measure hold the measure so that the required graduation is at eye level. Failure to do so can result in inaccurate readings as shown in Figure 9.2.
FIG. 9.1 A 5 mL medicine measure for measuring small volumes of oral drugs.

FIG. 9.2 Above eye level, a liquid appears to be more than it is — below, less.
With many drugs to be given by mouth, it is simply
a matter of checking the orders and label on the bottle
and measuring the required volume using the most suitable
sized measure for the dose required. Most oral drugs
are prescribed in multiples of 5 mL, e.g., 5 mL, 10 mL,
15 mL, 20 mL or 30 mL.
Take particular care with solutions in which part
of the ingredients are held in suspension. The
instruction "shake the bottle" is clearly shown on the
label of such mixtures. No dose should be poured from
that bottle until the contents are thoroughly mixed.
Because there are a few cases where you may need
to make some calculations we will look at some of the
problems with which you may be faced.
An antibiotic drug is labelled 125 mg in 4 mL.
Your patient is ordered 250 mg every six hours.
How much will you give at each dose?

STEP 1: Analyse the information you are given.
In every 4 mL of volume there are 125 mg of the
antibiotic drug.
Your patient is ordered 250 mg which is more than
125 mg, so you are dealing with a directly related
proportion.
The frequency of the dose does not enter into these
calculations.

STEP 2: Choose a variable such as an alphabetical letter
to represent the unknown quantity.
Let x represent the unknown volume.

STEP 3: In 4 mL there are 125 mg.
\[ \therefore \text{in } x \text{ mL there are } 250 \text{ mg.} \]

STEP 4: Ratio of volumes = ratio of corresponding masses.
\[ \frac{x}{4} = \frac{250}{125} \]

STEP 5:
\[ \frac{4 \times x}{4} = \frac{4 \times 250}{125} \]

9.3
\[
\frac{1}{x} \times \frac{x}{x} = \frac{4 \times 250}{125^2} \times \frac{1}{1}
\]

\[
x = 8
\]

.: For the required dose of 250 mg you will need to give 8 mL of the stock drug 125 mg in 4 mL.

Note that the answer is reasonable! The mass of the required dose is doubled, so therefore the volume to be given should be doubled.

If you prefer to work the problem an alternative way, you can again select a variable to represent the unknown volume and then set out your statement of facts.

125 mg in 4 mL

So 250 mg in \( x \) mL.

\[
125 \times x = 250 \times 4
\]

so \( x = \frac{250 \times 4}{125} \)

\[
= \frac{250}{125} \times \frac{4}{1}
\]

\[
x = 8
\]

.: From a stock drug of 125 mg in 4 mL you will need 8 mL for the required dose of 250 mg.

The stock mixture of aspirin and codeine contains 16 mg of codeine in every 15 mL. Doctor wishes the patient to have only 8 mg of codeine, so how much will you give?
Let $x$ represent the unknown volume.

Strength on hand = 16 mg in 15 mL

Strength required = 8 mg in $x$ mL.

Ratio of volumes = ratio of corresponding masses

\[
\frac{x}{15} = \frac{8}{16}
\]

\[
\frac{15 \times x}{15} = \frac{15 \times 8}{16}
\]

\[
\frac{x}{15} = \frac{1 \times 8}{16}
\]

\[
\frac{15 \times x}{16} = \frac{8}{2}
\]

\[
x = \frac{15}{2}
\]

\[
x = 7.5 \text{ mL}
\]

\[\therefore\text{ From a stock solution of 16 mg in 15 mL, give 7.5 mL for a dose of 8 mg.}\]

Note again that the answer is reasonable. The required dose is half of the stock drug, and the volume determined is half of the dosage volume.

For those who prefer the alternative method we will rework the same problem.

Let $x$ represent the unknown volume.

Strength on hand = 16 mg in 15 mL.

Strength required = 8 mg in $x$ mL.
\[
16 \times x = 8 \times 15
\]
\[
x = \frac{8 \times 15 \times 1}{16} \times \frac{1}{2}
\]
\[
= \frac{15}{2}
\]
\[
x = 7.5 \text{ mL}
\]

So 7.5 mL of the stock mixture of aspirin and codeine will contain the required dose of 8 mg codeine.

A suspension contains 100 mg of drug in 4 mL. How much will you give if the orders are for 75 mg three times per day. (The frequency of the dose does not enter into the calculations.)

Let \( x \) represent the unknown volume

Strength on hand = 100 mg in 4 mL

Strength required = 75 mg in \( x \) mL

Ratio of volumes = ratio of masses

\[
\frac{x}{4} = \frac{75}{100}
\]

\[
\frac{4 \times x}{4} = \frac{4 \times 75}{100}
\]

\[
\frac{1}{4} \times x = \frac{1 \times 3}{25}
\]

\[
x = 3 \text{ mL}
\]

\[
\therefore \text{ From a stock suspension of 100 mg in 4 mL, give 3 mL for the required dose of 75 mg.}
\]
Again the answer is reasonable.
Working by the alternative method:

Let \( x \) represent the unknown volume

Strength on hand = 100 mg in 4 mL
Strength required = 75 mg in \( x \) mL

\[
100 \times x = 75 \times 4
\]

\[
x = \frac{75 \times 4}{100} \quad \text{(Multiply by reciprocal of 100, i.e., } \frac{1}{100})
\]

\[
x = \frac{25 \times 4 \\ 3 \quad 1}{100 \\ 1}
\]

\[
x = \frac{25}{1} \\
\]

\[
x = 3 \text{ mL}
\]

\[\therefore 3 \text{ mL of a suspension of 100 mg in 4 mL will give the required dose of 75 mg.}\]

Most drugs in tablet form are made up in the strengths which are most frequently ordered. It is rarely necessary to calculate the required dose.

Tablets which are frequently ordered in fractions of a tablet are usually presented with a line scored across the middle of the tablet which allows it to be broken more readily into halves.

A few tablets may be double scored, allowing them to be broken into four parts. Any sized tablet may be scored. You may have seen the scoring on soluble aspirin tablets.

If you have tablets of soluble aspirin containing 0.3 gram of aspirin, how many tablets would you give to a patient who is ordered 450 mg aspirin?
The tablets are scored to break into halves.
Let \( x \) represent the unknown number of tablets (express both masses in the same units, e.g., change 0.3 gram to milligrams)

1 tablet contains 300 mg

\( x \) tablets contain 450 mg

Ratio of tablets = ratio of corresponding masses

\[
\frac{x}{1} = \frac{450}{300}
\]

\[
\frac{3}{45} = \frac{450}{300}
\]

\[
x = \frac{3 \times 450}{300} = \frac{1350}{300} = \frac{3}{2}
\]

\[
x = \frac{3}{2}
\]

\[
x = 1\frac{1}{2} \text{ tablets}
\]

\[
\therefore 1\frac{1}{2} \text{ tablets of aspirin strength 0.3 grams per tablet will give the required dose of 450 mg.}
\]

So long as the tablets ordered are soluble, these problems can be tackled in a different way. A suitable quantity of water may be chosen and the tablet or tablets dissolved in it. The calculations then proceed as for a drug in solution.

Choosing a suitable quantity of water may bother some students. There must be sufficient volume to ensure complete solution of the tablet or tablets. As a general rule, from 20 to 30 mL will be found satisfactory.

Instead of breaking one tablet in two as we did in the previous example, let us dissolve the aspirin tablets and find the required volume which gives the equivalent of 450 mg.

Consider the information given in the problem. You are told 1 tablet contains 0.3 g or 300 mg. The ordered dose is 450 mg so you can see you need to use more than one tablet.
Take 2 tablets of 300 mg each and thoroughly dissolve them in a suitable quantity of water - for example 20 mL of water. This gives a strength on hand of 600 mg in 20 mL.

Let \( x \) represent the unknown volume

so 600 mg in 20 mL

\[ 450 \text{ mg in } x \text{ mL} \]

Ratio of volumes = corresponding ratio of masses

\[ \frac{x}{20} = \frac{450}{600} \]

\[ \frac{1}{20} \times \frac{x}{1} = \frac{1}{20} \times \frac{450}{600} \]

\[ \frac{1}{20} \times \frac{x}{1} = \frac{1}{20} \times \frac{450}{600} \]

\[ x = 15 \]

\[ \cdot \cdot \cdot \text{To give 450 mg soluble aspirin, dissolve 2 tablets of 0.3 gram strength in 20 mL of water and give the patient 15 mL of the resulting solution. (To ensure the accuracy of the dose, transfer 15 mL to a second medicine measure and discard the 5 mL remaining.)} \]

When a wide range of doses may be ordered, you will often find tablets presented in different strengths. For example ascorbic acid (Vitamin C) tablets may be available in 10 mg, 25 mg and 50 mg strengths.

If a patient is ordered 75 mg ascorbic acid, show how you would give the correct dose.

If you have tablets in all three strengths, there are a number of alternative ways of giving the correct dose.

75 mg could be obtained from \( 7\frac{1}{2} \) of the 10 mg tablets, but few patients would be pleased to be asked to take so many tablets at the one time.

You could give 3 tablets of 25 mg each because
\[(3 \times 25) = 75 \text{ mg or you could give one of the 50 mg tablets and one of the 25 mg tablets, because } (50 + 25) = 75 \text{ mg. If the tablets are scored, you could give one and one-half of the 50 mg tablets. } \left(\frac{1}{2} \times 50\right) = 25 + 50 = 75 \text{ mg. Therefore, 75 mg of ascorbic acid could be given in either of the following ways.}\]

1. 1 tablet of 50 mg + 1 tablet of 25 mg.
2. 3 tablets of 25 mg each.
3. 1\frac{1}{2} tablets of 50 mg each.

A patient is ordered 1.25 mg of digoxin. The tablets available are in the strength of 250 micrograms. How will you give the ordered dose?

**STEP 1:** Express both strengths in the same unit.

If \(1000 \ \mu g = 1 \text{ mg}\)

\[1250 \ \mu g = 1.25 \text{ mg}\]

**STEP 2:** Choose a variable to represent the unknown number of tablets.

**STEP 3:** Set out the available information:

1 tablet contains 250 \(\mu g\)

\(x\) tablets contain 1250 \(\mu g\)

\[\text{Ratio of tablets} = \text{ratio of corresponding masses} \]

\[\frac{x}{1} = \frac{1250}{250}\]

\[x = 5\]

\[\therefore\] To give 1.25 mg of digoxin you would need 5 tablets of 250 \(\mu g\) each.
DRUGS FOR INJECTION

The majority of drugs for injection are in fluid form sealed into sterile containers. The most common container is a glass ampoule, with a restricted neck which can be broken off to open the ampoule. Although of varying sizes, these ampoules are designed to contain only one dose. Any of the contents which are not required when the ampoule is opened are discarded. Because of this, these ampoules are sometimes described as single dose containers.

If a container is designed to hold more than one dose it is sealed with a rubber diaphragm. To withdraw any of the contents, a sterile needle is inserted through the diaphragm and the required volume drawn into the syringe. These are often called "multidose containers".

FIG. 9.3 Syringes and single and multidose containers
Syringes come in many sizes, from a 1 mL capacity to 500 mL capacity. Most hospitals use disposable syringes. These are made in plastic and are supplied in a sterile wrap. After use the syringe is discarded.

Most one and two millilitre syringes are graduated so that tenths of a millilitre may be measured accurately. Some one millilitre syringes are especially graduated so that hundredths of a millilitre may be measured with accuracy. Most other syringes are graduated so that they are accurate to 0.2 of a millilitre.

Examples of syringes and single and multidose containers are shown in Figure 9.3.

When checking the volume of injectable fluid in a syringe, the syringe must be held so that the required graduation is at eye level. In Figure 9.4, a dose is being withdrawn from a multidose container.

**FIG. 9.4** Demonstration of withdrawal of drug solution from a multidose container.

Fluids for injection contain a stated quantity of active drug in a given volume of fluid. Drugs in which the active component cannot be effectively isolated are dispensed in units, while those in which the drug can be isolated are dispensed in terms of the mass of the drug.
present.

We will conclude this chapter with a few examples showing how the methods we have been using already can be applied to finding the correct volume of drug to be injected.

If you have a solution for injection labelled 10 units of drug in 1 mL, how will you obtain an ordered dose of 8 units?

STEP 1: Choose a variable to represent this unknown volume.

Let $x$ equal the unknown volume.

STEP 2: Summarise the information you have.

10 units are present in 1 mL.

8 units are present in $x$ mL.

STEP 3: Ratio of volumes = ratio of corresponding units.

STEP 4:

\[
\frac{x}{1} = \frac{8}{10}
\]

STEP 5:

\[
x = \frac{8}{10} \text{ mL}
\]

\[
x = 0.8 \text{ mL}
\]

\[\therefore\text{ To give 8 units when you have a solution of 10 units in 1 mL you would draw up and inject 0.8 mL (the remainder is discarded).}\]

Look again at the facts, and then examine the answer to see that it is reasonable.

8 units (the strength required) is less than 10 units (the strength on hand) and 0.8 mL is less than 1 mL, so the answer is reasonable.

The problem may also be worked by the alternative method from Step 2.

STEP 3: $10 \times x = 8 \times 1$

STEP 4: \[\therefore x = \frac{8 \times 1}{10}\] (Multiply by reciprocal of 10, i.e., \(\frac{1}{10}\))

9.13
STEP 5: \( x = \frac{8}{10} \)

STEP 6: \( x = 0.8 \)

\[ . \cdot \text{To give 8 units when you have a solution of 10 units}
\text{in 1 mL, inject 0.8 mL.} \]

What volume of a solution containing 40 units in 1 mL
will give the ordered dose of 30 units?

Let \( x \) represent the unknown volume

there are 40 units in 1 mL

and 30 units in \( x \) mL

Ratio of volumes = ratio of corresponding units

\[ \frac{x}{1} = \frac{30}{40} \]

\[ . \cdot \ x = \frac{3}{4} \]

\[ x = 0.75 \text{ mL} \]

\[ . \cdot \text{To give 30 units when you have a solution of 40 units}
\text{per mL inject 0.75 mL.} \]

If you are asked to express this answer correct to the first decimal place, you would inject 0.8 mL.

Refer back to Chapter 4.

A given solution contains 15 units of drug in every
millilitre, how much will contain 20 units?

Let \( x \) represent the unknown volume

there are 15 units in 1 mL

and 20 units in \( x \) mL
Ratio of volumes = ratio of corresponding units

\[
\frac{x}{1} = \frac{20}{15}
\]

\[
x = \frac{4}{3}
\]

\[
x = 1.3
\]

\[\therefore 1.3 \text{ mL of a solution containing 15 units per millilitre will give 20 units of the drug.}\]

Using the alternative method gives:-

\[
15 \times x = 20 \times 1
\]

\[\therefore x = \frac{20 \times 1 \times 1}{15}
\]

\[
x = \frac{4}{3}
\]

\[
x = 1.3
\]

\[\therefore 20 \text{ units will be contained in 1.3 mL of a solution of 15 units per millilitre.}\]

Please notice the way in which all these answers have been expressed. They contain three lots of information:

(i) The ordered dose.

(ii) The strength which is available for use (often called the stock strength or strength on hand).

(iii) The amount of the strength on hand which contains the ordered dose.

Make sure you include the same information in your answers.
Chapter 10

INSULIN AND OTHER DRUGS FOR INJECTION - DILUTION OF SOLUTIONS

Because the consequences of errors in drug dosages can be so drastic we will devote most of this chapter to looking at some further examples of the way in which drugs for injection may be ordered.

Until 1st August 1980, there were several strengths of insulin available in Australia. The most common strengths were U40 (40 units in 1 millilitre) and U80 (80 units in 1 millilitre). Doses were measured in special insulin syringes or standard 1 or 2 mL syringes. (None of these old insulins or insulin syringes should now be in use.)

Insulin is now produced in one strength which is known as U100, because it contains 100 units of insulin per millilitre.

Special U100 syringes are available for injecting the U100 insulin. Old insulin syringes must never be used with U100 insulin.

U100 insulin has some major benefits:

1. The single strength means less confusion and obviates dosage errors which occurred when a multiplicity of strengths and syringes were used.

2. There is greater flexibility of dosage. (If an insulin dependent diabetic needed frequent dosage adjustments they often had to change to different strengths e.g. from U40 to U80 or vice versa.)

3. Because U100 is stronger, the required dose is contained in a smaller volume, and less volume per injection means less pain and discomfort for the patient and fewer adverse changes to the tissues at the injection site.

Several types of insulin are available. Some types act quickly to lower the blood sugar level, but for a short duration. Others act more slowly but their effect
FIG. 10.1 Quickly Absorbed, Short Acting Insulins.
is prolonged. Figures 10.1 to 10.3 show some of these insulins in U100 strength. They have been produced by the Commonwealth Serum Laboratories.

FIG. 10.2 Less Quickly Absorbed, Longer Acting Insulin.
There are three types of U100 insulin syringes available. These are:

(i) a reusable 1 mL glass syringe marked in 2 unit divisions;

(ii) a disposable 1 mL plastic syringe (single use) marked in 2 unit divisions;

(iii) a disposable 0.5 mL plastic syringe (single use) marked in 1 unit divisions. This syringe is designed particularly for paediatric use. (Use with young children).
(i) reusable glass (1 mL) syringe.

(ii) disposable plastic (1 mL) syringe.

(iii) disposable plastic (0.5 mL) syringe.

FIG. 10.4 U100 Insulin Syringes
Most insulin-dependent diabetics require less than 100 units per dose and for these patients the U100 syringe should be used.

A few insulin dependent diabetics require more than 100 units per dose. They may use a standard 2 mL syringe which is graduated in tenths of a millilitre. The following calculations can show the volume to be injected.

EXAMPLE:

A patient requires 120 units of insulin. What volume of U100 would he inject if using a standard 2 mL syringe?

Strength on hand = 100 units in 1 mL
Strength required = 120 units
Let $x$ represent the unknown volume.

METHOD A

Ratio of volumes = ratio of corresponding units

\[
\frac{x}{1} = \frac{120}{100}
\]

\[
x = \frac{120}{100}
\]

\[
x = \frac{12}{10}
\]

\[
x = 1.2 \text{ mL}
\]

\[\therefore 120 \text{ units of insulin will be contained in } 1.2 \text{ mL if the available strength is U100 (i.e. 100 units in 1 mL).}\]

METHOD B

100 units in 1 mL
\[\therefore 120 \text{ units in } x \text{ mL}\]

\[
100 \times x = \frac{120 \times 1}{100}
\]

\[
x = \frac{120 \times 1}{100}
\]

\[
x = \frac{12}{10}
\]

\[
x = 1.2 \text{ mL} \quad \text{Therefore ...}
\]

10.7
\[
\therefore 120 \text{ units of insulin will be contained in } 1.2 \text{ mL if the available strength is U100.}
\]

Rarely does a patient require more than 200 units per injection. In these cases there are two options available:

(i) Use U100 insulin and a 5 mL syringe.

(ii) Use a special U300 insulin (where there are 300 units of insulin in 1 mL).

EXAMPLE:

A patient is ordered 240 units of insulin. Show the volume to be injected using:

(i) U100 insulin and (ii) U300 insulin.

(i)

Strength on hand = 100 units in 1 mL
Strength required = 240 units

Let \( x \) represent the unknown volume.

METHOD A

\[
\frac{x}{1} = \frac{240}{100}
\]

\[
x = 2.4 \text{ mL}
\]

\[
\therefore 240 \text{ units of insulin will be contained in } 2.4 \text{ mL if the available strength is U100.}
\]

METHOD B

\[
\begin{align*}
100 \text{ units in } 1 \text{ mL} \\
240 \text{ units in } x \text{ mL}
\end{align*}
\]

\[
100 \times x = 1 \times 240
\]

\[
x = \frac{1 \times 240}{100}
\]

\[
x = 2.4 \text{ mL}
\]

\[
\therefore 240 \text{ units of insulin will be contained in } 2.4 \text{ mL if the available strength is U100. (This dose could be drawn up into a standard 5 mL syringe.)}
\]
(ii) Strength on hand = 300 units in 1 mL
Strength required = 240 units
Let $x$ represent the unknown volume.

**METHOD A**

Ratio of volume = ratio of corresponding units

\[
\begin{align*}
\frac{x}{1} &= \frac{240}{300} \\
x &= \frac{24}{30} \\
x &= \frac{8}{10} \\
x &= 0.8 \text{ mL}
\end{align*}
\]

\[.\text{240 units of insulin will be contained in 0.8 mL if the available strength is U300.}\]

**METHOD B**

300 units in 1 mL
240 units in $x$ mL

\[
300 \times x = 240 \times 1
\]

\[
x = \frac{240 \times 1}{300}
\]

\[
x = \frac{24}{30}
\]

\[
x = \frac{8}{10}
\]

\[
x = 0.8 \text{ mL}
\]

\[.\text{240 units of insulin will be contained in 0.8 mL if the available strength is U300. (This dose could be drawn up into a standard one or two millilitre syringe.)}\]

You will notice how important it is to state clearly the strength of the insulin being used.

Whenever you write down a volume to be injected, immediately beside it you must write the strength of insulin to be used.
As with all other calculations involving drug dosages, the answer you obtain must contain three lots of information:

- the ordered dose
- the strength to be used
- the volume to be injected

**WHAT TO DO IF NO U100 SYRINGE IS AVAILABLE**

When giving insulin, if at all possible, the special insulin syringes should be used, but in an emergency standard 1 mL or 2 mL medical syringes may be used. Because U100 insulin is stronger, most doses will be contained within a small volume, so extreme care will be required when drawing up these doses into standard medical syringes. Choose syringes graduated in tenths of a millilitre.

**EXAMPLE:**

A patient is ordered 40 units of insulin and you must draw it up into a standard medical syringe. You have two options.

(i) To calculate the volume of U100 insulin which contains 40 units.

(ii) To examine the graduations on the syringe and relate each graduation to a number of units of insulin.

Option (i) Calculating the volume.

Strength on hand = 100 units in 1 mL
Strength required = 40 units

Let \( x \) represent the unknown volume.

**METHOD A**

\[
\frac{\text{Ratio of volumes}}{\text{ratio of corresponding units}} = \frac{40}{100} = \frac{4}{10} = 0.4 \text{ mL}
\]

Therefore ... 

\( 10 \times 10 \)
.

\[ 40 \text{ units of insulin will be contained in } 0.4 \text{ mL if the available strength is U100.} \]

**METHOD B**

\[ 100 \text{ units in } 1 \text{ mL} \]

\[ 40 \text{ units in } x \text{ mL} \]

\[ 100 \times x = 1 \times 40 \]

\[ x = \frac{1 \times 40}{100} \]

\[ x = \frac{4}{10} \]

\[ x = 0.4 \text{ mL} \]

\[ 40 \text{ units of insulin will be contained in } 0.4 \text{ mL if the available strength is U100.} \]

So, 0.4 mL of U100 insulin can be drawn into a 1 mL or 2 mL medical syringe. If this volume is injected, the patient will receive the ordered 40 units.

*Option (ii) Examining the syringe.*

Take the syringe and count the number of graduations in 1 millilitre. If there are ten graduations on the syringe barrel to each millilitre and you are to use U100 insulin you can make the following comparisons.

U100 insulin means 100 units in 1 mL so each graduation on the syringe represents 10 units or, each tenth of a millilitre of U100 insulin contains 10 units.

Therefore, four tenths of a millilitre (four graduations) would equal 40 units OR 40 units of insulin will be contained in 0.4 mL if the available strength is U100.

**THINGS YOU SHOULD DO**

- Examine a U100 insulin syringe. (If this is not possible, study the illustrations provided.)

- Notice that each graduation on the (1 mL) U100 syringe represents two units. (On the 0.5 mL paediatric insulin syringe each graduation represents one unit.)
If you have access to a U100 syringe, practise drawing up different volumes, for example, the volumes containing 20 units, 32 units, 40 units, 80 units, 100 units.

Remember to hold the required graduation at eye level to ensure accurate measurement. Refer back to Figure 9.4 of this textbook.

Penicillin is another drug which is usually ordered in units.

If you have penicillin in the strength of 100 000 units in 2 mL; what volume will contain an ordered dose of 75 000 units?

Let $x$ represent the unknown volume.

If 100 000 units in 2 mL
75 000 units in $x$ mL

$\frac{x}{2} = \frac{75000}{100000}$

$1 \times \frac{x}{2} = \frac{2 \times 75000}{100000}$

$\frac{2}{2} \times \frac{1}{2} = \frac{50000}{50}$

$x = \frac{3}{2}$

$x = 1.5$

From a stock solution of 100 000 units penicillin in 2 mL, give 1.5 mL for the ordered dose of 75 000 units.

The alternative method of working gives:-
\[100 \, 000 \times x = 75 \, 000 \times 2\]
\[x = \frac{75 \, 000 \times 2 \times 1}{100 \, 000}\]
\[x = \frac{75 \times 2 \times 1}{100}\]
\[x = \frac{150}{100} = 1.5\]

A multidose container holds 1 gram of drug in 5 mL. How much will you withdraw if you are to give a dose of 250 mg?

Express both the strength on hand and the strength required in the same units, i.e., 1 gram = 1000 mg.

Let \(x\) represent the unknown volume

1000 mg in 5 mL

250 mg in \(x\) mL

Ratio of volumes = ratio of corresponding masses

\[\frac{x}{5} = \frac{250}{1000}\]

\[\frac{1}{5} \times x = \frac{1}{5} \times \frac{250}{1000}\]

\[\frac{1}{5} \times x = \frac{1}{400}\]

\[x = \frac{5}{4}\]

\[x = 1.25\text{ mL}\]

Withdraw 1.25 mL for a dose of 250 mg if the multidose container contains 1 gram in 5 mL.
Or, alternatively:

\[1000 \times x = 250 \times 5 \times \frac{1}{4}\]

\[x = \frac{250 \times 5 \times 1}{1000} = \frac{5}{4}\]

\[x = 1.25 \text{ mL}\]

According to the agreement discussed in Chapter 4, 1.25 mL would be expressed as 1.2 mL, correct to the first decimal place.

If you examine the steps taken in working the previous examples, you will note that the volume required is finally derived from the product of the strength required and the volume of the stock drug, divided by the strength on hand. This has led to the adoption, by some people, of the following formula which they use when they are required to alter drug strengths:

\[\frac{\text{strength required} \times \text{volume of stock drug}}{\text{strength on hand}} = \text{volume to give patient}\]

Some people prefer to try and memorise the formula rather than work from basic mathematical principles. Difficulties and even serious errors may arise if the formula is forgotten or is incorrectly applied.

Whichever method you use, you should always examine the answer you obtain in the light of the information you were given to see whether the answer is reasonable.

Careful checking, together with repeated practice, is the surest way of maintaining accuracy.
CHILDREN'S DOSES

In most examples shown so far, the actual dose to be given to a patient has been stated.

The usual dose of a drug ordered for an adult may be far too much for a young child. Also, the tolerance to a drug may vary with changes in age and body build. So, in prescribing drugs for children, the doctor takes into account these factors. When this is done, the dose is related to the mass of the body. Under such circumstances the dose is prescribed as "so many milligrams or micrograms per kilogram of body mass". The required dose is then calculated from the recorded mass of the patient.

This method of relating dose to mass is used at times when prescribing adult dosages, but it is used most frequently when prescribing for babies and children. Refer back to page 7.2 for one example. Other examples follow:-

EXAMPLES:

1. A doctor orders a drug to be given at the rate of 1 milligram per kilogram. What dosage would you give to a baby with a recorded mass of 7 kilograms?

   1 mg for 1 kg
   x mg for 7 kg
   \[ x \times 1 = 1 \times 7 \]
   \[ x = 7 \text{ mg} \]

   Using the alternative method of working:
   ratio of drug dosages = corresponding ratio of masses

   \[ \frac{x}{1} = \frac{7}{1} \]
   \[ x = 7 \times 1 \]
   \[ x = 7 \text{ mg} \]

   If the prescribed dosage is 1 milligram per kilogram, 7 mg will be given to a baby whose mass is 7 kg.
2. A doctor orders a drug to be given at the rate of 20 micrograms per kilogram. What dosage would be given to a baby with a recorded mass of 5 kilograms?

\[
20 \, \mu g \text{ for 1 kg} \\
x \, \mu g \text{ for 5 kg} \\
x \times 1 = 20 \times 5 \\
x = 100 \, \mu g
\]

or

\[
\text{ratio of drug dosages} = \text{corresponding ratio of masses}
\]

\[
\frac{x}{20} = \frac{5}{1}
\]

\[
x = 20 \times 5
\]

\[
x = 100 \, \mu g
\]

If the prescribed dosage is 20 \( \mu g \) per kilogram, 100 \( \mu g \) will be given to a baby whose mass is 5 kilograms.

Some texts still introduce a formula based on Fried's Rule for calculating the dose for a child under twelve months.

Fried's Rule states that a suitable child's dose is calculated by multiplying the adult dose by the child's age in months and dividing by one hundred and fifty.

\[
\text{Child's dose} = \frac{\text{Adult dose} \times \text{age in months}}{150}
\]

EXAMPLES:

1. What is the dose for a babe of 9 months if the adult dose is 10 mg?

\[
\text{Child's dose} = \frac{10 \times 9}{150}
\]

\[
= \frac{1}{15} \times \frac{9}{5} = \frac{3}{5} = 0.6 \text{ mg}
\]

If the adult dose is 10 mg, a 9 month old babe should be given 0.6 mg.
2. The stated adult dose is 6 mL. What would be the dose for a babe of 4 months?

\[
\text{Child's dose} = \frac{6 \times 4}{150} = \frac{4}{25} = 0.16 \text{ mL}
\]

If the adult dose is 6 mL, a 4 months old babe should receive 0.16 mL.

For children over 12 months a similar formula based on Young's Rule may be used. Young's Rule states that a suitable children's dose may be obtained by multiplying the adult dose by the child's age in years and dividing this number by the age in years plus twelve.

\[
\text{Child's dose} = \frac{\text{adult's dose} \times \text{child's age in years}}{\text{(Child's age in years + 12)}}
\]

EXAMPLES:

1. How much morphine should be given to a child of 6 years if the adult dose is 15 mg?

\[
\text{Child's dose} = \frac{15 \times 6}{6 + 12} = \frac{15 \times 6}{18} = \frac{5 \times 1}{3} = 5 \text{ mg}
\]

If the adult dose is 15 mg a 6 year old child should receive 5 mg.
2. How much atropine should be given preoperatively to a child of 8 years if the adult dose is 0.6 mg?

Child's dose = \( \frac{0.6 \times 8}{8 + 12} \)
\[ = \frac{0.6 \times 8}{20} \]
\[ = \frac{0.6 \times 8}{20} = 0.24 \]
\[ = \frac{1.2}{5} \]
\[ = 0.24 \text{ mg} \]

If the adult dose is 0.6 mg a child of 8 years should receive 0.24 mg.

As it is generally felt that the more desirable practice is to relate dosage to mass, this is the method most commonly used. Some more examples will help reinforce the calculations involved.

EXAMPLES:

1. If the dosage rate is given as 6 mg per kilogram, how much will be given to a patient whose mass is given as 26 kg?

   \[
   6 \text{ mg for 1 kg} \\
   x \text{ mg for 25 kg} \\
   x \times 1 = 6 \times 25 \\
   x = 150 \text{ mg}
   \]

   OR

   ratio of drug dosages = corresponding ratio of masses

   \[
   \frac{x}{6} = \frac{25}{1} \\
   x = 25 \times 6 \\
   x = 150 \text{ mg}
   \]

   If the dosage rate is 6 mg per kg, 150 mg will be given to a patient whose mass is 25 kg.
2. If the dosage rate is given as 15 micrograms per kilogram, what will be the dose in milligrams given to a patient whose mass is 65 kg?

\[ 15 \, \mu g \text{ for 1 kilogram} \]

\[ x \, \mu g \text{ for 65 kilograms} \]

\[ x \times 1 = 15 \times 65 \]

\[ x = 975 \, \mu g \]

\[ 1000 \, \mu g = 1 \text{ mg} \]

\[ 975 \, \mu g = 0.975 \text{ mg} \]

\[ 0.975 \, \mu g = 1 \text{ mg approx.} \]

OR

ratio of drug dosages = corresponding ratio of masses

\[ \frac{x}{15} = \frac{65}{1} \]

\[ x = 65 \times 15 \]

\[ x = 975 \, \mu g \]

\[ x = 1 \text{ mg approx.} \]

If the dose rate is 15 \( \mu g \) per kg approx. 1 mg will be given to a 65 kg patient.
DILUTION OF FULL STRENGTH LOTIONS

If you examine a bottle of a common antiseptic or disinfectant, you will find instructions for diluting the solution. Relatively weak antiseptics may only need to have an equal volume of water added. Others need to be diluted further.

In a hospital there is considerable use of different solutions as decontaminants (disinfectants) and antiseptics. Sometimes the same solution may be used as either a decontaminant or an antiseptic. For decontamination it will need to be a stronger solution than is used as an antiseptic. Hospitals have a special department, the pharmacy or dispensary, to handle the majority of solution dilutions which saves the nursing staff a great deal of time. However, it is still essential for nurses to understand the meaning of solution or lotion strengths and to possess the ability to make up a solution of given strength when required.

It is obvious that the addition of water to some solution will produce a more dilute solution.

In Chapter 8 we saw how some solution strengths may be expressed in percentages. For example, 50% is a way of saying 50 parts in 100 parts. In other words, for every 100 parts of solution, 50 parts of it would be the pure solution. You know that an equivalent proportion to 50 parts in 100 parts is 1 part in 2 parts, which is written more simply as 1-2. If you take a measured quantity, e.g., 1 mL of some pure solution such as Dettol and add an equal quantity of water you have a 1-2 or a 50% solution. Further addition of water to double the volume of the 1-2 solution would give a 1-4 solution, i.e., in every 4 parts by volume, there is 1 part of pure solution.

A 1-10 solution is made by taking one part of the pure solution and adding water to make up the volume to 10 parts. A 1-10 solution can be expressed as a 10% solution.

Expressing this in units of volume would give 1 mL of pure solution in 10 mL - i.e., measuring 1 mL of the pure solution and adding water to bring the volume to 10 mL, or taking 1 litre of pure solution and adding water to bring the volume to 10 litres.

The best way to understand how to prepare an ordered solution is to actually practise making up solutions in varying strengths.
If you take your medicine measure and some coloured water which can be used to represent an antiseptic solution, we can make up solutions of varying strengths.

(a) Measure out 15 mL of the coloured water which represents your pure antiseptic solution. Now add water to the 30 mL mark. You have 30 mL of a 1-2 or a 50% solution.

You could put that aside in a jar which you label a 1-2 or a 50% solution.

(b) Now measure out 10 mL of your substitute pure solution and add water to the 40 mL mark. You have 40 mL of a 1-4 solution, or a 25% solution.

You could put this aside in another labelled jar.

(c) Frequently you will require larger volumes of diluted solutions. A measuring jug will be helpful, but if you haven’t one you can use the medicine measure and make up the larger volume in 40 mL lots.

Measure 5 mL of your substitute pure solution. If you have a measuring jug, transfer the 5 mL of substitute pure solution to the measuring jug and add water to bring the volume to 100 mL. You now have 100 mL of a 5% solution, or 100 mL of a 1-20 solution.

If you have no measuring jug you will have to place your 5 mL of substitute pure solution in a jar and then measure 95 mL with the medicine measure (40 + 40 + 15 mL) to give you a total of 100 mL in the jar. Label the solution.
So far you have made up these solutions from the volume of substitute pure solution and an added volume of water.

More frequently you are told:

(a) the total volume of solution needed;
(b) the strength of the solution.

You then have to calculate the volume of pure solution needed.

At the beginning of each problem you need to summarize the information you are given, and allocate an alphabetical letter to the unknown quantity. At the end, a detailed answer must be written down.

Consider the following request.

From a pure solution of Dettol, prepare 30 mL of a 1-2 solution.

Information
strength on hand = pure solution

Summary
strength required = 1-2
volume required = 30 mL

You will recall that in Chapter 7 we said that when direct proportions were involved, the products of the diagonally opposite numbers were equal.

The method below demonstrates this.

**METHOD A**

Let \( x \) represent the unknown volume of pure solution.

You know that a 1-2 solution means:

1 part of pure solution is present in 2 parts by volume
or 1 mL of pure solution is present in 2 mL
so \( x \) mL of pure solution is present in 30 mL

\[
x \times 2 = 1 \times 30
\]

\[
\therefore x = 1 \times 30 \times \frac{1}{2}
\]

\[
= 1 \times 15 \times \frac{1}{2}
\]

\[
x = 15 \text{ mL}
\]

10.22
Ans: To make 30 mL of 1-2 solution, take 15 mL of pure solution and add water to bring the volume to 30 mL.

Please note that this is the preferred way of expressing the answer. We do not say "Take 15 mL of pure solution and 15 mL of water".

Another method uses a formula which says -

Strength on hand × volume = strength required × volume

The strengths may be expressed as percentages or as ratios (written as fractions), but both must be in the same terms i.e. both percentages or both ratios.

The volumes must also be in the same units i.e. both in litres or both in millilitres.

Full strength solutions may be considered as 100%; 1:1 or 1-1; \( \frac{1}{2} \); or 1.

**METHOD B**

**Information**

- strength on hand = pure solution = \( \frac{1}{1} \)
- strength required = 1-2 = \( \frac{1}{2} \)
- volume required = 30 mL

**Summary**

Let \( x \) represent the unknown quantity (volume) of stock (pure) solution.

\[
\frac{1}{1} \times x = \frac{1}{2} \times 30 \]

so \( x = \frac{1}{2} \times 30 \times \frac{1}{1} \)

\( x = 15 \text{ mL} \)

Ans: To make 30 mL of 1-2 solution, take 15 mL of pure solution and add water to bring the volume to 30 mL.
Look back to (a) on page 10.21 where you made up 30 mL of a 1-2 solution from a substitute pure solution. The two methods we have just demonstrated show the calculations which can be used to prepare a required solution if you have pure solution on hand.

In (b) on page 10.21 you made 40 mL of a 25% solution. We will now use methods A and B to prepare this solution.

**From a pure solution of Dettol, prepare 40 mL of a 25% solution.**

(A 25% solution means 25 parts in 100 parts or 1 part in 4 parts).

**Information**

- Strength on hand = pure solution = 100% or $\frac{1}{1}$
- Strength required = 25% = $\frac{25}{100}$ or $\frac{1}{4}$
- Volume required = 40 mL

Let $x$ represent the unknown volume of pure solution.

**METHOD A**

If 1 mL of pure solution is present in 4 mL then $x$ mL of pure solution is present in 40 mL

$$x \times 4 = 1 \times 40$$

$$x = 1 \times 40 \times \frac{1}{4}$$

$$= 1 \times \frac{40}{4} \times \frac{1}{1}$$

$$x = 10 \text{ mL}$$

Ans: To make 40 mL of Dettol 25% (1-4) take 10 mL of pure solution and add water to bring the volume to 40 mL.

**METHOD B**

There are two possible approaches using Method B. In B(i) we will keep the strengths in percentages, while in B (ii) we will use ratios again.
B(i)

\[
\text{strength on hand } \times \text{ volume } = \text{ strength required } \times \text{ volume} \]

\[
100\% \times x = 25\% \times 40
\]

\[
x = \frac{25 \times 40}{100} = \frac{25 \times 40 \times 10}{180_4 1}
\]

\[
x = 10 \text{ mL}
\]

B(ii)

\[
\text{strength on hand } \times \text{ volume } = \text{ strength required } \times \text{ volume}
\]

\[
\frac{1}{4} \times x = \frac{1}{4} \times 40
\]

\[
x = \frac{1}{4} \times 40 \times 10
\]

\[
x = 10 \text{ mL}
\]

Ans: To make 40 mL of 25\% (1/4) solution take 10 mL of pure solution and add water to bring the volume to 40 mL.

From pure Dettol, prepare 100 mL of 5\% solution.

Information Summary

strength on hand = 100\% or \(\frac{1}{1}\)

strength required = 5\% = \(\frac{5}{100}\) or \(\frac{1}{20}\)

volume required = 100 mL

Let \(x\) represent the unknown volume of pure solution.

METHOD A

If 1 mL of pure solution is present in 20 mL
then \(x\) mL of pure solution is present in 100 mL

10.25
\[ x \times 20 = 1 \times 100 \]
\[ x = \frac{1 \times 100}{20} \times 1 \]
\[ x = \frac{1 \times 100}{20} \times 1 \]
\[ x = 5 \text{ mL} \]

**METHOD B(i)**

strength on hand \( \times \) volume = strength required \( \times \) volume

\[ 100\% \times x = 5\% \times 100 \]

\[ x = \frac{5}{1} \times \frac{100}{100} \]

\[ x = 5 \text{ mL} \]

**METHOD B(ii)**

strength on hand \( \times \) volume = strength required \( \times \) volume

\[ \frac{1}{1} \times x = \frac{1}{20} \times 100 \]

\[ x = \frac{1}{20} \times \frac{100}{100} \]

\[ x = 5 \text{ mL} \]

**Ans:** To make 100 mL of Dettol 5\% (1-20) take 5 mL of pure solution and add water to bring the volume to 100 mL.

The latter examples relate to the solution made in (c) on page 10.21.

We will now work some other examples. When you have found the volume of stock solution to use, you should endeavour to make up the required solution.
Prepare 1 litre of a 1-500 solution from pure solution.

Information Summary
strength on hand = pure solution = \( \frac{1}{1} \)

strength required = 1-500 = \( \frac{1}{500} \)

volume required = 1 litre = 1000 mL

Let \( x \) equal the unknown volume of pure solution.

METHOD A

If 1 mL of pure solution is present in 500 mL
then \( x \) mL of pure solution is present in 1000 mL

\[
x \times 500 = 1 \times 1000
\]

\[
x = \frac{1 \times 1000 \times 1}{500}
\]

\[
x = 2 \text{ mL}
\]

Ans. To make 1 litre of 1-500 solution, take 2 mL of pure solution and add water to bring the volume to 1000 mL.

METHOD B

strength on hand \( x \) volume = strength required \( x \) volume

\[
\frac{1}{1} \times x = \frac{1}{500} \times 1000
\]

\[
x = \frac{1 \times 1000}{500}
\]

\[
x = 2 \text{ mL}
\]

Ans. To make 1 litre of 1-500 solution, take 2 mL of pure solution and add water to bring the volume to 1000 mL.
From pure solution, prepare 600 mL of a 1-120 solution.

**Information**  
strength on hand = pure solution = \( \frac{1}{1} \)

**Summary**  
strength required = 1-120 = \( \frac{1}{120} \)

volume required = 600 mL

Let \( x \) equal the unknown volume of pure solution.

**METHOD A**

If 1 mL of pure solution is present in 120 mL  
then \( x \) mL of pure solution is present in 600 mL

\[
x \times 120 = 1 \times 600
\]

\[
x = \frac{1 \times 600 \times 1}{120} = \frac{600^5 \times 1}{120^1}
\]

\( x = 5 \) mL

*Ans.* To make 600 mL of 1-120 solution, take 5 mL of pure solution and add water to bring the volume to 600 mL.

**METHOD B**

strength on hand \( x \) volume = strength required \( x \) volume

\[
\frac{1}{1} \times x \times 600 = \frac{1}{120} 
\]

\[
x = \frac{1 \times 600}{120} = \frac{600^5}{120^1}
\]

\[ = 5 \text{ mL} \]

*Ans.* To make 600 mL of 1-120 solution, take 5 mL of pure solution and add water to bring the volume to 600 mL.
In the examples presented so far we have made up solutions by mixing a measured volume of one solution with the measured volume of another - usually water. These are sometimes referred to as volume/volume (V/V) solutions.

There are cases in which a solution is made by dissolving a measured quantity of solid in a measured quantity of water. These are sometimes referred to as weight/volume (W/V) solutions. Probably the commonest example of a W/V solution is adding sugar to a cup of tea.

In both types of solution the undiluted component is referred to as the SOLUTE and the fluid with which it is mixed is called the SOLVENT.

A very common weight/volume solution used in hospitals is saline solution in which a measured quantity of salt is dissolved in a measured quantity of water.

In V/V solutions the quantities of solute and solvent are both measured in units of volume. In W/V solutions the solute is a solid, usually in crystal or powder form and is measured in units of mass. The solvent is measured in units of volume but for the purposes of calculation we convert the volume to units of mass. This conversion is based on the metric relationship that 1 gram is the mass of 1 millilitre of water.

It is just as easy to find the ratio strength of a W/V solution as it is of a V/V solution.

**Find the ratio strength when 4 grams of salt are dissolved in 300 mL of water.**

Because 1 mL of water weighs 1 gram

300 mL of water weighs 300 grams.

Mass of total solution is therefore (300 + 4)

304 grams.

Ratio strength is therefore 4 gram in 304 gram.

This ratio may be expressed as 1:76 or 1:76.

From this you can see how necessary it is to read the question carefully. You should be able to see at a glance whether you have a V/V or a W/V solution. In addition, questions involving V/V solutions may be worded in different ways.
For example:

1. Find the ratio strength of a solution in which 7 mL of pure solution have been added to 203 mL of water.

   Quantity of solute = 7 mL
   Total volume of solution = (203 + 7) mL
   = 210 mL
   Ratio of solute to solvent = \( \frac{7}{210} \)
   = \( \frac{1}{30} \)

   '. Ratio strength is 1:30 or 1:30.

2. Find the ratio strength when 10 mL of a substance is made up with water to 1 litre.

   Quantity of solute = 10 mL
   Total volume of solution = 1000 mL (1 litre)
   Ratio of solute to solvent = \( \frac{10}{1000} \)
   = \( \frac{1}{100} \)

   '. Ratio strength is 1:100 or 1:100.

In example 1 the wording showed that it was necessary to find the total volume by adding the quantities of solute and solvent.
In Example 2 the wording made it clear that the total volume of solution was implied.
3. What is the percentage strength when 3 grams of silver nitrate is dissolved in 200 mL of distilled water?

Amount of solute = 3 gram

Total volume of solution = 200 mL

Total mass of solution = 200 gram

Ratio of solute to solvent = \( \frac{3}{200} \)

\[ \text{Percentage strength} = \frac{3}{200} \times 100 \]

\[ = \frac{3 \times 100}{200} \]

\[ = \frac{3}{2} \]

\[ = 1.5\% \]
Chapter 11

Further Examples in Dilution of Solutions

In the previous chapter we saw how some pure solutions could be diluted to varying strengths.
In this chapter we will be looking at ways of making an even weaker solution from an already diluted solution.
Initially we will use the solutions you made up in the previous chapter i.e. the 1-2 (50%) solution; the 1-4 (25%) solution and the 1-20 (5%) solutions.
In some circumstances, these solutions could be too strong for certain uses and so an even weaker solution must be prepared. The stock solution involved in all the following calculations will not be pure solution, but will be an already diluted solution.
As with the dilution from full strength solutions, there are several ways of tackling the necessary calculations. No matter which method you choose, there will be no variation in the information summary at the beginning, nor in the way in which the answer is expressed.
Method A involves lengthy setting out, and the calculations are performed in two stages. You are advised not to give in to the temptation to abbreviate the setting out. Only by following carefully the logical stages will you gain an adequate understanding of the procedures.
When working with pure solutions, Method A was worked on the basis that if quantities are directly proportional, "the products of the diagonally opposite values are equal".
We will now set down the information and calculations needed if you are asked to:

Make 50 mL of a 1-10 solution and you have on hand a 1-2 solution.
Information

<table>
<thead>
<tr>
<th>Strength on hand</th>
<th>1-2 = \frac{1}{2}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strength required</td>
<td>1-10 = \frac{1}{10}</td>
</tr>
<tr>
<td>Volume required</td>
<td>= 50 mL</td>
</tr>
</tbody>
</table>

Let \( x \) represent the unknown volumes.

**METHOD A (STAGE 1)**

In the first stage we imagine that we are able to use pure solution. So we act as though the problem is "prepare 50 mL of 1-10 solution from pure solution". Stage 1, is therefore the same as for those problems worked in Chapter 10.

You know that a 1-10 solution means

1 part of pure solution is present in 10 parts by volume
or 1 mL of pure solution is present in 10 mL by volume
so \( x \) mL of pure solution is present in 50 mL by volume.

\[
x \times 10 = 1 \times 50
\]
\[
\text{so } x = \frac{1 \times 50 \times 1}{10} = \frac{1 \times 5 \times 1}{10}
\]
\[
x = 5 \text{ mL}
\]

Stage 1 has told you that you could make 50 mL of 1-10 solution by using 5 mL of pure solution. But you do not have any pure solution. The available strength is 1-2. Stage 2 enables you to find the volume of 1-2 solution which contains the equivalent of 5 mL of pure solution. In Stage 2, the unknown quantity (represented by \( x \)) is the volume of 1-2 solution containing the equivalent of 5 mL of pure solution.

(C) BY

11.2
(STAGE 2)

You know that a 1-2 solution means

1 part of pure solution is present in 2 parts by volume
or 1 mL of pure solution is present in 2 mL by volume
so 5 mL of pure solution is present in x mL of 1-2.

\[ 5 \times 2 = 1 \times x \]
so \[ 10 = x \]
\[ .\] \[ 10 \text{ mL} = x \]

Ans. To make 50 mL of 1-10 solution, take 10 mL of 1-2 solution and add water to bring the volume to 50 mL.

Method B as used in the previous chapter can be used in this context and is in fact a shorter method.

METHOD B

\[ \frac{1}{2} \times x = \frac{1}{10} \times 50 \]
\[ x = \frac{1 \times 50 \times x}{10} \]
\[ x = \frac{1 \times \frac{50}{2}}{1} \]
\[ x = 5 \times 2 \]
soso \[ x = 10 \text{ mL} \]

Ans. To make 50 mL of 1-10 solution, take 10 mL of 1-2 solution and add water to bring the volume to 50 mL.

You should now take your medicine measure and pour into it 10 mL of the 1-2 solution you made in the previous chapter. Transfer it to a measuring jug and add water to bring the volume to 50 mL. Label the new solution "1-10 solution".
Some students may recognise that 1-2 is five times stronger than 1-10 \( \left( \frac{1}{10} \times 5 = \frac{1}{2} \right) \). One fifth of the desired volume will then be the stock solution of 1-2.

Referring back to the solutions made in Chapter 10, your second jar of diluted solution contains 40 mL of 1-4 solution.

Make 200 mL of a 1-20 solution using 1-4 solution.

**Information**
- strength on hand = 1-4 = \( \frac{1}{4} \)

**Summary**
- strength required = 1-20 = \( \frac{1}{20} \)
- volume required = 200 mL

Let \( x \) represent the unknown volume

**METHOD A (STAGE 1)**

You know that a 1-20 solution means
1 part of pure solution is present in 20 parts by volume
or 1 mL of pure solution is present in 20 mL by volume
so \( x \) mL of pure solution is present in 200 mL by volume.

\[
x \times 20 = 1 \times 200
\]
\[
x = \frac{1 \times 200 \times 1}{20} = \frac{1 \times 2000 \times 1}{20}
\]
\[
x = 10 \text{ mL}
\]

So, you could make the required volume of 200 mL of 1-20 solution by using 10 mL of pure solution. **But the strength available is 1-4.** You must now find the volume of 1-4 solution which contains the equivalent of 10 mL of pure solution.
(STAGE 2)

You know that a 1-4 solution means

1 part of pure solution is present in 4 parts by volume or 1 mL of pure solution is present in 4 mL by volume so 10 mL of pure solution is present in \( x \) mL of 1-4.

\[
10 \times 4 = 1 \times x \\
10 \times 4 = x \\
40 = x
\]

Ans. To make 200 mL of a 1-20 solution, take 40 mL of 1-4 solution and add water to bring the volume to 200 mL.

METHOD B

\[
\left( \frac{\text{strength on hand} \times \text{volume}}{4} \right) = \left( \frac{\text{strength required} \times \text{volume}}{20} \right) \\
\frac{1}{4} \times x = \frac{1}{20} \times 200 \\
x = \frac{1 \times 200 \times 4}{20} \\
= \frac{1 \times 200}{20} \\
x = 10 \times 4 \\
x = 40 \text{ mL}
\]

Ans. To make 200 mL of a 1-20 solution, take 40 mL of 1-4 solution and add water to bring the volume to 200 mL.

Take the 40 mL of the 1-4 solution you made previously. Add water to bring the volume to 200 mL so that you now have 200 mL of a 1-20 solution. Label the new solution.
Often when these problems are presented, either the strength on hand, or the strength required or both, are expressed as percentages. In order to undertake the calculations, both strengths need to be expressed in the same terms. Generally, it is preferable to change the percentage to "so many parts per hundred parts" or any equivalent proportion.

If using Method B, both strengths can remain as percentage strengths.

The following problem may appear difficult, but it is really very simple to solve:

"From a 25% solution of sodium chloride (salt solution) prepare 1 litre of 0.9% sodium chloride."

(When you prepare the information summary you can show the various ways in which the strengths can be expressed.)

**Information Summary**

- strength on hand = 25% = 25-100 = 1-4 = 1\( \div 4 \)
- strength required = 0.9% = 0.9-100
  = 9-1000 = 9\( \div 1000 \)

- volume required = 1 litre = 1000 mL

Let \( x \) represent the unknown volume.

**METHOD A (STAGE 1)**

(Find the volume of pure solution needed to make 1 litre (1000 mL of 0.9% solution.)

You know that 0.9% means

9 parts of pure solution are present in 1000 parts by volume or 9 mL of pure solution are present in 1000 mL by volume.

(As the required volume is 1000 mL, no further calculations are needed in stage 1.)

You could make 1 litre of 0.9% solution by taking 9 mL of pure solution and adding water to bring the volume to 1000 mL. But you do not have pure solution! The stock available is 25%.
(STAGE 2)

(Find the volume of 25% (1-4) solution which contains the equivalent of 9 mL of pure solution).
You know a 1-4 solution means

1 part of pure solution is present in 4 parts by volume
or 1 mL of pure solution is present in 4 mL by volume
so 9 mL of pure solution is present in x mL of 1-4

\[
9 \times 4 = 1 \times x \\
\text{so } 36 = x
\]

Ans. To make 1 litre of 0.9% sodium chloride, take 36 mL of 25% solution and add water to bring the volume to 1000 mL.

There are two possible approaches using Method B.
In B (i) we will keep the strengths in percentages and in B (ii) we will use ratios again.

METHOD B (i)

\[
\text{strength on hand} \times \text{volume} = \text{strength required} \times \text{volume} \\
25\% \times x = 0.9\% \times 1000 \\
x = \frac{0.9 \times 1000 \times 1}{25} \\
x = \frac{0.9 \times 1000}{25} \\
x = 0.9 \times 40 \\
x = (0.9 \times 10) \times 4 \\
x = 9 \times 4 \\
x = 36 \text{ mL}
\]

Ans. To make 1 litre of 0.9% solution, take 36 mL of the 25% solution and add water to bring the volume to 1000 mL.
METHOD B (ii)

\[
\text{strength on hand} \times \text{volume} = \text{strength required} \times \text{volume}
\]

\[
\frac{1}{4} \times x = \frac{9}{1000} \times 1000
\]

\[
x = \frac{9 \times 1000 \times 4}{1000^1} \times \frac{1}{1}
\]

\[
x = 9 \times 4
\]

\[
x = 36 \text{ mL}
\]

Ans. To make 1 litre of 0.9% solution, take 36 mL of the 25% solution and add water to bring the volume to 1000 mL.

In Chapter 10 we said that some people preferred to use a formula to calculate an appropriate dose. Over the years, formulae have been devised which have been used for dilution of solutions. You can probably see how one could be derived from Method B which we have been using.

Basically, the formula finds the volume of stock solution to use in the new solution by dividing the product of the fraction of pure solution in the required strength and the volume required by the fraction of pure solution in the strength on hand. It may be written as

\[
\text{volume of stock solution} = \frac{\text{fraction of pure solution in required strength} \times \text{volume required}}{\text{fraction of pure solution in stock strength}}
\]

We will demonstrate its use in solving the problem we have just worked.

The same information summary applies. The formula must then be written down.

strength on hand = 25% = 25-100; 1-4
strength required = 0.9% = 0.9-100; 9-1000
volume required = 1 litre = 1000 mL

11.8
Let $x$ represent the unknown volume of stock solution.

**FORMULA:**

\[
\text{volume of stock solution} = \frac{\text{fraction of pure solution in required strength} \times \text{volume required}}{\text{fraction of pure solution in stock strength}}
\]

The fraction of pure solution in the required strength is $\frac{9}{1000}$, the volume of solution required (i.e., the total volume) is 1000 mL and the fraction of pure solution in the strength on hand is $\frac{25}{100}$ or $\frac{1}{4}$.

\[
x = \frac{\frac{9}{1000} \times 1000}{\frac{1}{4} \times 1}
\]

\[
x = \frac{9 \times 1000 \times 4}{1000 \times 1 \times 1}
\]

(Reciprocal of $\frac{1}{4}$ is $\frac{4}{1}$)

\[
x = \frac{9 \times 1 \times 4}{1 \times 1 \times 1}
\]

\[
x = 36 \text{ mL}
\]

**Ans.** To make 1 litre of 0.9% sodium chloride, take 36 mL of the stock solution of 25% and add water to bring the volume to 1000 mL.

From time to time people have tried to abbreviate this formula. Some state it as:

\[
\frac{\text{strength required} \times \text{volume required}}{\text{strength on hand}} = \text{volume of stock solution to use}
\]

Again we will apply the values from the previous problem, and allow $x$ to represent the unknown volume of stock solution required.
\[
\frac{0.9\% \times 1000}{25\%} = x
\]
\[
= \frac{0.9 \times 100 \times 1000}{25 \times 100} = x
\]
\[
= \frac{0.9 \times 10000}{25} = x
\]
(The 100 in each line may be ignored because both proportions are expressed in the same terms)
\[
= \frac{0.9 \times 40}{25} = x
\]
\[
= 0.9 \times 40 = x
\]
\[
= (0.9 \times 10) \times 4 = 36 \text{ mL}
\]

Ans. To make 1 litre of 0.9\% sodium chloride take 36 mL of the 25\% solution and add water to bring the volume to 1000 mL.

This formula is often misapplied even by those who remember the wording of it. They say to themselves "what we want over what we've got, multiplied by volume gives volume of stock solution to use", yet they get their fractions confused and end up with the wrong answer.

We commend to you the longer methods of working by logical steps.

As a final example in this chapter we will show how the formula can be applied to the problem, "Prepare 200 mL of a 1-20 solution from a stock solution of 1-4". 

\[
\frac{\text{fraction of pure solution in strength required} \times \text{volume required}}{\text{fraction of pure solution in stock strength}} = \text{volume of stock solution}
\]

Let \(x\) represent the unknown volume of stock solution

\[
\frac{\frac{1}{20} \times 200}{\frac{1}{4}} = x
\]
\[
\therefore \frac{1 \times 4 \times 200}{20 \times 1} = x
\]
\[
\frac{1 \times 4 \times 26.10}{26} \times 1 = 40 \text{ mL}
\]

Ans. To make 200 mL of a 1-20 solution take 40 mL of the stock solution of 1-4 and add water to bring the volume to 200 mL.
Chapter 12

RECORDING OF DATA

Within hospitals a lot of information has to be recorded. Some of this recording is specifically the duty of the nurses, some is the responsibility of doctors, some is the responsibility of ward clerks and some the responsibility of administrative staff.

In the two chapters on measurement we discussed some of the things which nurses must record. Records may be kept on special charts and sheets (like those included with this text), on cards, in Report Books, in files, on microfilm or in the special storage banks of computers. Usually specialized staff transfer records from the former types mentioned to microfilm or into the computer bank.

In previous chapters there has been a good deal of emphasis placed on the need for accuracy in measurement and calculation. It is just as important that all information is accurately recorded. Records must be kept up to date and entries should be made immediately the measurement is taken or the information received. If it cannot be entered immediately, a written note must be made and the entry transferred to the appropriate record at the earliest possible time. Doctors depend upon the continuous records which nursing staff keep, and failure to record a vital piece of information may have serious repercussions for a patient.

There are several ways of recording data. We will discuss some of these in detail and provide some examples. The temperature, pulse and respiration rates of a patient can be written down, for example T° 37°C, pulse 98, resps. 18, or they may be plotted on a continuous graph. Sometimes both types of records are kept. The continuous graph is very useful because it shows at a glance any variations
which may be occurring. A graph type record is shown on a typical T.P.R. chart in Figure 12.1 (all charts are provided separately with the text). Charts used in hospitals vary slightly in the way in which they are set out, but in principle they are all similar. When temperature and pulse are recorded on the same graph it is usual to use a different colour to represent each record. Most temperature charts record the temperature at four-hourly intervals, e.g., 2 a.m., 6 a.m., 10 a.m., 2 p.m., 6 p.m., and 10 p.m. Sometimes permission is given to omit the 2 a.m., temperature so that sleep is not disturbed. During later stages of convalescence temperature may be recorded only twice daily.

It is essential to examine the chart carefully before making any entry on it. In the chart shown, temperature is recorded correct to the first decimal place - i.e., each mark represents 0.1 of a Celsius degree. Even in sickness, temperature variation is rarely more than two to three degrees Celsius. So only a limited temperature scale need be shown on the chart.

When drawing the pulse graph on this chart, each mark represents 2 beats. Check the graphical presentation with the numerical representation below the graph. For both the numerical representation of the pulse and respiration rates, the spaces above the diagonal lines record from left to right, the 2 a.m., 10 a.m., and 6 p.m., rates, while the 6 a.m., 2 p.m., and 10 p.m., rates are recorded in the space below the diagonal line.

The graphical record of temperature on the chart shows the temperature recorded strictly four-hourly for days 1, 2 and 3. On days 4 and 5, the 2 a.m., \( T^0 \) has been omitted and on days 6 and 7 the T.P.R. has been taken only twice daily. Notice the gradual decline in temperature and pulse rate as the child responds to treatment. Sometimes when antibiotics are given these are shown on the temperature chart, so that any changes in temperature in response to the antibiotic can be noted.

Most healthy people show a slight fluctuation in temperature over a 24-hour period. The temperature is lowest in the morning, rising a little toward evening and then dropping again to the lower morning reading. This pattern may be present to a greater degree during illness, or the pattern may alter during the acute stage of illness to show a continual pyrexia (elevated temperature).
On the weight (mass) chart shown as Figure 12.2 we can see the record of a baby admitted to hospital because of failure to gain mass. During the first three weeks, mass is recorded daily. During the fourth week and the following two days, mass is recorded every second day. It is again necessary to examine the chart before making any entry or attempting to read the graph. Each division represents 10 grams. The baby's mass on admission is recorded as 7.95 kg. There is a slight loss on the two days following admission which is followed by a gradual gain in mass over the remaining weeks.

With adults, gain or loss of a few grams is not very significant and their mass may only be recorded at weekly intervals. A graph type record is still very useful for adults needing to gain or lose mass. Imagine the psychological effect of a falling graph on a person sticking to a rigid diet in an effort to lose mass! Any slight upward trend could indicate that food intake needed to be controlled more carefully.

The mass of every patient admitted to a hospital is recorded as part of the admission routine. A patient is again weighed pre-operatively and at regular intervals if his or her hospitalization is prolonged.

Figure 12.3 shows a treatment and medication sheet on which the doctor writes his orders about treatment and drugs for the particular patient. Compare the information to be shown on this sheet with the examples given in Chapters 9 and 10. Note that the strength to be given is to be clearly specified, the route by which the drug is to be given, i.e., by mouth (orally), by injection (hypodermically, subcutaneously, intramuscularly or intravenously) or by external application. In some cases drugs may also be given per rectum - the majority of these are in the form of specially prepared suppositories. Doctors may also, in certain cases, introduce other drugs into intravenous infusions. The doctor must also state the times when the medication is to be given. It may be a single dose to be given pre-operatively or it may be ordered strictly six hourly or eight hourly. Other likely instructions may be "twice daily" or "three times daily", "before meals" or "after meals".

Fluid Balance Charts are a means of recording the fluid intake of a patient and the fluids lost from the body. Over a 24 hour period, the fluid balance chart should show whether output equals input, whether output is greater than input, or whether input is greater than
output. Fluid loss occurs from various routes. It is easy to measure urinary output from an older child and an adult, but from babies it is often necessary to estimate the output rather than be able to record an actual volume. Fluid loss also occurs normally from the colon or large bowel in the faeces; from the lungs as water vapour and from the skin as sweat. In abnormal conditions fluid may be lost through vomiting, diarrhoea, haemorrhage or drainage from a wound or body cavity.

A person in normal health gets his required fluid intake from the various liquids he drinks and from the water present in foods. Some water is also manufactured in the body as a result of its metabolic activity. During illness, or following surgery or accident a person may not be able to take in enough fluid by mouth to meet his or her needs. Alternative routes must then be found. Sometimes the gastro-intestinal tract has to be completely by-passed, while in other cases a limited oral intake is supplemented by other methods of giving fluid. In some cases a tube is passed into the stomach and measured amounts of fluid are introduced through the tube at regular intervals. Premature babies and unconscious patients are often fed this way. This tube in the stomach is the intragastric (I.G.) tube referred to in the chart shown as Figure 12.5. The other most common route is the intravenous route. There is an illustration of a patient receiving fluid by the intravenous route in the frontispiece to Chapter 5. There are many fluids suitable for introduction by the intravenous route. Some are: whole blood, blood plasma, dextrose and saline.

Smaller volumes of fluid can be absorbed from the subcutaneous tissues. In these cases the needle conveying the fluid (which must be of water-like consistency) is not actually placed in the vein, but allowed to remain in loose subcutaneous tissues. Fluid may also be absorbed from the large bowel, so a rectal drip is another means of improving the fluid intake of a sick person.

Exact measurement of fluid taken into the body is not difficult, however it is not always so easy to measure with accuracy the fluid lost from the body. With very sick patients it is essential to know how the fluid intake compares with the fluid loss. If fluids lost cannot be measured accurately an intelligent estimation must be made. An ability to estimate with accuracy only comes with practice.
To gain such experience you need to take measured quantities of various fluids and then try to simulate the type of situation in which you may need to estimate fluid volumes. For example, pour out 5 mL of water onto a cement or tiled floor. On an adjacent area pour out 10 mL and then a variety of larger volumes. Next take a towel and pour various measured volumes on to it. Do the same with a large piece of cotton wool or any other absorbent material. Generally, fluids spilt or absorbed by materials appear to be of greater volume than they really are.

Figure 12.4 shows a typical fluid chart for a young child suffering from mild diarrhea and vomiting.

Figure 12.5 shows a more complicated chart used when patients are receiving fluid via intravenous, subcutaneous, or rectal infusions, or when an intragastric tube is used alone or in conjunction with the other methods mentioned above.

While each fluid balance chart shows the information about fluid intake and output for a day it is sometimes desirable to show a comparison of several days' totals. This time we will make our own graph. Refer Figs. 12.6 & 12.7.

We make the graph by representing quantity on a vertical scale with the date on a horizontal scale. A key indicates that red represents fluid intake while blue represents fluid output.

The information to be recorded on the graph is set out below.

<table>
<thead>
<tr>
<th>DATE</th>
<th>INTAKE mL</th>
<th>OUTPUT mL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1st</td>
<td>1734</td>
<td>1495</td>
</tr>
<tr>
<td>2nd</td>
<td>2102</td>
<td>1704</td>
</tr>
<tr>
<td>3rd</td>
<td>1591</td>
<td>1020</td>
</tr>
<tr>
<td>4th</td>
<td>1910</td>
<td>1903</td>
</tr>
<tr>
<td>5th</td>
<td>2272</td>
<td>1780</td>
</tr>
<tr>
<td>6th</td>
<td>1875</td>
<td>1648</td>
</tr>
<tr>
<td>7th</td>
<td>1875</td>
<td>1761</td>
</tr>
<tr>
<td>8th</td>
<td>2000</td>
<td>2000</td>
</tr>
</tbody>
</table>

If ruled graph paper is available the graph can be constructed with ease.

First, examine the graph paper and insert the vertical and horizontal scales. To do this you must work out the value of each square. Refer Fig. 12.6.
Looking at the vertical scale each square represents 20 mL. On the horizontal scale, six squares are used for each day.

Only the first day's input and output are shown in Fig. 12.6 leaving room for you to complete the graph.

STATISTICS

STATISTICS is a science involved with the collection, representation and interpretation of data. It is used to assist those responsible for making estimates or forecasts which allow an adequate health care system to be devised and maintained.

DATA is a name given to a collection of facts. The items, results, quantities involved are referred to as SCORES.

Many new terms are applied to the study of statistics. We will attempt to introduce only a few in this chapter.

Some simple examples will best demonstrate certain of the important facts to be gleaned from statistical data.

Take the information plotted on the graphs showing the relationship of fluid intake to output.

The statistician will want to calculate what is the average or mean of those scores. The MEAN of a group of scores is the sum of the scores divided by the number of scores. The number of scores is eight and the sum of those scores can be found by adding the eight numbers. The indicated sum is 15359. The mean then is 15359 ÷ 8, i.e., 1919.875

The MODE is the most frequent score in the given data. Only one score is repeated more than once so the mode is 1875.

The MEDIAN is the middle score of the given data when it is arranged in numerical order. The median has as many scores above it as it has below it.

In data which has an odd number of scores there is one median. The data we are working with at present has an even number of scores so there will be two middle scores. The mean of the two middle scores is then found.

First we will arrange the scores in ascending numerical order.
The two middle scores are 1875 and 1910.
The mean of 1875 and 1910 is:

\[
\frac{1875 + 1910}{2} = \frac{3785}{2}
\]

The median of the group of eight scores is then 1892.5.

THE NORMAL CURVE

When information from a large number of persons is plotted on a graph, the scores are found to follow a similar pattern. If we were to draw a graph showing the relative masses of all babies born in this state in one year we would find the graph looked like a bell - see Figure 12.8. This shows that the numbers at the extremes are similar and the greater majority are in the middle ranges.

Gradually more and more of our hospital records will be kept by electronic computers. Computer programming is a specialized field of study and nurses will not be required to undertake complicated work with computers. It is possible that input consoles will become part of the nurses' station in the hospital ward and nurses will be required to feed in data through these consoles.

It is because of the computer that we are going to explore some further facts about numbers. In Chapter 1 there is a statement - "In a later chapter we will look at other numerical systems with place value, but in which the grouping is not based on ten". That time has now come! Some students will have explored some of these systems already. It is not intended that students should study these other numerical systems in depth. The following pages are included more for interest. Any student who wants to pursue the subject can obtain extra details from the many books written on automatic data processing (A.D.P.) systems.
FIG. 12.8 Normal Distribution of Mass of Babies at Birth

\[ P_{3}^{97} = 94\% \text{ of all babies} \]

\[ P_{10}^{90} = 80\% \text{ of all babies} \]

\[ P_{25}^{75} = 50\% \text{ of all babies} \]

We described our decimal system as a place value system with a base of ten. We also saw how the same number can be expressed by different numerals or numerical expressions. For example, the number twelve can be represented by many numerals such as 12; 3 \times 4; 10 + 2; 15 - 3, etc. The NUMERALS are not the same, but they all represent the same number. Numerals are only symbols.

So far, all the numerals we have used to represent numbers have been from the decimal system or base ten system, where we group in tens. However, it is possible for any number greater than one to be used as a base.

If we group in twos we can have a numerical system with a base of two and with place value. This is known as the base two or BINARY SYSTEM.
If we group in threes we can have a numerical system with a base of three and with place value.
If we group in fours we can have a numerical system with a base of four and with place value.
If we group in fives we can have a numerical system with a base of five and with place value; and so on.
In the decimal system we grouped in tens. Thus the crosses xxxxxxxxxxxxx are grouped as:

\[
\begin{array}{ccccccccccccccc}
\times & \times & \times & \times & \times & \times & \times & \times & \times & \times & \times & \times \\
& & & & & & & & & & & & \\
\end{array}
\]

and we represent this number of crosses as:

<table>
<thead>
<tr>
<th>2nd place</th>
<th>1st place</th>
</tr>
</thead>
<tbody>
<tr>
<td>tens</td>
<td>ones</td>
</tr>
<tr>
<td>1</td>
<td>3</td>
</tr>
</tbody>
</table>

The numeral could be written as \(13_{\text{ten}}\).

Now let us group the same number of crosses in fives. We get:

\[
\begin{array}{ccccccccccccccc}
\times & \times & \times & \times & \times & \times & \times & \times & \times & \times & \times & \times \\
& & & & & & & & & & & & \\
\end{array}
\]

This grouping shows two groups of five and three single crosses, and we represent this number of crosses as:

<table>
<thead>
<tr>
<th>2nd place</th>
<th>1st place</th>
</tr>
</thead>
<tbody>
<tr>
<td>fives</td>
<td>ones</td>
</tr>
<tr>
<td>2</td>
<td>3</td>
</tr>
</tbody>
</table>

This numeral is written as \(23_{\text{five}}\).

It is read as "two, three base five". It tells us there are two groups of five and three ones.
In \(23_{\text{five}}\) the subscript "five" tells us that the numeral is written in base five or that we are grouping in FIVES. It is read as "two, three base five" so that it cannot be confused with twenty three.
Now let us group the same number of crosses in fours.

We get:

```
  x xxx  x xxx  x xxx  x
```

This grouping shows three groups of four and one single cross and we represent this as:

<table>
<thead>
<tr>
<th>2nd place</th>
<th>1st place</th>
</tr>
</thead>
<tbody>
<tr>
<td>fours</td>
<td>ones</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
</tr>
</tbody>
</table>

This numeral is written 31 four.

It is read as "three, one base four". It tells us there are three groups of four and one one.

In base five we have five digits: 0 1 2 3 and 4 called by their usual names, zero, one, two, three and four. We do not need 5 as it is shown by one group of five and none over, i.e., 10 five.

In base four we have four digits - 0, 1, 2 and 3. We don't need 4 as it is shown as one group of four and none over - i.e., 10 four.

In base three we have three digits - 0, 1 and 2. We do not need 3 as it is shown by one group of three and none over - i.e., 10 three.

If we try to group the previously used number of crosses in threes we have:

```
  xxx  xxx  xxx  xxx  x
```

i.e., four groups of three and one over.

We cannot write 41 three because the symbol 4 is not used in base three.

However, since we are grouping in threes we can group three of the groups into a larger group of three threes.

This is shown below:

```
  xxx  xxx  xxx  xxx  x
```

12.10
You can now see that we have 1 group of three threes, 1 group of three and 1 single cross.

<table>
<thead>
<tr>
<th>3rd place</th>
<th>2nd place</th>
<th>1st place</th>
</tr>
</thead>
<tbody>
<tr>
<td>$3^2$</td>
<td>$3^1$</td>
<td>(ones)</td>
</tr>
<tr>
<td>(three threes)</td>
<td>(threes)</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

This numeral is written as 111 three.

It is read as "one one one base three". It tells us there is one group of three threes, one group of three and 1 group of ones.

The left-hand 1 is in the third place and means 1 group of three threes, i.e., $3^2$.

The middle 1 is in the second place and means 1 group of three, i.e., $3^1$.

The right-hand 1 is in the first place and means 1 group of one.

Thus $111_{three} = (1 \times three \times three) + (1 \times three) + (1 \times one)$

= $(1 \times 3 \times 3) + (1 \times 3) + (1 \times 1)$

= $9 + 3 + 1$

= 13 (which is the equivalent decimal numeral)

The place values in the decimal system are:

1st place = ones or 1
2nd place = tens or ten or $10^1$
3rd place = hundreds or (ten x ten) or $10^2$
4th place = thousands or (tenxtenxten) or $10^3$

and so on.

The place values in the base three system are:
1st place = ones or 1
2nd place = threes or three
3rd place = three threes or three or (three x three)
4th place = (three x three x three) or three

No doubt you are wondering what practical value could be derived from studying other than the base ten or decimal system which we use in everyday life. Electronic computers can make use of the binary or base two system. The binary system uses only two digits 0 and 1 which can be likened to the two positions of a simple electrical switch, i.e., ON or OFF.

Current flow, i.e., "ON" is represented by the digit 1.

Absence of current flow, i.e., "OFF" is represented by the digit 0.

Any number may be expressed in terms of a base two numeral and then be recorded by allowing current to flow when the digit 1 is to be recorded and stopping current flow when the digit 0 is to be recorded.

If you wish to record the number ten, its base two equivalent is \(1010_{\text{two}}\) and so it could be recorded by ON OFF ON OFF.

The apparent disadvantage of the base two system is that so many digits are required to represent quite low numbers expressed in the base ten system. You need a base two numeral with five places to express the number twenty-five, i.e., \(11001_{\text{two}}\).

5th place 4th place 3rd place 2nd place 1st place
\(2^4\) \(2^3\) \(2^2\) \(2^1\)
\(2x2x2x2\) \((2x2)\) \((2x2)\) \((2)\)

25 = 1 1 0 0 1 (base two)

\(= (1x2x2x2x2) + (1x2x2x2) + (0x2x2) + (0x2) + (1x1)\)

\(= 16 + 8 + 0 + 0 + 1\)

\(= 25_{\text{ten}}\)

12.12
Just as in the base ten system, negative powers of ten were used to represent decimal numbers and mixed decimals - i.e., a whole number and a decimal, so in the base two system negative powers of two can be used to represent numerals of less than one. The point separating positive and negative powers of two is called the binary point.

\[
0.1011_{\text{two}} = (1 \times 2^{-1}) + (0 \times 2^{-2}) + (1 \times 2^{-3}) + (1 \times 2^{-4})
\]

\[
= \frac{1}{2} + 0 + \frac{1}{8} + \frac{1}{16}
\]

\[
= 0.5 + 0 + 0.125 + 0.0625
\]

\[
= 0.6875_{\text{ten}}
\]

In order to simplify these large strings of ones and zeros representing larger binary numbers, there is a base sixteen or hexadecimal system used. Each hexadecimal digit stands for four binary units.

Hexadecimal notation requires the use of sixteen symbols to represent sixteen number values. The decimal system provides ten number symbols 0 to 9, but six extra symbols are needed to represent the remaining values. The letters A, B, C, D, E, F have been adopted for this purpose.

In decimal notation, upon reaching the symbol 9, the decimal symbols are exhausted and so a "1 carry" is placed in front of each decimal symbol during the second cycle from 10 to 19.

In binary notation, the "1 carry" comes into effect after the second of the binary symbols has been used.

In hexadecimal notation, the "1 carry" comes into effect when the sixteen symbols have been exhausted.

For interest, a table of equivalent, decimal, binary and hexadecimal numerals is included.

Because each hexadecimal unit stands for four binary units, all binary units are expressed to a minimum of four places.
<table>
<thead>
<tr>
<th>DECIMAL</th>
<th>BINARY</th>
<th>HEXADECIMAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0000</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>0001</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0010</td>
<td>2</td>
</tr>
<tr>
<td>3</td>
<td>0011</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
<td>0100</td>
<td>4</td>
</tr>
<tr>
<td>5</td>
<td>0101</td>
<td>5</td>
</tr>
<tr>
<td>6</td>
<td>0110</td>
<td>6</td>
</tr>
<tr>
<td>7</td>
<td>0111</td>
<td>7</td>
</tr>
<tr>
<td>8</td>
<td>1000</td>
<td>8</td>
</tr>
<tr>
<td>9</td>
<td>1001</td>
<td>9</td>
</tr>
<tr>
<td>10</td>
<td>1010</td>
<td>A</td>
</tr>
<tr>
<td>11</td>
<td>1011</td>
<td>B</td>
</tr>
<tr>
<td>12</td>
<td>1100</td>
<td>C</td>
</tr>
<tr>
<td>13</td>
<td>1101</td>
<td>D</td>
</tr>
<tr>
<td>14</td>
<td>1110</td>
<td>E</td>
</tr>
<tr>
<td>15</td>
<td>1111</td>
<td>F</td>
</tr>
<tr>
<td>16</td>
<td>10000</td>
<td>10</td>
</tr>
<tr>
<td>17</td>
<td>10001</td>
<td>11</td>
</tr>
<tr>
<td>18</td>
<td>10010</td>
<td>12</td>
</tr>
<tr>
<td>19</td>
<td>10011</td>
<td>13</td>
</tr>
<tr>
<td>20</td>
<td>10100</td>
<td>14</td>
</tr>
<tr>
<td>21</td>
<td>10101</td>
<td>15</td>
</tr>
<tr>
<td>22</td>
<td>10110</td>
<td>16</td>
</tr>
<tr>
<td>23</td>
<td>10111</td>
<td>17</td>
</tr>
<tr>
<td>24</td>
<td>11000</td>
<td>18</td>
</tr>
<tr>
<td>25</td>
<td>11001</td>
<td>19</td>
</tr>
<tr>
<td>26</td>
<td>11010</td>
<td>1A</td>
</tr>
<tr>
<td>27</td>
<td>11011</td>
<td>1B</td>
</tr>
<tr>
<td>28</td>
<td>11100</td>
<td>1C</td>
</tr>
<tr>
<td>29</td>
<td>11101</td>
<td>1D</td>
</tr>
<tr>
<td>30</td>
<td>11110</td>
<td>1E</td>
</tr>
<tr>
<td>31</td>
<td>11111</td>
<td>1F</td>
</tr>
</tbody>
</table>
CONVERSION OF DECIMAL NUMERALS TO BINARY NUMERALS

Two easy methods of conversion may be used, but before we discuss these we must become familiar with two new terms.

We know that with a place value system, the value of the number depends on the relative position of the individual digits as well as on the digits themselves.

In any numerical system with place value, the digit in the extreme right is called the "least significant digit" or L.S.D. because it is the digit of least value. The digit on the extreme left is called the "most significant digit" or M.S.D. because it is the digit of highest value.

In the decimal numeral 25, the two is the M.S.D. and the five is the L.S.D.
In the binary numeral 1010, the left-hand one is the M.S.D. and the right-hand zero is the L.S.D.
In the hexadecimal numeral 1F, the one is the M.S.D. and the F is the L.S.D.

METHOD A

Divide the decimal number repeatedly by 2, until the quotient of 0 is obtained. The equivalent binary numeral is composed of the remainders. The first remainder is the least significant digit and the last remainder is the most significant digit.

The declining quotients may be set out one under the other and the remainders along side.
Convert the decimal number 10 to a binary number.

<table>
<thead>
<tr>
<th>QUOTIENTS</th>
<th>REMAINDERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>10\text{ten}</td>
<td>0 (L.S.D.)</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1 (M.S.D.)</td>
</tr>
<tr>
<td>0</td>
<td>1010\text{two}</td>
</tr>
</tbody>
</table>

12.15
Convert the decimal number 25 to a binary number.

<table>
<thead>
<tr>
<th>QUOTIENTS</th>
<th>REMAINDERS</th>
</tr>
</thead>
<tbody>
<tr>
<td>25&lt;sub&gt;ten&lt;/sub&gt;</td>
<td></td>
</tr>
<tr>
<td>12</td>
<td>1 (L.S.D.)</td>
</tr>
<tr>
<td>6</td>
<td>0</td>
</tr>
<tr>
<td>3</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>0</td>
<td>1 (M.S.D.)</td>
</tr>
</tbody>
</table>

25<sub>ten</sub> = 11001<sub>two</sub>

**METHOD B**

In this method we seek the highest power of two in the decimal number. This gives the place value of the most significant digit. The highest power of two is subtracted from the number and the next highest power of two applied to the remainder. The process again continues till the lowest power of two is applied which determines the position of the least significant digit.

Using Method B we will convert the decimal number 10 to a binary number.

<table>
<thead>
<tr>
<th>DECIMAL</th>
<th>BINARY DIGITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>Highest power of 2 in 10:</td>
<td>10 = (1 \times 2^3) 1 (M.S.D.)</td>
</tr>
<tr>
<td>remainder:</td>
<td>8</td>
</tr>
<tr>
<td>Next highest power of 2:</td>
<td>0 = (0 \times 2^2) 0</td>
</tr>
<tr>
<td>remainder:</td>
<td>2</td>
</tr>
<tr>
<td>Next highest power of 2:</td>
<td>2 = (1 \times 2^1) 1</td>
</tr>
<tr>
<td>remainder:</td>
<td>0</td>
</tr>
<tr>
<td>Lowest power of 2:</td>
<td>0 = (0 \times 2^0) 0 (L.S.D.)</td>
</tr>
<tr>
<td>remainder:</td>
<td>0</td>
</tr>
</tbody>
</table>

'. Decimal number 10 = binary number 1010

12.16
Convert the decimal number 25 to a binary number.

<table>
<thead>
<tr>
<th>DECIMAL</th>
<th>BINARY DIGITS</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>1 (M.S.D.)</td>
</tr>
<tr>
<td>16</td>
<td>1</td>
</tr>
<tr>
<td>8</td>
<td>0</td>
</tr>
<tr>
<td>4</td>
<td>0</td>
</tr>
<tr>
<td>2</td>
<td>0</td>
</tr>
<tr>
<td>1</td>
<td>1 (L.S.D.)</td>
</tr>
</tbody>
</table>

Remainder:

Highest power of 2 in 25: \( \frac{25}{16} = 1 \times 2^4 \) remainder: \( 9 \)
Next highest power of 2: \( \frac{9}{8} = 1 \times 2^3 \) remainder: \( 1 \)
Next highest power of 2: \( \frac{1}{4} = 0 \times 2^2 \) remainder: \( 1 \)
Next highest power of 2: \( \frac{1}{2} = 0 \times 2^1 \) remainder: \( 1 \)
Lowest power of 2: \( \frac{1}{0} = 1 \times 2^0 \) remainder: \( 0 \)

.'. Decimal number 25 = binary number 11001

CONVERSION OF DECIMAL NUMERALS TO HEXADECIMAL NUMERALS

The decimal number is divided repeatedly by 16 until a zero quotient is obtained. Decimal remainders from 10 to 15 are converted to hexadecimal symbols A-F. The first remainder is the least significant digit and the last remainder is the most significant digit.

Convert the decimal number 195 to a hexadecimal number.

Divide by base 16 = Quotient + Remainder (= Hexadecimal digits)

195 ÷ 16 = 12 + 3 = 3 (L.S.D.)
12 ÷ 16 = 0 + 12 = C (M.S.D.)

.'. Decimal number 195 = hexadecimal number C3.

Convert the decimal number 1710 to a hexadecimal number.
Divide by base 16 = Quotient + Remainder = Hexadecimal digits

$1710 \div 16 = 106 + 14 = E \ (L.S.D.)$

$106 \div 16 = 6 + 10 + A$

$6 \div 16 = 0 + 6 = 6 \ (M.S.D.)$

\therefore \text{Decimal number } 1710 = \text{Hexadecimal number } 6AE$

OR $1710_{\text{ten}} = 6AE_{\text{sixteen}}$

We do not intend to pursue this adventure into other numerical systems any further. If at any time you wish, or need, to find out more, there are many books which will provide you with the knowledge you require. T.E.S. Library may be able to help you.
FIG. 12.2
### Treatment & Medication Sheet

<table>
<thead>
<tr>
<th>DATE</th>
<th>DRUG NAME</th>
<th>DRUG STRENGTH</th>
<th>ROUTE</th>
<th>ORDERED SPECIFIED DOSAGE</th>
<th>SIGN AM PM AM PM AM PM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
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<td></td>
</tr>
</tbody>
</table>

**FIG. 12.3**

**MEDICAL RECORD No.**

**SEX/DATE OF BIRTH**

**WEIGHT:**

**ADDRESS:**

**PATIENT'S NAME:**

**WARD:**

HEALTH MATHEMATICS  
CHAPTER 12
<table>
<thead>
<tr>
<th>Time</th>
<th>Fluid In</th>
<th>Fluid Out</th>
<th>Urine</th>
<th>Vomitus</th>
<th>Faeces</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 a.m.</td>
<td>100</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 a.m.</td>
<td>120</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 a.m.</td>
<td>140</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 a.m.</td>
<td>160</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 a.m.</td>
<td>180</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 a.m.</td>
<td>200</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7 a.m.</td>
<td>220</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 a.m.</td>
<td>240</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9 a.m.</td>
<td>260</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 a.m.</td>
<td>280</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11 a.m.</td>
<td>300</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12 noon</td>
<td>320</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1 p.m.</td>
<td>340</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 p.m.</td>
<td>360</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3 p.m.</td>
<td>380</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4 p.m.</td>
<td>400</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5 p.m.</td>
<td>420</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6 p.m.</td>
<td>440</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7 p.m.</td>
<td>460</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8 p.m.</td>
<td>480</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9 p.m.</td>
<td>500</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10 p.m.</td>
<td>520</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11 p.m.</td>
<td>540</td>
<td>0</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Fig. 12.4**

**Fluid Chart**

**Medical Officer**
**Patient's Name**
**Address**

**Chapter 12**

**Health Mathematics**
### FLUID INTAKE

<table>
<thead>
<tr>
<th>Time</th>
<th>Progressive Readings</th>
<th>Amount in Each Hour</th>
<th>Nature of Fluid &amp; Rate</th>
<th>By Mouth or I.G. Tube</th>
<th>Remarks e.g. B.P., Pupil, Drugs, etc.</th>
<th>T.</th>
<th>P.</th>
<th>E.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.15 p.m.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3.45</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.00</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.30</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4.45</td>
<td></td>
<td></td>
<td></td>
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### FLUID LOSS

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**FIG. 12.5**
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FIG. 12.1
Graph showing fluid intake and output over 8 days

FIG. 12.6
Graph showing fluid intake and output over 8 days

**FIG. 12.7**
HEALTH MATHEMATICS

DESCRIPTION
Topics include: The Language of Mathematics; Measurement; Quantities and Their Measures; Calculation of Drug Dosages; Recording of Data. Study Guide 52-409 Health Mathematics is used with this text.

CATEGORY
Health and Community Services