TRADE CALCULATIONS FOR FABRICATORS
TRADE CALCULATIONS FOR FABRICATORS

By

Bryan Shatlock

A C.R. & D. Project

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### COMPETENCY PROFILE FOR MEM2.7C10A Perform Computations – Basic

<table>
<thead>
<tr>
<th>A competent person who can:</th>
<th>Will be able to perform these skills/tasks</th>
<th>To produce/service or applicable to:</th>
<th>Supported by underpinning knowledge of:</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEM2.7C10A Perform Computations – Basic (Prerequisites: Nil)</td>
<td>Without the aid of a calculator, perform the following mathematical functions: Add, subtract, multiply, divide whole numbers. Describe the <strong>S.I. System</strong>. State the basic and derived units that comprise the S.I. System. Convert units within the S.I. System (eg m to mm etc). Solve problems involving the use of proper and improper <strong>fractions</strong> and mixed numbers (ie +, −, ×, /). Convert fractions into decimals, decimals into fractions and <strong>decimals</strong> and fractions into <strong>percentages</strong> and vice versa. Solve problems involving percentages. Simple calculations are performed using the four basic rules, addition, subtraction, multiplication and division.</td>
<td>Convert units within the S.I. System to produce more usable/realistic values. Establish the relationship between fractions, decimals, and percentages. Solve simple workshop problems related to the above.</td>
<td>Basic mathematical processes. The S.I. System. The decimal system. Percentages.</td>
</tr>
</tbody>
</table>

**Competency dimensions**
- 1, 2
- 1, 2, 4
- 3, 4

**Sources of evidence/assessment tool**
- 2.7C10A Workbook
- Work diary/folio
- STAR sheets
- Student performance rating sheet
- 2.7C10A Maths test
### COMPETENCY PROFILE FOR MEM2.8C10A Perform Computations

<table>
<thead>
<tr>
<th>A competent person who can:</th>
<th>Will be able to (perform these skills/tasks):</th>
<th>To produce/service or applicable to:</th>
<th>Supported by underpinning knowledge of:</th>
<th>Located in these learning resources:</th>
</tr>
</thead>
<tbody>
<tr>
<td>MEM2.8C10A Perform Computations (Prerequisites: 2.7C10A)</td>
<td>Carry out basic mathematical tasks using a <strong>calculator</strong>. With the aid of a calculator: solve mathematical problems involving <strong>ratios</strong>; solve mathematical problems involving direct and inverse <strong>proportion</strong>; <strong>apply formulae</strong> to calculate derived data as required; calculate area, volume, mass of plates, prisms, pyramids, spheres and steel sections, and solve related problems; interpret information from tables and charts.</td>
<td>Carry out typical workshop calculations related to the fabrication trade. Should include but not be limited to: costing of materials and processes; mass/SWL calculation; determination of lengths and diameters using proportion; determination of data using ratios.</td>
<td>Standard scientific calculator operations and functions. Basic mathematical functions (BOMDAS). Basic mathematical procedures when using formulas. Percentages. Decimals. S.I. System.</td>
<td>Mathematics for Technicians Alidis, B and Pantlin, K MEM98/010 Maths Worksheets WestOne Pub</td>
</tr>
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</table>

### Competency dimensions

<table>
<thead>
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<th>1, 2</th>
<th>1, 2, 4</th>
<th>3, 4</th>
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### Sources of evidence/assessment tool

<table>
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<tr>
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<table>
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<tr>
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<th>Student performance rating sheet</th>
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</table>

| STAR sheets |  |
## COMPETENCY PROFILE FOR MEM2.13C5A  Perform Mathematical Computations

A competent person who can:

<table>
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<th>To produce/service or applicable to:</th>
<th>Supported by underpinning knowledge of:</th>
<th>Located in these learning resources:</th>
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<td>With the aid of a calculator:</td>
<td>Solving problems related to surface</td>
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</tr>
<tr>
<td>Perform</td>
<td>perform calculations using Pythagoras'</td>
<td>area, volume, mass, capacity, SWL and</td>
<td>Aldis, B and Pantlin, K</td>
</tr>
<tr>
<td>Mathematical</td>
<td>Theorem;</td>
<td>costs of plates, tanks, spheres,</td>
<td>MEM98/010 Maths Worksheets</td>
</tr>
<tr>
<td>Computations</td>
<td>solve problems using the intersecting</td>
<td>structural members, cones, diagonal</td>
<td>WestOne Pub</td>
</tr>
<tr>
<td>(Prerequisites:</td>
<td>chords formula; transpose formulae</td>
<td>measurements, etc. Solving workshop</td>
<td></td>
</tr>
<tr>
<td>2.7C10A,</td>
<td>to solve workshop problems as</td>
<td>problems involving non-right triangles</td>
<td></td>
</tr>
<tr>
<td>2.8C10A)</td>
<td>required; apply SIN, COS, and TAN</td>
<td>where included angles are &lt;90 degrees.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>ratios to solve problems involving</td>
<td>Functions including +, −, ×, /, roots,</td>
<td></td>
</tr>
<tr>
<td></td>
<td>right-angled triangles and non-right</td>
<td>powers, parenthesis. Geometrical</td>
<td></td>
</tr>
<tr>
<td></td>
<td>triangles where included angles are</td>
<td>drawing. Marking out.</td>
<td></td>
</tr>
<tr>
<td></td>
<td>&lt;90 degrees.</td>
<td></td>
<td></td>
</tr>
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<td></td>
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<td></td>
</tr>
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<td></td>
<td>Competency dimensions</td>
<td>1, 2, 4</td>
<td>3, 4</td>
</tr>
<tr>
<td></td>
<td>Sources of evidence/assessment tool</td>
<td>2.13C5A Worksheets Structural projects, piping projects and other projects to be identified. 2.13C5A Maths test</td>
<td></td>
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<tr>
<td></td>
<td>2.13C5A Maths test</td>
<td>Work diary/folio</td>
<td>Student performance rating sheet</td>
</tr>
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Prerequisites: 2.7C10A, 2.8C10A
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PART 1

MEM2.7C10A
Perform Computations – Basic
SECTION 1

Decimals
OBJECTIVE 1

TO STATE THE PLACE VALUE OF A DIGIT IN A DECIMAL

The decimal 0.3 means \( \frac{3}{10} \), i.e. three-tenths.

The decimal 0.27 means \( \frac{27}{100} \), i.e. twenty-seven hundredths.

This can be written as:

\[
\frac{27}{100} = \frac{20}{100} + \frac{7}{100} = \frac{2}{10} + \frac{7}{100}
\]

by reducing the \( \frac{20}{100} \) to the simpler form \( \frac{2}{10} \).

So, 0.27 means \( \frac{2}{10} + \frac{7}{100} \).

Similarly,

\[
0.518 = \frac{518}{1000} = \frac{500}{1000} + \frac{10}{1000} + \frac{8}{1000} = \frac{5}{10} + \frac{1}{100} + \frac{8}{1000}
\]

From these examples, we can see that the digit in the first decimal place, i.e. the first digit to the right of the decimal point, represents tenths (e.g. in 0.3 the 3 represents \( \frac{3}{10} \), in 0.27 the 2 represents \( \frac{2}{10} \)) and in 0.518 the 5 represents \( \frac{5}{10} \).

Similarly, the digit in the second decimal place represents hundredths (e.g. the 7 in 0.27 and the 1 in 0.518) while the digit in the third decimal place represents thousandths (e.g. the 8 in 0.518).

This can be summarised as follows:

<table>
<thead>
<tr>
<th>Place</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>First decimal place</td>
<td>tenths</td>
</tr>
<tr>
<td>Second decimal place</td>
<td>hundredths</td>
</tr>
<tr>
<td>Third decimal place</td>
<td>thousandths</td>
</tr>
<tr>
<td>Fourth decimal place</td>
<td>ten-thousandths</td>
</tr>
<tr>
<td>Fifth decimal place</td>
<td>hundred-thousandths</td>
</tr>
</tbody>
</table>
Consider the number 3.85. The 8 is in the first decimal place, so its *place value* is one tenth. However, its *value*, or the number, which it represents is 8. Similarly, the 5 is the second decimal place, so its place value is one hundredth. The number which the 5 represents is \( \frac{5}{100} \).

*Example 1*  
What is the place value of the 7 in 3.867?  
The 7 is in the third decimal place, so its place value is one thousandth.

*Example 2*  
What is the place value of the 3 in 2.345?  
The 3 is in the first decimal place, so its place value is one tenth.

*Example 3*  
What is the value of the 6 in 3.867?  
The 6 is in the second decimal place, so its value is 6 hundredths or \( \frac{6}{100} \).

*Example 4*  
What is the value of the 3 in 8.356?  
The 3 is in the first decimal place, so its value is 3 tenths or \( \frac{3}{10} \).

*Example 5*  
What does each numeral represent (what is its value) in the number 576.238?  
The 5 represents 5 hundred or 500.  
The 7 represents 7 tens or 70.  
The 6 represents 6 units or 6.  
The 2 represents 2 tenths or \( \frac{2}{10} \).  
The 3 represents 3 hundredths or \( \frac{3}{100} \).  
The 8 represents 8 thousandths or \( \frac{8}{1000} \).

**EXERCISE**

What does each numeral represent in 68.57?
OBJECTIVE 2

TO ROUND A GIVEN DECIMAL TO THE NEAREST WHOLE NUMBER, TENTH, HUNDREDTH OR THOUSANDTH

Example 1  Round 2.4 to the nearest whole number.

This means, ‘find the whole number, which is nearest to 2.41’. If we look at the diagram above, we can clearly see that the whole number closest to 2.4 is 2 so, 2.4, rounded to the nearest whole number is 2.

Example 2  Round 2.8 to the nearest whole number.

This means, ‘find the whole number, which is nearest to 2.8’. We clearly have to choose between 2 and 3 and, from the diagram, we can see that 3 is the one we require. So, 2.8 rounded to the nearest whole number is 3.

Example 3  Round 2.5 to the nearest whole number.

This means, ‘find the whole number, which is nearest to 2.5’. Looking at the diagram, we can see that 2.5 is exactly halfway between 2 and 3, so there is no ‘nearest whole number’ to 2.5. This situation requires special treatment and we will look at this problem again later.

Example 4  Round 2.64 to the nearest tenth.

Clearly, 2.64 is between 2.6 and 2.7 – these are the ‘tenths’ nearest to 2.64. Which of these is nearer to 2.64?

Looking at this expanded diagram, we can see that 2.64 is closer to 2.6. So, 2.64 rounded to the nearest tenth is 2.6.
Example 5  
Round 2.67 to the nearest tenth.

From the diagram, we can easily see that 2.67 is nearer to 2.7 than it is to 2.6. So, 2.67, rounded to the nearest tenth, is 2.7.

Summarising these examples, we find:

- 2.4 rounded to the nearest whole number is 2.
- 2.8 rounded to the nearest whole number is 3.
- 2.5 rounded to the nearest whole number is ?
- 2.64 rounded to the nearest tenth is 2.6.
- 2.67 rounded to the nearest tenth is 2.7.

A quick method of rounding (which can be easily justified by using diagrams similar to those given) is shown in the following examples.

**RULES**

1. Determine the number of decimal places in the final result.

<table>
<thead>
<tr>
<th>ROUND TO THE NEAREST</th>
<th>NUMBER OF DECIMAL PLACES</th>
</tr>
</thead>
<tbody>
<tr>
<td>whole number</td>
<td>0</td>
</tr>
<tr>
<td>tenth</td>
<td>1</td>
</tr>
<tr>
<td>hundredth</td>
<td>2</td>
</tr>
<tr>
<td>thousandth</td>
<td>3</td>
</tr>
<tr>
<td>ten-thousandth</td>
<td>4</td>
</tr>
<tr>
<td>etc.</td>
<td></td>
</tr>
</tbody>
</table>

2. Draw a dotted line immediately to the right of these decimal places – these will be the retained digits.

3. (a) If the first digit to the right of the dotted line is 0, 1, 2, 3 or 4, the last retained digit is unaltered.

(b) If the first digit to the right of the dotted line is 6, 7, 8 or 9, or if it is a 5 followed by any non-zero digits, the last retained digit is increased by 1.

(c) If the only digit to the right of the dotted line is a 5, then the last retained digit is unaltered if it is even, but is increased by one if it is odd. Notice that in this case, the last retained digit will always finish up being an even digit.

Example 1  
Round 6.314 to the nearest tenth.

\[
6.3 \div 14 \rightarrow 6.3 \text{ using } 3(a)
\]

Example 2  
Round 7.2867 to the nearest hundredth.

\[
7.28 \div 67 \rightarrow 7.29 \text{ using } 3(b)
\]
Example 3  Round 8.152 to the nearest tenth.
8.1 | 52  →  8.2 using 3(b)

Example 4  Round 4.2509 to the nearest hundredth.
4.25 | 09  →  4.25 using 3(b)

Example 5  Round 2.796 to the nearest hundredth.
2.79 | 6  →  2.80 using 3(b)

Example 6  Round 3.997 to the nearest hundredth.
3.99 | 7  →  4.00 using 3(b)

Example 7  Round 2.45 to the nearest tenth.
2.4 | 5  →  2.4 using 3(c)

Example 8  Round 2.55 to the nearest tenth.
2.5 | 5  →  2.6 using 3(c)

Example 9  Round 2.995 to the nearest hundredth.
2.99 | 5  →  3.00

Important note in rounding
Never round ‘one step at a time’, eg round 2.345 8 to the nearest tenth.

Remember!  This means ‘find the number, with one decimal place’. (ie with tenths only, not hundredths, thousandths, etc), which is nearest to 2.345 8. If we look at the diagram below, we see that 2.345 8 is closer to 2.3 than 2.4, so, 2.345 8 rounded to the nearest tenth is 2.3.

<table>
<thead>
<tr>
<th>2.28</th>
<th>2.29</th>
<th>2.3</th>
<th>2.31</th>
<th>2.32</th>
<th>2.33</th>
<th>2.34</th>
<th>2.35</th>
<th>2.36</th>
<th>2.37</th>
<th>2.38</th>
<th>2.39</th>
<th>2.4</th>
<th>2.41</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.3458</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

However, a common error is to round ‘one step at a time’. For example, first round 2.3458 to the nearest thousandth.

2.345 | 8  →  2.346

Then, round 2.346 to the nearest hundredth.

2.34 | 6  →  2.35
Then, round 2.35 to the nearest tenth.

\[ 2.35 \rightarrow 2.4 \]

\textit{ie} 2.3458 is incorrectly rounded to 2.4

\[ \therefore \text{ never round 'one step at a time'.} \]
OBJECTIVE 3

TO CONVERT A FRACTION OR A MIXED NUMBER INTO A DECIMAL

To convert a fraction to a decimal, divide the denominator into the numerator.

Example 1

Convert \( \frac{3}{4} \) into a decimal.

\[
4 \overline{)3.000}
\]

Put a decimal point after the 3, and then three or four zeroes after the decimal point (more zeroes may be required – this will become evident as we proceed).

\[
4 \overline{)3.000}
\]

Put a decimal point in the answer immediately above the other decimal point.

\[
0.7
\]

Now, we can start the division. ‘4 into 3 won’t go’, so put a zero above the 3.

\[
0.7\overline{0}
\]

Then, ‘4 into 30 goes 7 times’, so put a 7 in the answer.

\[
0.7\overline{0}
\]

Multiply 7 by 4, giving 28; write 28 under the 30, and subtract the 28 from the 30, giving 2.
Now, divide 4 into 20. This gives 5, so write 5 beside the 7 in the answer.

\[
\begin{array}{c}
0.75 \\
4) 3.000 \\
2 8** \\
20 \\
\hline
0
\end{array}
\]

Multiply 5 by 4, giving 20. Write this under the other 20, and subtract.

\[
\begin{array}{c}
0.75 \\
4) 3.000 \\
2 8** \\
20 \\
20 \\
\hline
0
\end{array}
\]

Since the result of this subtraction is zero, we are finished.

So, \( \frac{3}{4} = 0.75 \).

Notice, in this example, we used only two of the three zeroes, which we originally put after the decimal point.

**Example 2** Convert \( \frac{5}{8} \) into a decimal.

Divide 8 into 5.

\[
\begin{array}{c}
0.625 \\
8) 5.000 \\
4 8** \\
20 \\
16 \\
40 \\
40 \\
\hline
0
\end{array}
\]

So, \( \frac{5}{8} = 0.625 \).
Example 3  Convert \(\frac{7}{16}\) into a decimal.

\[
\begin{array}{c|c}
& 0.437 \\
16 & 7.000 \\
6 & 4 \ldots \\
60 & 60 \\
48 & 48 \\
120 & 120 \\
112 & 112 \\
8 & 80 \\
80 & 80 \\
0 & 0 \\
\end{array}
\]

When we get this far, we have to put extra zeroes in the original number, so that we can proceed. As many zeroes as you like can be put in. It is customary to start with 3 or 4 zeroes, and then put extra zeroes one at a time, as needed.

\[
\begin{array}{c|c}
& 0.4375 \\
16 & 7.0000 \\
6 & 4 \ldots \\
60 & 60 \\
48 & 48 \\
120 & 120 \\
112 & 112 \\
80 & 80 \\
80 & 80 \\
0 & 0 \\
\end{array}
\]

So, \(\frac{7}{16} = 0.4375\).

Example 4  Convert \(\frac{5}{6}\) into a decimal.

Divide 6 into 5, as before.

\[
\begin{array}{c|c}
& 0.83333 \ldots \\
6 & 5.0000 \\
4 & 8 \ldots \\
20 & 20 \\
18 & 18 \\
20 & 20 \\
18 & 18 \\
2 & 2 \\
\end{array}
\]
Notice now, though, that as we do the subtraction, the remainder is always 2, so the result of the divisions will always be 3, and this will continue indefinitely.

So, \( \frac{5}{6} = 0.833 3 \ldots \) with the 3 repeated indefinitely. A decimal like 0.833 3 is called a recurring, or a repeating decimal. Sometimes a special notation is used, and 0.833 3 \ldots \) is written as:

\[
0.83 \text{ or } 0.\overline{83}
\]

**Example 5**  
Convert \( \frac{3}{7} \) into a decimal.

When we divide 7 into 3, we obtain 0.428 571 428 571 \ldots \) with the six digits 428 571 all repeating, as a group. We write this as:

\[
0.428 \text{ or } 0.\overline{428\,571}
\]

To correct a mixed number to a decimal, first convert the fractional part to a decimal.

**Example 6.**  
Convert \( 3\frac{7}{8} \) into a decimal.

\[
3\frac{7}{8} = 3 + \frac{7}{8} \\
= 3 + 0.875 \\
= 3.875
\]

Convert \( \frac{7}{8} \) to a decimal, as before, by dividing the denominator (8) into the numerator (7).

**Example 7.**  
Convert \( 4\frac{2}{3} \) into a decimal.

\[
4\frac{2}{3} = 4 + \frac{2}{3} \\
= 4 + 0.666 \ldots \\
= 4 + 0.\overline{6} \\
= 4.\overline{6} \text{ or } 4.666 \ldots
\]

**EXERCISE**

Express each of the following as a decimal.

\[
\frac{3}{25}, \frac{7}{10}, \frac{15}{100}, \frac{7}{125}, \frac{9}{20}, \frac{3}{8}, \frac{1}{3}, \frac{4}{5}, \frac{7}{8}
\]
OBJECTIVE 4

TO CONVERT A TERMINATING DECIMAL INTO A FRACTION OR A MIXED NUMBER

Example 1  Convert 0.3 into a fraction.

0.3 means three-tenths, \( ie \)  \( 0.3 = \frac{3}{10} \)

Example 2  Convert 0.27 into a fraction.

0.27 means twenty-seven hundredths, \( ie \)  \( 0.27 = \frac{27}{100} \)

Example 3  Convert 0.353 into a fraction.

0.353 means three hundred and fifty-three thousandths,

\( ie \)  \( 0.353 = \frac{353}{1000} \)

Example 4  Convert 0.28 into a fraction.

0.28 means twenty-eight hundredths \( ie \)  \( 0.28 = \frac{28}{100} \)

**NOTE:** The \( \frac{28}{100} \) can be reduced to a simpler form.

So, \( 0.28 = \frac{28}{100} = \frac{7}{25} \).

Example 5  Convert 2.37 into a mixed number.

2.37 means 2 and thirty-seven hundredths,

\( ie \)  \( 2.37 = 2 \frac{37}{100} \)

Example 6  Convert 3.058 into a mixed number.

3.058 means 3 and fifty-eight thousandths,

\( ie \)  \( 3.058 = 3 \frac{58}{1000} \)

\[ = 3 \frac{29}{500} \]
EXERCISE

Express each of the following as a mixed number.

9.6, 3.5, 2.25, 6.14, 7.125, 1.06
OBJECTIVE 5

TO ADD TWO OR MORE DECIMALS

Example 1  Add 3.29, 4.16 and 2.08.

\[
\begin{align*}
3.29 \\
4.16 \\
2.08 \\
\hline
9.53
\end{align*}
\]

Arrange the numbers in a column, with the decimal points all lined up vertically.

Put a decimal point in the answer under all the other decimal points, then add, just as with whole numbers, starting from the right hand side.

Example 2  Find the sum of 2.85, 3.1, 9.723 and 1.341.

\[
\begin{align*}
2.850 \\
3.100 \\
9.723 \\
1.341 \\
\hline
17.014
\end{align*}
\]

It might be helpful to put zeroes (0s) in these places, especially in the earlier stages.

EXERCISE 1

Add the following.

1. 2.46 + 3.19 + 2.84 + 1.72
2. 6.81 + 2.9 + 3.04 + 11.5
3. 2.97 + 1.8 + 3.004 + 1.6
4. 1.65 + 3.64 + 5.63
5. 4.008 + 3.07 + 1

Example 3  Multiply 2.684 by 100.

To multiply by 100, shift the decimal point 2 places to the right.

\[
\begin{align*}
2.684 \\
\hline
268.4
\end{align*}
\]

So, 2.684 \times 100 = 268.4.
Example 4 Multiply 3.14 by 1 000.

To multiply by 1000, shift the decimal point 3 places to the right.

\[ 3.140 \]

So, \( 3.14 \times 1000 = 3140 \).

Notice, that we need to insert a zero, in order to allow the third shift.

**EXERCISE 2**

Simplify the following.

1. \( 9.85 \times 100 \)
2. \( 0.087 \times 10000 \)
3. \( 2.763 \times 100 \)
4. \( 8.8 \times 100000 \)
OBJECTIVE 6

TO MULTIPLY TWO OR MORE DECIMALS

Example 1  Multiply 2.68 by 3.1.

Step 1  Ignore the decimal point, and multiply 268 by 31.

\[
\begin{array}{c}
263 \\
\times 31 \\
268 \\
804 \\
8308 \\
\end{array}
\]

Step 2  Find the total number of decimal places in the numbers being multiplied.

2.68 has 2 decimal places.
3.1 has 1 decimal place.
So, the total is 3 decimal places.

Step 3  Insert a decimal point, so that the product has 3 decimal places.

\[ie \; 8.308\]
\[\therefore \; 2.68 \times 3.1 = 8.308\]

Example 2  Multiply 1.05 by 0.23.

Step 1  \(105 \times 23 = 2415\)

Step 2  

\[
\begin{array}{c}
1.05 \\
\quad 2 \text{ decimal places} \\
0.23 \\
\quad 2 \text{ decimal places} \\
\hline
\text{Total} \\
\quad 4 \text{ decimal places} \\
\end{array}
\]

So, \(1.05 \times 0.23 = 0.2415\).
EXERCISE

Simplify:

1. $6.8 \times 2.3$
2. $3.14 \times 2.07$
3. $4.02 \times 2.04$
4. $10.03 \times 3.4$
5. $27.6 \times 0.00349$
OBJECTIVE 7

TO DIVIDE A DECIMAL BY 10, 100, 1 000, etc

Example 1  Divide 12.3 by 10.

\[
12.3 \div 10 = 12 \frac{3}{10} \div 10
\]

\[
= \frac{123}{10} \times \frac{1}{10}
\]

\[
= \frac{123}{100}
\]

\[
= 1.23
\]

So, \( 12.3 \div 10 = 1.23 \).

Notice that the decimal point has been shifted one place to the left.

Remember, when we multiplied a decimal by 10, 100, or 1 000, we simply shifted the decimal point 1, 2 or 3 places to the right.

Now, when we divide a decimal by 10, 100 or 1 000, we simply shift the decimal point 1, 2 or 3 places to the left.

Example 2  Divide 3.6 by 100.

To divide by 100, move the decimal point 2 places to the left. (This requires putting in an extra zero.)

\( 03.6 \)

So, \( 3.6 \div 100 = .036 \).

EXERCISE

Simplify:

1. \( 17.42 \times 10 \)
2. \( 43.61 \times 1 \ 000 \)
3. \( 0.00 \ 165 \times 1 \ 000 \)
4. \( 0.081 \ 6 \times 10 \)
5. \( 2.493 \times 100 \)
6. 6.42 ÷ 10
7. 33.97 ÷ 100
8. 7.23 ÷ 1,000
9. 0.03 ÷ 100
10. 1.003 ÷ 100
OBJECTIVE 8

TO DIVIDE ONE DECIMAL BY ANOTHER

Example 1  Divide 6.51 by 0.7.

\[
0.7 \)
6.51
\]

The division must not be attempted with a ‘decimal divisor’ (0.7 is the divisor in this example). First, we must multiply 0.7 by 10 to obtain a whole number \(- 0.7 \times 10 = 7\). We must also multiply 6.51 by 10 (whatever we do to the divisor – the 0.7 – we must also do to the dividend – the 6.51).

\[
6.51 \times 10 = 65.1
\]

So, the question now becomes:

\[
7 \)
65.1
\]

Put a decimal point in the answer, directly over the decimal point in 65.1. Then divide, just as with whole numbers:

\[
9.3
7 \)
65.1
- 63
- 21
\]

\[6.51 \div 0.7 = 9.3\]

Example 2  12.038 \div 26

\[
0.26 \)
12.038
\]

Step 1  The divisor must be a whole number, so multiply 0.26 by 100, ie shift the decimal point two places to the right.

\[0.26 \times 100 = 26\]

Step 2  Do the same to the dividend.

\[12.038 \times 100 = 1203.8\]
Step 3 Put the decimal point in the answer, and divide as with whole numbers.

\[ \begin{array}{c}
46.3 \\
26 \overline{1203.8} \\
104 \\
163 \\
156 \\
78 \\
78 \\
\ldots
\end{array} \]

So, \( 12.038 \div 0.26 = 46.3 \).

Sometimes extra zeroes may need to be placed in the dividend.

Example 3 Divide 537.6 by 0.32.

\[ \begin{array}{c}
0.32 \overline{537.6} \\
\end{array} \]

Step 1 Multiply 0.32 by 100.

\[ 0.32 \times 100 = 32 \]

Step 2 Do the same to 537.6.

\[ 537.6 \times 100 = 53,760 \]

So, the problem becomes:

\[ \begin{array}{c}
1 \, 680 \\
32 \overline{53,760} \\
32 \\
217 \\
192 \\
256 \\
256 \\
\ldots
\end{array} \]

\[ ie \ 537.6 \div 0.32 = 1,680 \]

Example 4 \( 7.95 \div 2.5 \)

\[ 2.5 \overline{7.95} \quad (Multiply \ both \ parts \ by \ 10.) \]

\[ \begin{array}{c}
\end{array} \]
Sometimes, the division does not conclude early (as in the previous examples) or even at all, no matter how many extra zeroes we use. In such cases, we usually round the answers to the nearest tenth, or hundredth, or thousandth, depending on the particular problem.

**Example 5**

Divide 13.21 by 3.5, and round the answer to the nearest tenth.

\[
3.5 \overline{)13.21}
\]

becomes \(35 \overline{)132.1}\) as before.

Since we have to give our answer rounded to the nearest tenth, we usually calculate the answer to the next decimal place, i.e., hundredths. Thus, we need two decimal places in the answer (before rounding) so we need two decimal places in the dividend – we put in an extra zero giving 132.10. Then, we can divide as usual. See below.

\[
3.77
35 \overline{)132.10}
\]

\[
105
\]

\[
271
\]

\[
245
\]

\[
260
\]

\[
245
\]

\[
15
\]

So, \(13.21 \div 3.5 = 3.77\) which rounds to 3.8 to the nearest tenth.

**EXERCISE**

Simplify:

1. \(0.6 \div 2.4\)
2. \(0.056 \div 8.0\)
3. \(1.6 \div 0.4\)
4. \(0.0125 \div 0.5\)
SECTION 2

Fractions
INTRODUCTION

In a fraction, like \( \frac{2}{5} \), the top part (the 2) is called the **numerator** and the bottom part (the 5) is called the **denominator**.

It is frequently convenient to draw graphical representations of fractions. For example, to graph the fraction \( \frac{2}{5} \), first draw a bar and divide it into 5 parts.

Each part of the box represents \( \frac{1}{5} \) (one fifth) of the bar. We want a picture of \( \frac{2}{5} \), so shade two of these parts:

![Diagram showing 2 out of 5 parts shaded]

**NOTE:** When drawing the bar, the length of the bar does not matter, but you should choose a length, which is convenient for dividing into parts. For example, if the fraction has a denominator of 5, then a bar of length 5 cm, or 10 cm, or 15 cm would be convenient. Similarly, if the denominator is 5, lengths of 6 cm, 12 cm, etc would be convenient.

A fraction, in which the numerator is smaller than the denominator is called a **proper fraction**, or a **vulgar fraction** OR **common fraction**.

\( \frac{2}{5}, \frac{1}{3}, \frac{3}{4}, \frac{17}{19}, \frac{1}{2} \) are examples of proper fractions.

A fraction, in which the numerator is larger than the denominator is called an **improper fraction**.

\( \frac{7}{4}, \frac{5}{2}, \frac{13}{12}, \frac{3}{1} \) are examples of improper fractions.

Notice that proper fractions are always less than 1, while improper fractions are always greater than 1.

A numeral like \( 2\frac{1}{4} \) is called a **mixed number** (sometimes called a **mixed numeral**).

Note that \( 2\frac{1}{4} \) means \( 2 + \frac{1}{4} \).
OBJECTIVE 1

TO IDENTIFY EQUIVALENT FRACTIONS

The diagram shows a bar divided into six parts. Two of these parts have been shaded. Thus, the shaded portion represents $\frac{2}{6}$ (two-sixths) of the bar.

$\frac{2}{6}$ of the whole bar

The next diagram shows the same bar, this time divided into three parts, with one of those shaded, $\frac{1}{3}$ (one third) of the bar is shaded.

$\frac{1}{3}$ of the whole bar

Notice that in both diagrams the bars are the same length, and the shaded parts are the same length. In other words,

$\frac{1}{3}$ of the whole bar $= \frac{2}{6}$ of the whole bar

or,

$\frac{1}{3} = \frac{2}{6}$

Fractions like $\frac{2}{6}$ and $\frac{1}{3}$ which represent the same part of the whole unit, are called equivalent fractions.

How can we decide whether two fractions are equivalent? For example, are $\frac{2}{5}$ and $\frac{4}{10}$ equivalent?

One way to decide is to draw a pair of diagrams similar to those above.
Divide the bar into 5 pieces, and shade two of these.

\[ \frac{2}{5} \text{ of the bar} \]

Now divide another bar (the same length as the first one) and divide it into 10 pieces. Shade four of these.

\[ \frac{4}{10} \text{ of the bar} \]

Since the two shaded areas have the same length, we can say that \( \frac{2}{5} \) and \( \frac{4}{10} \) are equivalent, \( \text{i.e.} \ \frac{2}{5} = \frac{4}{10} \).

This method of testing the equivalence of two fractions is clearly not very efficient, especially with fractions like \( \frac{15}{37} \) and \( \frac{13}{31} \). There are several methods, but one of the quickest methods is the \textit{method of cross-multiplication}.

For example, consider the fractions \( \frac{2}{5} \) and \( \frac{4}{10} \).

\[ \frac{2}{5} \times \frac{4}{10} \]

Multiply the numbers indicated by the arrows, \( \text{i.e.} \ 2 \times 10 \) and \( 4 \times 5 \). If the results of these two multiplications are equal, then the fractions are equivalent.

Check with:

\[ \frac{2}{6} \text{ and } \frac{1}{3} \]

\[ \frac{2}{6} \times \frac{1}{3} \]

So, \( 2 \times 3 = 6 \) and \( 6 \times 1 = 6 \). So the fractions are equivalent, \( \text{i.e.} \ \frac{2}{6} = \frac{1}{3} \).

Now, consider the fractions \( \frac{15}{37} \) and \( \frac{13}{31} \).
\[ \frac{15}{37} \times \frac{14}{31} \]

\[ 15 \times 31 = 465 \text{ and } 37 \times 13 = 481. \]

Since \( 465 \neq 481 \), we conclude that the fractions \( \frac{15}{37} \) and \( \frac{13}{31} \) are not equivalent, i.e.

\[ \frac{15}{37} \neq \frac{13}{31}. \]

**NOTE:** This method of cross-multiplication will be justified later in this booklet.

---

**EXERCISE**

Check each of the following pairs of fractions, and state whether they are equivalent or not.

1. \( \frac{2}{3}, \frac{9}{12} \)
2. \( \frac{3}{4}, \frac{7}{10} \)
3. \( \frac{2}{5}, \frac{8}{20} \)
4. \( \frac{5}{7}, \frac{25}{35} \)
OBJECTIVE 2

TO FIND A FRACTION EQUIVALENT TO A GIVEN FRACTION

How can we find a fraction, with a denominator of 12, which is equivalent to $\frac{1}{4}$?

A graphical method, similar to that used in the previous section, can be used for questions like this one.

Draw a bar, divide it into four sections and shade one (1) of these, *ie* the shaded part represents $\frac{1}{4}$.

![Diagram showing $\frac{1}{4}$ of the whole bar.]

The next diagram shows the same bar, this time divided into three parts, with one of those shaded, *ie* $\frac{1}{3}$ (one third) of the bar is shaded.

![Diagram showing $\frac{1}{3}$ of the whole bar.]

We must now decide how many of these parts to shade, in order to obtain a shaded portion. Looking at the two diagrams, we can see that if we shade three of these parts, we will have the shaded parts of the two bars the same length. Shading three of the 12 parts means that we have shaded $\frac{3}{12}$ (three twelfths).

So, $\frac{3}{12} = \frac{1}{4}$.

This graphical method is very time-consuming, and can be replaced by the method of cross-multiplication, as follows:

We need to find a fraction with a denominator of 12, which is equivalent to $\frac{1}{4}$. In the fraction, which we are trying to find, we know the denominator is 12, but we do not know the numerator. Call the numerator ‘X’. Then the fraction we are trying to find can be written as $\frac{X}{12}$, and all we need to do now is find X.
Since we know that \( \frac{X}{12} \) and \( \frac{1}{4} \) are equivalent, \( ie \ \frac{X}{12} = \frac{1}{4} \), we can ‘cross-multiply’ and use the fact that the results of the multiplications will be equal.

\[
\frac{1}{4} \times \frac{X}{12} = \frac{X}{4} = \frac{1}{4} \times 12
\]

\( X \times 4 = 4X \) and \( 12 \times 1 = 12 \) and these two are equal.

So, \( 4X = 12 \).

If \( 4X = 12 \), then \( X = 3 \). So, the fraction we want is \( \frac{3}{12} \).

**EXERCISE**

Find the equivalent fraction to \( \frac{2}{3} \) using the denominators shown.

\[
\frac{2}{3} = \frac{6}{6} = \frac{12}{12} = \frac{15}{15} = \frac{9}{9} = \frac{27}{27} = \frac{81}{81}
\]
OBJECTIVE 3

TO WRITE A FRACTION IN ITS SIMPLEST FORM

The fractions \( \frac{1}{3}, \frac{2}{6}, \frac{3}{9}, \frac{6}{18}, \frac{11}{33} \) are all equivalent.

(These can be easily checked using diagrams, or by considering the fractions in pairs and using the cross-multiplication method.) The fraction \( \frac{1}{3} \) is the simplest form of these equivalent fractions.

**Example 1**

Write \( \frac{12}{14} \) in the simplest form.

\[
\frac{12}{14} = \frac{6 \times 2}{7 \times 2} = \frac{6}{7} \times \frac{2}{2} = \frac{6}{7} \times 1 = \frac{6}{7}
\]

Notice that what we have done is simply divided the numerator and denominator by 2.

**Example 2**

Simplify \( \frac{12}{16} \).

\[
\frac{12}{16} = \frac{12 + 2}{16 + 2} = \frac{6}{8} = \frac{6 + 2}{8 + 2} = \frac{3}{4}
\]

(\( \text{Divide the numerator and the denominator by 2.} \))

\[
\frac{12}{16} = \frac{3}{4}
\]

(\( \text{Divide the numerator and the denominator by 2 again.} \))

Notice that we could have saved some time by dividing the numerator and denominator both by 4 (instead of 1 and then 2 again):

\[
\frac{12}{16} = \frac{3}{4}
\]

You should try to divide by the largest number possible.
Example 2. Simplify \( \frac{36}{84} \).

\[
\frac{36}{84} = \frac{36 \div 12}{84 \div 12} = \frac{3}{7}
\]

We can divide both the numerator and the denominator by 2, 3, 6 and 12. It will save time if we use 12.

EXERCISE

Simplify each of the following.

1. \( \frac{15}{20} \)
2. \( \frac{18}{14} \)
3. \( \frac{9}{12} \)
4. \( \frac{10}{15} \)
5. \( \frac{12}{18} \)
OBJECTIVE 4

TO CHANGE A MIXED NUMBER INTO AN IMPROPER FRACTION

Example 1  Change $2\frac{1}{4}$ into an improper fraction.

$2\frac{1}{4}$ means $2 + \frac{1}{4}$ which is the same as $1 + 1 + \frac{1}{4}$.

Since 1 can be written as $\frac{4}{4}$, we can then write $2\frac{1}{4}$ as:

$$\frac{4}{4} + \frac{4}{4} + \frac{1}{4} \text{ which total } \frac{9}{4}.$$ 

This can be easily checked with diagrams.

Notice in this example that we firstly changed the two whole numbers into quarters, then added the $\frac{1}{4}$. Since each whole number contains four quarters, then clearly 2 contains $2 \times 4$ or 8 quarters.

So, $2\frac{1}{4} = 8$ quarters + 1 quarter = 9 quarters.

This means that we have an easy way of converting a mixed number to an improper fraction.

Example 1 (again)  Change $2\frac{1}{4}$ into an improper fraction.

$$2\frac{1}{4} = \frac{9}{4} \quad \left( \text{Multiply } 2 \times 4 \text{ and add 1, giving 9 quarters, ie } \frac{9}{4}. \right)$$

Example 2  Change $3\frac{2}{5}$ into an improper fraction.

$$3\frac{2}{5} = \frac{17}{5} \quad \left( \text{Multiply } 3 \times 5 \text{ and add 2, giving 17 fifths, ie } \frac{17}{5} \right)$$
Example 3  Change $5\frac{3}{4}$ into an improper fraction.

$$5\frac{3}{4} = \frac{23}{4} \quad \left( \text{Multiply } 5 \times 4 \text{ and add } 3, \text{ giving } 23 \text{ quarters, i.e. } \frac{23}{4}. \right)$$

**EXERCISE**

Change the following mixed numbers to improper fractions.

1. $2\frac{1}{3}$
2. $6\frac{1}{4}$
3. $1\frac{7}{8}$
4. $1\frac{3}{7}$
5. $7\frac{1}{2}$
OBJECTIVE 5

TO CHANGE AN IMPROPER FRACTION INTO A MIXED NUMBER

Example 1  Change $\frac{10}{7}$ into a mixed number.

We can write $\frac{10}{7}$ as $\frac{7}{7} + \frac{3}{7}$ and then as $1 + \frac{3}{7}$. So $\frac{10}{7} = 1\frac{3}{7}$.

Example 2  Change $\frac{13}{4}$ into a mixed number.

$$\frac{13}{4} = \frac{4}{4} + \frac{4}{4} + \frac{1}{4} = 1 + 1 + \frac{1}{4} = 3\frac{1}{4}$$

Example 3  Change $\frac{157}{5}$ into a mixed number.

Clearly the method used in Examples 1 and 2 would be unsuitable here, because of the large numerator. We want a more efficient method.

Notice in Example 2, we were able to take out 4 quarters three times, with one quarter left over; ie $\frac{13}{4}$ is equal to three whole numbers, with one quarter left over. To decide how many whole numbers can be taken out, simply divide the denominator (the 4) into the numerator (13). 4 goes into 13 three times, with a remainder of 1.

$$ie \quad \frac{13}{4} = 3 + \frac{1}{4}$$

Example 3  Change $\frac{157}{5}$ into a mixed number.

(again)

$$\frac{157}{5} = 31 + \frac{2}{5} = 31\frac{2}{5}$$ (Divide 5 into 157, giving 31, with a remainder of 2, ie 31 whole numbers, with 2 fifths left over.)
**Example 4**
Change $\frac{65}{6}$ into a mixed number.

\[
\frac{65}{6} = 10 \frac{5}{6}
\]

(Divide 6 into 65, giving 10, with a remainder of 5.)

\[
= 10 \frac{5}{6}
\]

**Example 5**
Change $\frac{15}{6}$ into a mixed number.

\[
\frac{15}{6} = 2 \frac{3}{6}
\]

(Divide 6 into 15, giving 2, with a remainder of 3.)

\[
= 2 \frac{3}{6}
\]

Notice that the $\frac{3}{6}$ can be reduced to $\frac{1}{2}$.

\[
= 2 \frac{1}{2}
\]

\[
= 2 \frac{1}{2}
\]

**EXERCISE**

Change the following improper fractions into mixed numbers.

1. $\frac{7}{2}$
2. $\frac{13}{5}$
3. $\frac{23}{3}$
4. $\frac{18}{4}$
5. $\frac{16}{6}$
6. $\frac{19}{7}$
7. $\frac{25}{2}$
8. $\frac{53}{4}$
OBJECTIVE 6

TO ADD OR SUBTRACT FRACTIONS

If the fractions have identical denominators, the addition or subtraction can be performed immediately.

Example 1 \[ \frac{2}{7} + \frac{3}{7} = \frac{5}{7} \] (ie 2 sevenths + 3 sevenths = 5 sevenths)

Example 2 \[ \frac{8}{11} + \frac{7}{11} = \frac{15}{11} \] (Since the result is an improper fraction, it should be converted to a mixed number.)

Example 3 Subtract \( \frac{3}{8} \) from \( \frac{7}{8} \).

\[ \frac{7}{8} - \frac{3}{8} = \frac{4}{8} \] (ie 7 eighths – 3 eighths = 4 eighths)

\[ = \frac{1}{2} \] (since \( \frac{4}{8} \) can be reduced to \( \frac{1}{2} \))

Example 4 Find the difference between \( \frac{11}{12} \) and \( \frac{1}{12} \).

\[ \frac{11}{12} - \frac{1}{12} = \frac{10}{12} \]

\[ = \frac{5}{6} \]

Example 5 Find the sum of \( \frac{1}{8} \), \( \frac{3}{8} \) and \( \frac{7}{8} \).

\[ \frac{1}{8} + \frac{3}{8} + \frac{7}{8} = \frac{11}{8} \]

\[ = 1 \frac{3}{8} \]

Example 6 Simplify \( \frac{1}{9} + \frac{5}{9} - \frac{2}{9} \).

\[ \frac{1}{9} + \frac{5}{9} - \frac{2}{9} = \frac{4}{9} \] (1 + 5 – 2 = 6 – 2 = 4)
If the fractions have different denominators, we must first replace all of the fractions by new fractions. These new fractions must (a) all have the same denominator and (b) each be equivalent to the fraction they replace. The new fractions can then be added, or subtracted easily.

**Example 7**

Add $\frac{1}{3}$ and $\frac{1}{4}$.

\[
\frac{1}{3} + \frac{1}{4}
\]

The denominators are different, so we must replace $\frac{1}{3}$ and $\frac{1}{4}$ by new fractions. What denominator will these new fractions have? The 'new' denominator must be a number into which both 3 and 4 will divide evenly. Clearly 12 will be a suitable number. (24, 36, 38 and so on would also be suitable, but our work is made simpler if we use the smallest of these numbers.)

So $\frac{1}{3}$ must be replaced by an equivalent fraction with a denominator of 12.

ie $\frac{1}{3} = \frac{?}{12}$. The fraction we want is $\frac{4}{12}$.

Similarly, $\frac{1}{4}$ must be replaced by an equivalent fraction with a denominator of 12.

ie $\frac{1}{4} = \frac{?}{12}$. The fraction we want is $\frac{3}{12}$.

Thus,

\[
\frac{1}{3} + \frac{1}{4} = \frac{4}{12} + \frac{3}{12} = \frac{7}{12}
\]

**Example 8**

Add $\frac{3}{5} + \frac{2}{3}$.

(A suitable new denominator is 15, since 5 and 3 both divide evenly into 15.)

\[
\frac{3}{5} + \frac{2}{3} = \frac{9}{15} + \frac{10}{15} \quad \left(\text{We then need to find equivalent fractions, } \frac{3}{5} = \frac{?}{15} \text{ and } \frac{2}{3} = \frac{?}{15}\right)
\]

= $\frac{19}{15}$

These are $\frac{3}{5} = \frac{9}{15}$ and $\frac{2}{3} = \frac{10}{15}$. 


Example 9  
Simplify \( \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \).

\[
\frac{1}{2} + \frac{1}{3} - \frac{1}{4} = \frac{6}{12} + \frac{4}{12} - \frac{3}{12} = \frac{7}{12}.
\]

A suitable new denominator is 12.

**EXERCISE**

Simplify each of the following.

1. \( \frac{2}{5} + \frac{4}{5} \)
2. \( \frac{5}{7} - \frac{3}{7} \)
3. \( \frac{3}{4} + \frac{2}{3} \)
4. \( \frac{5}{7} - \frac{2}{3} \)
5. \( \frac{3}{8} + \frac{2}{3} \)
6. \( \frac{5}{6} - \frac{3}{8} \)
OBJECTIVE 7

TO ADD OR SUBTRACT MIXED NUMBERS

Example 1  Add $1\frac{1}{2}$ and $2\frac{1}{4}$.

\[ 1\frac{1}{2} + 2\frac{1}{4} \]

\[ = 1 + \frac{1}{2} + 2 + \frac{1}{4} \]

\[ = 3 + \frac{1}{2} + \frac{1}{4} \]

\[ = 3\frac{3}{4} \]

(Add the whole numbers, $1 + 2 = 3$.)

(Now add the fractions – in this case, because the denominators are different, we must replace the $\frac{1}{2}$ by $\frac{1}{4}$.)

Example 2  Simplify $2\frac{2}{3} + 3\frac{5}{6}$.

\[ 2\frac{2}{3} + 3\frac{5}{6} \]

\[ = 2 + \frac{2}{3} + 3 + \frac{5}{6} \]

\[ = 2 + 3 + \frac{2}{3} + \frac{5}{6} \]

\[ = 5 + \frac{4}{6} + \frac{5}{6} \]

\[ = 5 + \frac{9}{6} \]

\[ = 5 + \frac{3}{2} \]

\[ = 5 + 1\frac{1}{2} \]

\[ = 5 + 1 + \frac{1}{2} \]

\[ = 6 + \frac{1}{2} \]

\[ = 6\frac{1}{2} \]
Example 3

From $3\frac{3}{7}$ subtract $2\frac{1}{5}$.

\[
3\frac{3}{7} - 2\frac{5}{8}
\]

\[
= 3 + \frac{3}{4} - 2 - \frac{1}{8}
\]

\[
= 3 - 2 + \frac{3}{4} - \frac{1}{8}
\]

\[
= 1 + \frac{6}{8} - \frac{1}{8}
\]

\[
= 1 + \frac{5}{8}
\]

\[
= 1\frac{5}{8}
\]

Notice that we are subtracting more than 2, i.e., we are subtracting 2 and we are also subtracting 1/8.

Hence $-2 - \frac{1}{8}$.

Example 4

Simplify $5\frac{1}{3} - 2\frac{5}{6}$.

\[
5\frac{1}{3} - 2\frac{5}{6}
\]

\[
= 5 + \frac{1}{3} - 2 - \frac{5}{6}
\]

\[
= 5 - 2 + \frac{1}{3} - \frac{5}{6}
\]

\[
= 3 + \frac{2}{6} - \frac{5}{6}
\]

\[
= 3 - \frac{3}{6}
\]

\[
= 2 + \frac{6}{6} - \frac{3}{6}
\]

\[
= 2 + \frac{3}{6}
\]

\[
= 2 + \frac{1}{2}
\]

\[
= 2\frac{1}{2}
\]

Change the 1/3 to 2/6.

To resolve the 3/6, we separate the 3 into 2 + 6/6 and then continue the subtraction.
Example 5  Subtract $3\frac{3}{8}$ from 9.

\[
9 - 3\frac{3}{8} = 9 - 3 - \frac{3}{8} = 6 - \frac{3}{8} = 5 + 1 - \frac{3}{8} = 5 + 8 - \frac{3}{8} = 5 + 3 = 5\frac{3}{8}
\]

EXERCISE

Simplify each of the following.

1. $2\frac{1}{3} + 5\frac{3}{4}$
2. $2\frac{5}{8} + 5\frac{3}{4}$
3. $4\frac{1}{3} + 3\frac{4}{5}$
4. $7\frac{3}{4} - 3\frac{1}{2}$
5. $12\frac{1}{4} - 4\frac{1}{2}$
6. $9\frac{5}{12} - 4\frac{3}{8}$
OBJECTIVE 8

TO MULTIPLY FRACTIONS OR MIXED NUMBERS

Example 1 \[ \frac{1}{2} \times \frac{3}{4} \]

To multiply two fractions, first multiply the numerators and then multiply the denominators.

So, \[ \frac{1 \times 3}{2 \times 4} = \frac{3}{8} \]

Notice that in this final step, we reduced the fraction \( \frac{35}{60} \) to its simplest form \( \frac{7}{12} \) by dividing the numerator and the denominator by 5. We could have saved ourselves some work by doing this division at an earlier stage.

\[ \text{ie} \quad \frac{5 \times 7}{6 \times 10} = \frac{5 \times 7}{6 \times 10} \]

and we can divide numerator and denominator by 5 at this stage.

\[ \frac{\frac{1}{2} \times 7}{6 \times \frac{3}{2}} \]

\[ = \frac{7}{12} \]
Example 3

\[
\frac{10}{21} \times \frac{7}{15}
\]

\[
\frac{10}{21} \times \frac{7}{15} = \frac{2 \times 7 \times 3}{2 \times 3 \times 7} = \frac{2}{9}
\]

Example 4

Multiply \(3\frac{1}{3}\) by \(1\frac{3}{5}\).

\[
3\frac{1}{3} \times 1\frac{3}{5} = \frac{15}{4} \times \frac{8}{5} = \frac{3 \times 8^2}{1 \times 5^2} = \frac{6}{1} = 6
\]

Example 5

Multiply \(3\frac{2}{5}\) by 4.

\[
3\frac{2}{5} \times 4 = \frac{31}{8} \times \frac{4}{1} = \frac{31 \times 4}{8 \times 1} = \frac{31}{2} = 15\frac{1}{2}
\]
EXERCISE

Simplify each of the following.

1. $1\frac{1}{5} \times 3\frac{1}{3}$
2. $2\frac{3}{4} \times 2\frac{2}{3}$
3. $1\frac{2}{5} \times 3\frac{1}{3}$
4. $4\frac{2}{3} \times 2\frac{1}{7}$
5. $4\frac{3}{4} \times 1\frac{5}{7}$
6. $3\frac{1}{4} \times 1\frac{3}{4}$
OBJECTIVE 9

TO FIND THE RECIPROCAL OF A GIVEN NUMBER

The reciprocal of a given number is found by:

(a) expressing that number as a fraction, then
(b) inverting this fraction, so that when the two numbers are multiplied together, the result is 1.

For example, the reciprocal of 2 is \(\frac{1}{2}\) since \(2 \times \frac{1}{2} = 1\).

Similarly, the reciprocal of 7 is \(\frac{1}{7}\) since \(7 \times \frac{1}{7} = 1\)

and the reciprocal of \(\frac{1}{3}\) is 3 since \(\frac{1}{3} \times 3 = 1\).

Example 1  What is the reciprocal of 4?

The reciprocal of 4 is \(\frac{1}{4}\) (since \(4 \times \frac{1}{4} = 1\)).

Example 2  What is the reciprocal of \(\frac{1}{5}\)?

The reciprocal of \(\frac{1}{5}\) is 5 (since \(\frac{1}{5} \times 5 = 1\)).

Example 3  What is the reciprocal of \(1\frac{1}{4}\)?

The reciprocal of \(1\frac{1}{4}\), i.e. \(\frac{5}{4}\), is \(\frac{4}{5}\) (since \(\frac{5}{4} \times \frac{4}{5} = 1\)).

Example 4  What is the reciprocal of 0?

Note that 0 does not have a reciprocal, since we cannot find a number such that \(0 \times \text{(the number)} = 1\), because \(0 \times \text{any number} = 0\).
**EXERCISE**

Simplify each of the following.

1. $\frac{4}{5}$
2. $\frac{7}{3}$
3. $2\frac{1}{2}$
4. $1\frac{2}{3}$
OBJECTIVE 10

TO DIVIDE ONE FRACTION OR MIXED NUMBER BY ANOTHER

Example 1

Simplify \( \frac{1}{2} + \frac{1}{3} \)

\[
\frac{1}{2} + \frac{1}{3}
\]

This can be written as:

\[
\frac{1}{2} \quad \frac{1}{3}
\]

If we now multiply the numerator and the denominator of the fraction by 3, we obtain:

\[
\frac{1 \times 3}{2 \times 3} \quad \frac{1 \times 3}{3 \times 3}
\]

which is better written as:

\[
\frac{1}{2} \times \frac{3}{1} \quad \frac{1}{3} \times \frac{3}{1}
\]

Now, each part of this fraction, i.e., the numerator and the denominator, consists of a product of two fractions. These are multiplied in the usual way, so:

\[
\frac{1}{2} \times \frac{3}{1} = \frac{1 \times 3}{2 \times 1} = \frac{3}{2}
\]

and

\[
\frac{1}{3} \times \frac{3}{1} = \frac{1 \times 3}{3 \times 1} = \frac{3}{3} = 1
\]

So, our fraction now becomes:

\[
\frac{3}{2} \quad \frac{3}{1}
\]

\[
\frac{3}{2} \times \frac{1}{3} = \frac{3}{2} \quad \frac{1}{3} \times \frac{3}{1} = \frac{3}{1}
\]
and this is equal to \( \frac{3}{2} \).

\[
\text{ie } \quad \frac{1}{2} + \frac{1}{3} = \frac{3}{2} \text{ or } 1\frac{1}{2}
\]

Notice that since we made the denominator of the fraction 1, the significant part of the fraction is the numerator.

\[
\text{ie } \quad \frac{1}{2} \div \frac{1}{3} = \frac{1}{2} \times \frac{3}{1}
\]

and this is equal to \( \frac{3}{2} \).

\[
\text{ie } \quad \frac{1}{2} + \frac{1}{3} = \frac{3}{2} \text{ or } 1\frac{1}{2}
\]

So, in order to divide by \( \frac{1}{3} \), we simply multiply by the reciprocal of \( \frac{1}{3} \).

**Example 2**

\[
\frac{2}{3} + \frac{8}{9} = \frac{2}{3} \times \frac{9}{8}
\]

\[
= \frac{1}{3} \times \frac{3^3}{3^3}
\]

\[
= \frac{1}{3} \times \frac{3}{3^3}
\]

\[
= \frac{3}{4}
\]

**Example 3**

\[
2\frac{1}{2} + 3\frac{1}{4} = \frac{5}{2} + \frac{15}{4}
\]

\[
= \frac{5}{2} \times \frac{4}{15}
\]

\[
\text{Convert the mixed numbers into improper fractions.}
\]

\[
= \frac{1}{2} \times \frac{1}{3}
\]

\[
= \frac{2}{3}
\]
Example 4  
Divide $1\frac{5}{7}$ by $2\frac{1}{7}$.

\[
1\frac{5}{7} + 2\frac{1}{7} = \frac{14}{9} + \frac{7}{3} = \frac{14}{9} + \frac{3}{7} = \frac{2 \times 14 \times 7}{9 \times 3} = \frac{2}{3}
\]

Example 5  
Divide 4 by $2\frac{2}{3}$.

\[
4 \div 2\frac{2}{3} = \frac{4}{1} \div \frac{8}{3} = \frac{4}{1} \times \frac{3}{8} = \frac{1 \times 3}{1 \times \frac{8}{2}} = \frac{3}{2}
\]

Example 6  
Divide $\frac{3}{7}$ by 5.

\[
\frac{3}{7} \div 5 = \frac{3}{7} \div 1 = \frac{3}{7} \times \frac{1}{5} = \frac{3 \times 1}{7 \times 5} = \frac{3}{35}
\]
EXERCISE

Simplify each of the following.

1. $4\frac{1}{5} + \frac{3}{20}$
2. $4\frac{2}{5} + \frac{11}{15}$
3. $3\frac{3}{8} + \frac{3}{10}$
4. $4\frac{1}{5} + 1\frac{3}{5}$
5. $5\frac{1}{4} + 1\frac{1}{8}$
6. $6\frac{3}{4} + 7\frac{1}{2}$
SECTION 3

Per Cent
INTRODUCTION

‘Forty per cent of the students at a certain school are girls.’ What does this mean?

Forty per cent, or 40%, means 40 per hundred, or 40 out of every hundred.

That is, of every hundred students at the school, 40 are girls.

Therefore, if the school had a total of 800 students, then there would be:

\[
\frac{8}{100} \times 40 = 320 \text{ girls}
\]

In practice we rarely use 40% by itself, but in the context of ‘40% of something’, for example, 40% of 800 students. Since 40% means 40 out of every hundred, we can write it as \( \frac{40}{100} \). This will be convenient to think of per cent as meaning hundredths.

\[
ie 40\% = \frac{40}{100}
\]
OBJECTIVE 1

TO WRITE A COMMON FRACTION AS A PER CENT

Example 1  Write $\frac{3}{5}$ as a per cent.

Solution  We need to find a fraction, with a denominator of 100, which is equivalent to $\frac{3}{5}$.

\[
\frac{X}{100} = \frac{3}{5} \Rightarrow 5X = 3 \times 100 \Rightarrow X = \frac{300}{5} = 60 \Rightarrow \frac{3}{5} = \frac{60}{100} = 60\%
\]

Example 2  Write $\frac{7}{8}$ as a per cent.

Solution

\[
\frac{X}{100} = \frac{7}{8} \Rightarrow 8X = 7 \times 100 \Rightarrow X = \frac{700}{8} = 87\frac{1}{2} \Rightarrow \frac{7}{8} = \frac{87\frac{1}{2}}{100} = 87\frac{1}{2}\%
\]
Example 3  Convert $\frac{1}{3}$ into a per cent.

**Solution**

\[
\frac{X}{100} = \frac{1}{3}
\]

\[\therefore 3 \times X = 1 \times 100\]

\[
3X = 700
\]

\[
X = \frac{100}{3}
\]

\[
= 33\frac{1}{3}
\]

\[ie\frac{1}{3} = \frac{33\frac{1}{3}}{100} = 33\frac{1}{3}\%\]

**EXERCISE**

Write each of the following as a per cent.

1. $\frac{12}{50}$
2. $\frac{3}{10}$
3. $\frac{7}{20}$
4. $\frac{12}{25}$
OBJECTIVE 2

TO WRITE A DECIMAL AS A PER CENT

Example 1  Convert 0.85 into a per cent.

Solution  0.85 means \( \frac{85}{100} = 85\% \).

Example 2  Convert 0.625 into a per cent.

Solution  0.625 means \( \frac{625}{1000} \).

We now need to find an equivalent fraction with a denominator of 100.

\[
\frac{X}{100} = \frac{625}{1000}
\]

\[
\therefore 1000 \times X = 625 \times 100
\]

\[
1000X = 62500
\]

\[
X = \frac{62500}{1000}
\]

\[
= \frac{62.5}{100}
\]

\[
= 62.5\%
\]

Example 3  Convert 0.8 into a per cent.

Solution  0.8 means \( \frac{8}{10} \).

We now need to find an equivalent fraction with a denominator of 100.

\[
\frac{X}{100} = \frac{8}{10}
\]

\[
\therefore 10X = 800
\]

\[
\therefore X = \frac{800}{10}
\]

\[
= 80
\]

\[
\therefore 0.8 = \frac{80}{100}
\]

\[
= 80\%
\]
Looking back at these three (3) examples, we can see that converting a decimal to a per cent is basically a matter of moving the decimal point two (2) positions to the right.

**EXERCISE**

Write the following decimals as a per cent.

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>0.53</td>
<td>5.</td>
<td>0.007</td>
</tr>
<tr>
<td>2.</td>
<td>9.54</td>
<td>6.</td>
<td>0.66</td>
</tr>
<tr>
<td>3.</td>
<td>0.08</td>
<td>7.</td>
<td>18.6</td>
</tr>
<tr>
<td>4.</td>
<td>3.82</td>
<td>8.</td>
<td>15</td>
</tr>
</tbody>
</table>
OBJECTIVE 3

TO WRITE A PER CENT AS A COMMON FRACTION

Example 1  Write 70% as a common fraction, in its lowest terms.
Solution  70% means 70 hundredths.

\[
\text{ie } 70\% = \frac{70}{100} \quad (\text{Divide top and bottom by 10.})
\]

\[
= \frac{7}{10}
\]

Example 2  Write 25% as a common fraction, in its lowest terms.
Solution

\[
25\% = \frac{25}{100} \quad (\text{Divide top and bottom by 5.})
\]

\[
= \frac{5}{20}
\]

\[
= \frac{1}{4}
\]

Example 3  Write 150% as a common fraction, in its lowest terms.
Solution

\[
150\% = \frac{150}{100}
\]

\[
= \frac{3}{2}
\]

or \(= 1\frac{1}{2}\)

Example 4  Write 12\(\frac{1}{2}\)% as a common fraction, in its lowest terms.
Solution

\[
12\frac{1}{2}\% = 12\frac{1}{2} \div 100
\]

\[
= \frac{25}{2} \times \frac{1}{100}
\]

\[
= \frac{1}{8}
\]
**EXERCISE**

Write each of the following as a common fraction, in its lowest terms.

1. 25%  
2. 63%  
3. 142%  
4. 6\(\frac{1}{2}\)%  
5. 5.3%  
6. 80%
OBJECTIVE 4

TO WRITE A PER CENT AS A DECIMAL

Example 1  Write 68% as a decimal.

Solution

\[ 68\% = \frac{68}{100} \]
\[ = 0.68 \]

Example 2  Write 30% as a decimal.

Solution

\[ 30\% = \frac{30}{100} \]
\[ = \frac{3}{10} \]
\[ = 0.3 \]

Example 3  Write 62.5% as a decimal.

Solution

\[ 62.5\% = \frac{62.5}{100} \]
\[ = 0.625 \]

Example 4  Write 7% as a decimal.

Solution

\[ 7\% = \frac{7}{100} \]
\[ = 0.07 \]

Example 5  Write 27 1/2% as a decimal.

Solution

\[ 27 \frac{1}{2}\% = \frac{27 \frac{1}{2}}{100} \]
\[ = \frac{27.5}{100} \]
\[ = 0.275 \]
EXERCISE

Write each of the following as a decimal.

1. 85%  
2. 135%  
3. 8.5%  
4. 61%  
5. 38\frac{1}{2}%  
6. \frac{1}{4}%
OBJECTIVE 5

TO CALCULATE A GIVEN PER CENT OF A GIVEN NUMBER

Example 1  Find 15% of 60.

Solution

15% of 60 means \( \frac{15}{100} \) of 60

\[
\begin{align*}
15\% \text{ of } 60 &= \frac{15}{100} \times 60 \\
&= \frac{15}{100} \times \frac{60}{1} \\
&= 9
\end{align*}
\]

Example 2  There are 240 people in a motoring club. 68% of these are younger than 25 years. How many members are under 25?

Solution

60% of 240 = \( \frac{60}{100} \times \frac{240}{1} \)

\[
\begin{align*}
&= 144 \\
\text{ie } 144 \text{ members are under 25.}
\end{align*}
\]

EXERCISE

Write each of the following as a decimal.

1. Find 12% of 75.

2. Find 8% of 142.

3. Find 113% of 52.

4. On a test, Jim got 85% of the questions correct. If there were 300 questions, how many did he get correct?

5. A family spends 12% of its income on groceries. If the yearly income is $11 500, how much is spent on groceries?
OBJECTIVE 6

TO CALCULATE WHAT PER CENT ONE NUMBER IS OF ANOTHER

Example 1  In a group of 40 people, 25 are women. What per cent of the group is women?

Solution  We are trying to find what per cent 25 is of 40.

First write a fraction \(rac{25}{40}\) (ie twenty-five fortieths of the group are female) and then convert this fraction into a per cent:

\[
\frac{X}{100} = \frac{25}{40}
\]

\[
40 \times X = 25 \times 100
\]

\[
40X = 2500
\]

\[
X = \frac{2500}{40}
\]

\[
X = 62\frac{1}{2}
\]

\[
ie \ \frac{25}{40} = \frac{62\frac{1}{2}}{100} = 62\frac{1}{2}\%
\]

So, \(62\frac{1}{2}\)% of the group is women.

Example 2  What percentage is 7 of 10?

Solution  First, the fraction is \(\frac{7}{10}\) and then we convert this fraction into a per cent.

\[
\frac{X}{100} = \frac{7}{10}
\]

\[
10X = 700
\]

\[
X = \frac{700}{10}
\]

\[
= 70
\]

\[
ie \ 7 \text{ is } 70\% \text{ of } 10.
\]
EXERCISE

1. What percentage is 12 of 20?
2. What percentage is 13 of 25?
3. What percentage of 40 is 15?
4. What percentage of 10 is 13?
OBJECTIVE 7

TO CALCULATE A NUMBER WHEN A PER CENT OF IT IS KNOWN

Example 1  12 is 40% of what number?
Solution  40% of the number is 12.

∴ 1% of the number is $\frac{1}{40} \times \frac{12}{1} = \frac{12}{40}$

∴ 100% of the number is $100 \times \frac{12}{40} = 30$

ie the number is 30.

Example 2  A school football team won 60% of the matches they played. They won a total of 9 games. How many did they play?
Solution  60% of the games played is 9.

∴ 1% of the number is $\frac{1}{60} \times \frac{9}{1} = \frac{9}{60}$

∴ 100% of the number is $\frac{100}{1} \times \frac{9}{60} = 15$

ie the team played 15 games.

EXERCISE

1.  4 is 25% of what number?
2.  26 is 2% of what number?
3.  50 is 20% of what number?
4.  75% of a number is 0.06. What is the number?
5.  Tom spent $46.50, which was 10% of his savings. What were the total savings?
PART 2

MEM2.8C10A
Perform Computations
SECTION 4

Measurement 1
THE S.I. SYSTEM

BASIC UNITS

<table>
<thead>
<tr>
<th>QUANTITY</th>
<th>UNIT NAME</th>
<th>SYMBOL</th>
</tr>
</thead>
<tbody>
<tr>
<td>length</td>
<td>metre</td>
<td>m</td>
</tr>
<tr>
<td>mass</td>
<td>kilogram</td>
<td>kg</td>
</tr>
<tr>
<td>time</td>
<td>second</td>
<td>s</td>
</tr>
<tr>
<td>force</td>
<td>Newton</td>
<td>N</td>
</tr>
</tbody>
</table>

The SI system is based on the above basic units each of which is the basis for measurement units. Symbols use lower case letters except where the symbol is named after a person

\[ \text{eg} \quad \text{Newton} = \text{N} \]
\[ \text{Pascal} = \text{P}. \]

The magnitude of measurement can be further described by using symbols for multiples and sub-multiples each of which is one thousand times more or less than the one adjacent.

<table>
<thead>
<tr>
<th>symbol</th>
<th>value</th>
<th>symbol</th>
</tr>
</thead>
<tbody>
<tr>
<td>mega</td>
<td>( \times 1000000 )</td>
<td>M</td>
</tr>
<tr>
<td>kilo</td>
<td>( \times 1000 )</td>
<td>k</td>
</tr>
<tr>
<td>basic unit</td>
<td>( \times 1 )</td>
<td></td>
</tr>
<tr>
<td>milli</td>
<td>( + 1000 )</td>
<td>m</td>
</tr>
<tr>
<td>micro</td>
<td>( + 1000000 )</td>
<td>( \mu )</td>
</tr>
</tbody>
</table>

**NOTE:** Centi (100) and Deci (10) are not used in the SI System.

If we were to take the basic unit for length, the metre (m), we could:

- multiply it by 1000 to convert it into kilometres (km)
- divide it by 1000 to convert it into millimetres (mm).

These units are sufficient for most linear measurements. However, micrometres (\( \mu \)m) are commonly used for precision measurement in machining applications.

Convert the following.

28.9 m into mm _________________________
0.003 79 km into mm _________________________
0.080 m into mm _________________________
0.05 km into mm _________________________
0.000 397 km into mm _________________________
8 014 m into km  
748 943 mm into km  
907 m into km  
70.5 m into km  

49 034 mm into m  
0.384 km into m  
7.049 km into m  
0.003 12 km into m  
7162 mm into m  

**BASIC SI UNIT FOR MASS**

Mass is the quantity of matter in an object. Weight is the effect that gravity has upon this matter, but because gravity exerts a force of 1 kg upon 1 kg of matter the two terms are generally taken to be the same.

The basic SI Unit for mass is the kilogram (kg).

The kilogram is divided by 1 000 to give grams and multiplied by 1000 to give tonnes i.e

\[ 1 \text{ kg} \div 1 000 = 1 \text{ g} \]

\[ 1 \text{ kg} \times 1 000 = 1 \text{ tonne (t)} \]

A useful although not perfectly accurate relationship exists between volume and mass in that:

1 litre of pure water at 4° Celsius has a mass of 1kg.

Convert the following.

34 kg into g  
0.034 t into g  
0.894 kg into g  

307 g into kg  
0.481 t into kg  
0.003 97 t into kg  

47 348 kg into t  
409 364 8 g into t  
347 kg into t
OBJECTIVE 1

TO CALCULATE THE PERIMETER OF A RECTANGLE

A rectangle is a figure similar to the following diagram.

You should notice that the sides opposite each other are both equal in length and parallel. Also, each angle is 90°.

For establishing formulas, we label the longest side with $L$ (for length) and the shortest side with $W$ (for width or breadth).

The term perimeter refers to the total distance or length measurement of a closed boundary interface.

Then if $P$ stands for the perimeter, we can write

$$P\text{ (rectangle)} = L + W + L + W$$

$$= 2L + 2W$$

$$= 2(L + W)$$

So:

$$P\text{ (rectangle)} = 2(L + W)$$

Example 1
Find the perimeter of the following rectangle.

\[
\begin{array}{c}
6 \text{ cm} \\
14 \text{ cm}
\end{array}
\]
Solution

\[ P \text{ (rectangle)} = 2(L + W) \]
\[ = 2(14 + 6) \text{ cm} \]
\[ = 2(20) \text{ cm} \]
\[ = 40 \text{ cm} \]

Example 2
Find the perimeter of the following square.

Solution

A square is simply a special type of rectangle (one in which \( L = W \)), so we can still use the same formula.

\[ P \text{ (rectangle)} = 2(L + W) \]
\[ = 2(5 + 5) \text{ cm} \]
\[ = 2(10) \text{ cm} \]
\[ = 2(20) \text{ cm} \]
\[ = 20 \text{ cm} \]

Example 3
Find the perimeter of a rectangle whose length is 8 m and whose width is 4 m.

Solution

Step 1
Draw a diagram.

Step 2
Write down the appropriate formula.

\[ P \text{ (rectangle)} = 2(L + W) \]

Step 3
Substitute numbers into the formula and calculate the answer.

\[ P \text{ (rectangle)} = 2(8 + 4) \text{ m} \]
\[ = 2(12) \text{ m} \]
\[ = 24 \text{ m} \]
EXERCISE

1. Find the perimeter of a rectangle with a base of 4.5 mm and a breadth of 3.5 mm.

2. Find the perimeter of a rectangle with a length of 1 cm and a height of 5 cm.

3. Find the perimeter of a rectangle with a length of 1200 cm and a width of 40 cm.

4. Find the perimeter of a square with sides of 2.7 mm.

5. A table measuring 82 cm by 124 cm has a thin jarrah beading around its edge. What length of beading is used for this table?
OBJECTIVE 2

TO CALCULATE THE CIRCUMFERENCE OF A CIRCLE GIVEN EITHER THE DIAMETER OR THE RADIUS

The perimeter of a circle is called its circumference. The distance from the centre of a circle to any point on the circumference is called the radius. The distance across the circle is called the diameter (and equals twice the radius).

So, if the radius $r$ in our circle above, is 3 cm, then the diameter $d$, would be 6 cm ($2 \times 3$ cm).

If a great number of circles were drawn, and their circumferences measured, it would be found in every case, that the circumferences were approximately 3.14 times the diameter. This number, whose approximate value is 3.14, cannot be found exactly and so we refer to it as $\pi$, standing for the Greek letter ‘pi’.

The formula then for finding the circumference of a circle is:

$$C = \pi \times d$$

where $C =$ circumference

$d =$ diameter

$\pi = 3.14.$

Example 1  Find the circumference of a circle of diameter 5 cm.

Solution  Step 1  Draw a diagram.

Step 2  Write down the appropriate formula.

$$C = \pi \times d$$
Step 3  Substitute numbers into the formula and calculate the answer.

\[ C = 3.14 \times 5 \text{ cm} \]
\[ = 15.70 \text{ cm} \]

Example 2  Find the circumference of a circle of radius \( 3\frac{1}{2} \) cm.

Solution  
The radius = \( 3\frac{1}{2} \) cm  
\[ = 3.5 \text{ cm} \]
\[ \therefore \text{ the diameter} = 2 \times (3.5) \text{ cm} \]
\[ = 7 \text{ cm} \]

\[ \text{Now } C = \pi \times d \]
\[ = 3.14 \times 7 \text{ cm} \]
\[ = 21.98 \text{ cm} \]

Example 3  What is the circumference of a car wheel whose diameter is 90 cm?

Solution  
\[ C = \pi \times d \]
\[ = 3.14 \times 90 \text{ cm} \]
\[ = 282.6 \text{ cm} \]
EXERCISE

1. Find the circumference of a circle whose diameter is 10 cm.
2. What is the circumference of a circle with a diameter of 14 cm?
3. Find the circumference of a circle with radius 25 m.
4. Find the circumference of a circle with radius 2.5 cm.
5. A circle has a diameter of 280 mm. Find its circumference.
6. A circular pond is 15 m across. Find the circumference of this pond.
OBJECTIVE 3

TO CALCULATE THE AREA OF A RECTANGLE

The area of a figure is the amount of surface it covers.

This is a 1 centimetre square; it covers one square centimetre (1 cm$^2$) of the surface of this page.

This rectangle is 10 cm by 7 cm; it covers 70 square centimetres (70 cm$^2$) of the surface of this page.

This rectangle is 10 cm by 7 cm; it covers 70 square centimetres (70 cm$^2$) of the surface of this age.
The formula for finding the area of a rectangle is:

\[
A \text{ (rectangle)} = B \times H
\]

where

- \( A \) = number of square units in the area
- \( B \) = number of units in the base
- \( H \) = number of units in the height.

and the base and height must be in units of the same name.

**Example 1**  Find the area of a rectangle which is 5 cm along the base, and has a height of 3 cm.

**Solution**

\[
\begin{align*}
H &= 3 \text{ cm} \\
B &= 5 \text{ cm} \\
A \text{ (rectangle)} &= B \times H \\
&= (5 \times 3) \text{ cm}^2 \\
&= 15 \text{ cm}^2
\end{align*}
\]

**Example 2**  Find the area of a rectangle which is 2.5 m long and 0.5 m high.

**Solution**

\[
\begin{align*}
H &= 0.5 \text{ m} \\
B &= 2.5 \text{ m} \\
A \text{ (rectangle)} &= B \times H \\
&= (2.5 \times 0.5) \text{ m}^2 \\
&= 1.25 \text{ m}^2
\end{align*}
\]

**Example 3**  Find the area of a rectangle which is 12 cm long and 0.5 mm high.

**Solution**  First, both measurements must be in the same units (either cm or mm). Millimetres have been chosen here.

\[
\begin{align*}
H &= 0.5 \text{ mm} \\
B &= 12 \text{ cm} = 120 \text{ mm}
\end{align*}
\]
\[
A \text{ (rectangle)} = B \times H
\]
\[
= (120 \times 0.5) \text{ mm}^2
\]
\[
= 60 \text{ mm}^2
\]

**Example 4**
Find the area of a rectangular field which is 2 km by 1.5 km.

**Solution**

\[
\begin{align*}
H &= 1.5 \text{ km} \\
B &= 2 \text{ km}
\end{align*}
\]

\[
A \text{ (rectangle)} = B \times H
\]
\[
= (2 \times 1.5) \text{ km}^2
\]
\[
= 3 \text{ km}^2
\]

**Example 5**
A square piece of metal has a side of 2.5 cm. Find its area.

**Solution**
Remember! A square is just a special rectangle.

\[
\begin{align*}
H &= 2.5 \text{ cm} \\
B &= 2.5 \text{ cm}
\end{align*}
\]

\[
A \text{ (rectangle)} = B \times H
\]
\[
= (2.5 \times 2.5) \text{ cm}^2
\]
\[
= 6.25 \text{ cm}^2
\]

**EXERCISE**

Complete the following table by finding the area for each of the given rectangles.

<table>
<thead>
<tr>
<th>BASE</th>
<th>HEIGHT</th>
<th>AREA</th>
</tr>
</thead>
<tbody>
<tr>
<td>7 cm</td>
<td>1 cm</td>
<td>? cm²</td>
</tr>
<tr>
<td>15 cm</td>
<td>6 cm</td>
<td>? cm²</td>
</tr>
<tr>
<td>17 m</td>
<td>9 m</td>
<td>? m²</td>
</tr>
<tr>
<td>2 cm</td>
<td>2 mm</td>
<td>? mm²</td>
</tr>
<tr>
<td>0.5 cm</td>
<td>0.6 cm</td>
<td>? cm²</td>
</tr>
<tr>
<td>3.5 m</td>
<td>3.5 m</td>
<td>? m²</td>
</tr>
</tbody>
</table>
OBJECTIVE 4

TO CALCULATE THE AREA OF A PARALLELOGRAM

Let’s start with the parallelogram (‘a pushed-over rectangle’) in Figure 1, having one pair of sides with length \( B \) and the distance between those sides \( H \).

If we now draw the vertical line shown in Figure 2, we can then ‘cut off’ the shaded triangle on the left, and ‘put it on’ the other end – see Figure 3. This results in a rectangle with base \( B \) and height \( H \) (Figure 4).

We can now find the area of the original parallelogram by simply finding the area of the rectangle.

Now \( A \) (rectangle) = \( B \times H \)
\[
A \text{ (parallelogram)} = B \times H
\]

But remember that height \( H \) is a vertical height, not the slant height, \textit{ie} it is the distance between the base and the side opposite the base, \textit{not} the length of the other sides to the parallelogram.

Example 1  
Find the area of a parallelogram with a base of 5 cm and a height of 2.3 cm.

Solution

\[
A \text{ (parallelogram)} = B \times H
\]

\[
= (5 \times 2.3) \text{ cm}^2
\]

\[
= 11.5 \text{ cm}^2
\]

Example 2  
Find the area of the following parallelogram.

Solution  
Since the only ‘height’ given (\textit{ie} the distance between two opposite sides) is 25 mm, then we must consider 12 mm as our ‘base’ and realise that the measurement of 27 mm (the ‘slant height’) is not relevant and hence not necessary for our particular calculations.

So,  
\[
A \text{ (parallelogram)} = B \times H
\]

\[
= (12 \times 25) \text{ mm}^2
\]

\[
= 300 \text{ m}^2
\]
Example 3  Find the area of the following parallelogram.

Solution  Again, we can see that the only ‘height’ given is 15 m (since 17 m is actually the ‘slant height’, not the ‘vertical height’) and hence our base is 26 m.

So, \[ A \text{ (parallelogram)} = B \times H \]
\[ = (26 \times 15) \text{ m}^2 \]
\[ = 390 \text{ m}^2 \]

Remember then, that you may often be given more measurements than you actually require, and care must be taken in selecting the correct ones for your particular formula.

EXERCISE

1. Find the area of a parallelogram with a base of 12 mm and a height of 11 mm.
2. Find the area of a parallelogram with a base of 2.3 m and a height of 1.2 m.
3. Find the areas of the following parallelogram.

4. Find the area of the following parallelogram.

5. Find the area of the following parallelogram.
OBJECTIVE 5

TO CALCULATE THE AREA OF A TRIANGLE

A parallelogram whose diagonal is drawn in always forms two equal triangles.

So, since $A$ (parallelogram) = $B \times H$

then

$$A \text{ (triangle)} = \frac{1}{2} (B \times H)$$

**Example 1**  
Find the area of a triangle with a base of 16 m and a height of 5 m.

**Solution**

$$A \text{ (triangle)} = \frac{1}{2} B \times H$$

$$= \left( \frac{1}{2} \times \frac{16}{1} \times \frac{5}{1} \right) \text{ m}^2$$

$$= 40 \text{ m}^2$$

**Example 2**  
Find the area of a triangle with a base of 4.3 cm and a height of 1.2 cm.

**Solution**

$$A \text{ (triangle)} = \frac{1}{2} B \times H$$

$$= \left( \frac{1}{2} \times \frac{4.3}{1} \times \frac{1.2}{1} \right) \text{ cm}^2$$

$$= 2.58 \text{ cm}^2$$
Example 3  Find the area of the triangle below.

Solution

\[ A \text{ (triangle)} = \frac{1}{2} (B \times H) \]
\[ = \frac{1}{2} (4.2 \times 1.5) \text{ m}^2 \]
\[ = 3.15 \text{ m}^2 \]

Example 4  Find the area of the triangle below.

Solution  Since this is a right-angled triangle, then the side marked 5 mm is also our height.

\[ A \text{ (triangle)} = \frac{1}{2} (B \times H) \]
\[ = \frac{1}{2} (12 \times 5) \text{ mm}^2 \]
\[ = 30 \text{ mm}^2 \]

Notice that in both of these last examples, you were supplied with more information than you actually required.
EXERCISE

1. Calculate the area of a triangle with a base of 52 mm and a height of 36 mm.
2. Calculate the area of a triangle with a base of 22 cm and a height of 38 cm.
3. Calculate the area of a right triangle with a base of 40 mm which has a vertical side of length 28 mm.
4. Calculate the area of the triangle below.
OBJECTIVE 6

TO CALCULATE THE AREA OF A CIRCLE

The formula for the area of a circle is:

\[ A \text{ (circle)} = \pi r^2 \]
\[ = 3.14 \times r \times r \]

Example 1  Find the area of a circle with a radius of 10 cm.

Solution

\[ A \text{ (circle)} = \pi r^2 \]
\[ = (3.14 \times 10 \times 10) \text{ cm}^2 \]
\[ = 314 \text{ cm}^2 \]

Example 2  Find the area of a circle with a radius of 2.6 mm.

Solution

\[ A \text{ (circle)} = \pi r^2 \]
\[ = (3.14 \times 2.6 \times 2.6) \text{ mm}^2 \]
\[ = 21.2264 \text{ mm}^2 \]

Example 3  Find the area of a circle with a diameter of 7 m.

Solution

Since the diameter = 7
then the radius \[ r = \frac{7}{2} = 3.5 \]
\[ \therefore A \text{ (circle)} = \pi r^2 \]
\[ = (3.14 \times 3.5 \times 3.5) \text{ m}^2 \]
\[ = 38.465 \text{ m}^2 \]
EXERCISE

Find the areas of the following circles.

1. radius of 7 cm
2. radius of 20 cm
3. radius of 1 m
4. radius of 4.2 m
5. radius of 21 m
6. diameter of 200 km
7. radius of 1.75 m
OBJECTIVE 7

TO CALCULATE THE AREA OF A TRAPEZOID

A trapezoid or trapezium is a four-sided figure in which one pair of opposite sides is parallel.

Sides $A$ and $B$ are the parallel sides and $H$ is the height between these two sides. It follows that the area is given by:

$$A \text{ (trapezoid)} = \frac{(A + B)}{2} \times H$$

From this you can see that the area is found by:

(i) adding the parallel sides and dividing by 2, then
(ii) multiplying this by the height $H$.

Example 1  Find the area of the trapezoid below.

Solution

$$A \text{ (trapezoid)} = \frac{(A + B)}{2} \times H$$

$$= \frac{(8 + 12)}{2} \times 4 \text{ cm}^2$$

$$= 40 \text{ cm}^2$$
Example 2  
Find the area of the trapezoid below.

Solution

\[
A \text{ (trapezoid)} = \frac{(A + B)}{2} \times H
\]

\[
= \frac{(3 + 7)}{2} \times 5 \text{ m}^2
\]

\[
= 25 \text{ m}^2
\]

EXERCISE

Find the area for each of the following trapezoids.

1.  

2.  

3.  

4.  

5.
OBJECTIVE 8

TO CALCULATE THE AREA OF COMPOSITE REGIONS

The areas of many complicated figures may be calculated by dividing them into figures whose area can be found easily *i.e.* rectangle, circles, triangles, etc.

*Example 1*  
Find the area of the figure drawn.

By dividing the figure as shown we have reduced it to two separate rectangles whose areas can be calculated.

**Solution**

Area of rectangle $A$ (rectangle) $= Base \times Height$

$= (3 \times 2.5) \text{ m}^2$

$= 7.5 \text{ m}^2$

Area of rectangle $B = Base \times Height$

$= (6 \times 1.5) \text{ m}^2$ (since the base $= 2 + 3 + 1 \text{ m}$)

$= 9 \text{ m}^2$

$\therefore$ Total Area $= (7.5 + 9) \text{ m}^2$

$= 16.5 \text{ m}^2$

*Example 2*  
Find the area of the figure drawn.
We should treat the region as consisting of a rectangle (in the middle), together with two semi-circles (one on each end of the rectangle).

The dimension of 70 m gives us the radius of our semi-circles. It should be noted that the diameter of these semi-circles (140 m) is the dimension for the ends of our rectangle. So we have:

\[ \text{Area of region (i)} = \frac{1}{2} A \text{ (circle)} \]

\[ = \frac{1}{2} \times \pi r^2 \]

\[ = \left( \frac{1}{2} \times \frac{3.14}{1} \times \frac{70}{1} \times \frac{70}{1} \right) \text{ m}^2 \]

\[ = 7693 \text{ m}^2 \]

Area of region (ii) = \( A \) (rectangle)

\[ = B \times H \]

\[ = (450 \times 140) \text{ m}^2 \]

\[ = 63000 \text{ m}^2 \]

And Area of region (iii) = Area of region (i) = 7693 m²

\[ \therefore \text{Total Area} = (7693 + 63000 + 7693) \text{ m}^2 \]

\[ = 78386 \text{ m}^2 \]

Example 3 Find the shaded region’s area in the figure below, if the circle has a radius of 3 cm.

Since the circle has a radius of 3 cm, its diameter must be 6 cm.

Furthermore, since the circle fits snugly inside the square, then the sides of this square must be 6 cm long.
Now, Area of square \( = B \times H \)
\[ = 6 \times 6 \text{ cm}^2 \]
\[ = 36 \text{ cm}^2 \]

and, Area of circle \( = \pi r^2 \)
\[ = 3.14 \times 3 \times 3 \text{ cm}^2 \]
\[ = 28.26 \text{ cm}^2 \]

\[ \therefore \text{ Area of shaded region} = \text{Area of square} - \text{Area of circle} \]
\[ = (36 - 28.26) \text{ cm}^2 \]
\[ = 7.74 \text{ cm}^2 \]
**EXERCISE**

1. Find the area of:

   ![Diagram 1]

2. Find the area of:

   ![Diagram 2]

3. Find the area of:

   ![Diagram 3]

4. Find the shaded area.

   ![Diagram 4]
5. Find the shaded area.
OBJECTIVE 9

TO CALCULATE THE SURFACE AREA OF A RECTANGULAR SOLID

A rectangular solid is a figure whose shape is similar to that of a shoe box. To find its surface area we must find the area for each of the six faces (4 sides, top and bottom).

Example 1 Find the outside surface area of the box below.

Solution This problem is easy if we find the total area of the outside (ie surface area) in stages, recognising that the opposite faces are congruent.

(i) The area of the top of the box is given by:

\[ A \text{(rect)} = B \times H \]

\[ = 10 \times 7 \text{ cm}^2 \]

\[ = 70 \text{ cm}^2 \]

Since the bottom is congruent to the top, the area of the bottom is also 70 cm². Therefore, the area of the top and bottom is 140 cm².

(ii) The area of one of the two shaded areas is given by:

\[ A \text{(rect)} = B \times H \]

\[ = 7 \times 8 \text{ cm}^2 \]

\[ = 56 \text{ cm}^2 \]

Therefore the area of both sides is 112 cm².
(iii) The area of the front of the box is given by:

\[ A \text{ (rect)} = B \times H \]

\[ = 10 \times 8 \text{ cm}^2 \]

\[ = 80 \text{ cm}^2 \]

Therefore the area of front and back is 160 cm\(^2\).

(iv) Thus the required surface area is:

\[ 140 \text{ cm}^2 + 112 \text{ cm}^2 + 160 \text{ cm}^2 = 412 \text{ cm}^2. \]

**EXERCISE**

1. Calculate the total surface area of a rectangular block which measures 5 cm × 4 cm × 3 cm.
2. Calculate the surface area of a rectangular block whose length, width and depth are 250 mm, 10 mm and 20 mm respectively.
3. Calculate the surface area of a blackboard duster which measures 12 cm × 8 m × 5 cm.
4. How many square metres of material are needed to cover a rectangular box which is 2 m × 1 m × 0.5 m?
OBJECTIVE 10

TO CALCULATE THE SURFACE AREA OF A CYLINDER
A cylinder is a figure whose shape is similar to a pipe. Its ends are usually closed, as in the case of a tin of beans.

The total surface area of a cylinder is made up of two separate parts:
(i) the curved surface (where the label is) – C.S.A.
(ii) the end pieces (lid and bottom).

So, total surface area (T.S.A.) of cylinder = C.S.A. + area of ends.

*It can be shown that the C.S.A. = \( \pi \times d \times H \)

and we know that the \( A \) (circle) = \( \pi r^2 \) and both ends have the same area.

\[
\text{T.S.A. (cylinder)} = (\pi \times d \times H) + (\pi r^2)
\]

Example 1 Calculate the total surface area (T.S.A.) of a tin of fruit with a diameter of 10 cm and a height of 20 cm.
Solution

(i) C.S.A. = π × d × H

= (3.14 × 10 × 20) cm²

= 628 cm²

(ii) Area of one end = πr²

= (3.14 × 5 × 5) cm²

= 78.5 cm²

∴ Area of two ends = 2(78.5) cm²

= 157 cm²

∴ T.S.A. (cylinder) = (628 + 157) cm²

= 785 cm²

Example 2
Calculate the curved surface area of a pipe with a diameter of 4 cm and a length of 100 cm.

Solution

C.S.A. = π × d × H

= (3.14 × 4 × 100) cm²

= 1256 cm²

Example 3
A cylindrical bucket (no lid) has a radius of 20 cm and a height of 75 cm. Find its external surface area.

Solution

C.S.A. = π × d × H

= (3.14 × 40 × 75) cm²

= 9420 cm²

Area of bottom = πr²

= (3.14 × 20 × 20) cm²

= 1256 cm²

∴ External Surface Area = (9420 + 1256) cm²

= 10676 cm²
EXERCISE

1. The outside section of a water pipe needs maintenance. If it has a diameter of 30 cm and a length of 100 cm, find the surface area of this curved section.

2. A cylindrical water tank, with no top, requires cleaning. Find the surface area (internal only) that must be attended to. The diameter of the tank is 2.5 m and its height is 4 m.

3. A paint roller is 20 cm long and has a radius of 5 cm. Find the curved surface area of the roller.

4. A tank on a railroad truck has the shape of a cylinder and is made of aluminium. The diameter of its base is 2.2 m and it is 11 m long. How many square metres of aluminium would have been used to make the tank?
SECTION 5

Measurement 2
OBJECTIVE 1

TO CALCULATE THE SURFACE AREA OF A CONE, GIVEN THE FORMULA \[ A \text{ (cone)} = \pi r^2 + \pi rS \]

A regular circular cone is a solid, similar to a regular pyramid, whose base is a circle with a radius \( r \). The slant height \( S \), is the distance measured from the vertex (top point) down along the sloping side of the cone to its base. (You must be careful not to confuse this with the vertical height \( H \).)

THE TOTAL SURFACE AREA (CONE) = AREA OF BASE + AREA OF CURVED SURFACE

\[ \therefore A \text{ (cone)} = (\pi r.r) + (\pi r.S) \]

Example 1 Find the surface area of a cone whose base is 10 cm in radius and slant height is 102 cm.

Solution

\[ A \text{ (cone)} = (\pi r.r) + (\pi r.S) \]
\[ = (3.14 \times 10 \times 10) + (3.14 \times 10 \times 102) \]
\[ = 3.14 + 202.8 \text{ cm}^2 \]
\[ = 3516.8 \text{ cm}^2 \]

Example 2 Calculate the surface area of the cone shown, using \( A = (\pi r^2) + (\pi rS) \)

where \( \pi = 3.14 \)

\( r = \) radius of base

\( S = \) slant height
Solution

\[ A \text{(cone)} = \pi r^2 + \pi rS \]
\[ = (3.14 \times 4 \times 4) + (3.14 \times 4 \times 5) \]
\[ = 50.24 + 62.8 \text{ m}^2 \]
\[ = 113.04 \text{ m}^2 \]

Example 3 Calculate the surface area of the cone shown, using

\[ A = (\pi r^2) + (\pi rS) \]

where \( \pi = 3.14 \)

\[ r = \text{radius of base} \]

\[ S = \text{slant height} \]

Solution

Diameter of base = 6 m
\[ \therefore \text{Radius } r \text{ of base} = 3 \text{ m} \]

also slant height \( S = 7 \text{ m} \)

(the vertical height \( H = 5 \text{ m} \) but is not required for this problem)

\[ A \text{(cone)} = \pi r^2 + \pi rS \]
\[ = (3.14 \times 3 \times 3) + (3.14 \times 3 \times 7) \text{ m}^2 \]
\[ = 28.26 + 65.94 \text{ m}^2 \]
\[ = 94.2 \text{ m}^2 \]

EXERCISE

1. Calculate the surface area of a cone whose slant height is 44 nm and whose base radius is 10 mm.

   Use \( A = \pi r^2 + \pi rS \) where \( \pi = 3.14, r = \text{radius of base}, S = \text{slant height} \).

2. Calculate the surface area of a conical wheat silo with a slant height of 20 m and base radius of 6 m.

   Use \( A = \pi r^2 + \pi rS \) where \( \pi = 3.14, r = \text{radius of base}, S = \text{slant height} \).

3. Calculate the surface area of a conical heap of fine beach sand with a slant height of 5 m and base diameter of 20 m.

   Use \( A = \pi r^2 + \pi rS \) where \( \pi = 3.14, r = \text{radius of base}, S = \text{slant height} \).

4. Calculate the surface area of a cone which has a base diameter of 12 cm, slant height of 11 cm and a vertical height of 5 cm.

   Use \( A = \pi r^2 + \pi rS \) where \( \pi = 3.14, r = \text{radius of base}, S = \text{slant height} \).
OBJECTIVE 2

TO CALCULATE THE SURFACE AREA OF A SPHERE, GIVEN THE FORMULA \( A \text{(sphere)} = 4\pi r^2 \)

A sphere is a solid on whose surface every point is an equal distance from the centre of the sphere. Examples would be: a cricket ball, baseball, orange, the moon, etc.

The radius \( r \) of the sphere is the distance from the centre to any point on the surface.

\[
A \text{(sphere)} = 4\pi r^2
\]

\[
\therefore A \text{(sphere)} = 4\pi r^2
\]

Example 1  Find the surface area of a sphere with a radius of 5 cm.

Solution

\[
A \text{(sphere)} = 4\pi r^2
\]

\[
= (4 \times 3.14 \times 5 \times 5) \text{ cm}^2
\]

\[
= 314 \text{ cm}^2
\]

Example 2  Find the surface area of a beach ball with a diameter of 60 cm.

Solution

\[
A \text{(sphere)} = 4\pi r^2
\]

\[
= (4 \times 3.14 \times 30 \times 30) \text{ cm}^2
\]

\[
= 11304 \text{ cm}^2
\]
EXERCISE

1. Find the surface area of a sphere which has a radius of 7 m.
   Use $A = 4\pi r^2$, where $\pi = 3.14$ and $r = \text{radius of sphere}$.

2. Find the surface area of a ball which has a radius of 3 cm.
   Use $A = 4\pi r^2$, where $\pi = 3.14$ and $r = \text{radius of sphere}$.

3. A ball has a diameter of 16 cm. Calculate its surface area.
   Use $A = 4\pi r^2$, where $\pi = 3.14$ and $r = \text{radius of sphere}$.

4. An orange has a diameter of 8 cm. Calculate the area of its peel.
   Use $A (\text{sphere}) = 4\pi r^2$, where $\pi = 3.14$ and $r = \text{radius of sphere}$.
OBJECTIVE 3

TO CALCULATE THE VOLUME OF A RECTANGULAR SOLID

A rectangular solid is a regular prism whose ends are equal and parallel, and whose sides are rectangles. The ends are called the bases, and the distance between the bases is the length, altitude or height $H$. A common example of this shape is that of a shoe box.

To measure volume, it is necessary to consider 3 dimensional units. Such a unit is pictured here.

This is a cube (a square block) whose sides are all one centimetre. Its volume is 1 cubic centimetre or 1 cm$^3$.

The size of the cubic unit can vary. It could be cm$^3$, m$^3$, km$^3$, etc.

This is a rectangular prism, which has been divided into one centimetre cubes. The number of cubes in the bottom layer is $2 \times 3 = 6$. There are 4 layers, each the same as the bottom layer.

Volume = 24 cm$^3$. 
The volume of any prism can be found in this way, *ie* by finding the volume of the bottom layer (base) of the solid, and multiplying this by the number of layers (height). This can be expressed briefly by:

\[
\text{Volume of a prism} = \text{area of base} \times \text{height}.
\]

for a rectangular solid, this becomes:

\[
V (\text{rectangular solid}) = L \times B \times H
\]

since the base is a rectangle, and \( A (\text{rectangle}) = L \times B \).

**Example 1**  
Calculate the volume of a rectangular box whose dimensions are shown.

**Solution**

\[
V (\text{rectangular solid}) = L \times B \times H = 3 \times 7 \times 11 \text{ cm}^3 = 231 \text{ cm}^3
\]

**Example 2**  
Calculate the volume of sand needed to fill a trench 5 m long, 1.5 m wide and 0.5 m deep.

**Solution**

\[
V (\text{rectangular solid}) = L \times B \times H = (0.5 \times 1.5 \times 5) \text{ m}^3 = 3.75 \text{ m}^3
\]
EXERCISE

1. Calculate the volume of a rectangular box whose dimensions are shown.

2. Calculate the volume of a matchbox which measures 4 cm long, 2 cm wide and 1 cm high.

3. Calculate the amount of cement needed for a driveway which is 10 m long, 3 m wide and 0.1 m deep.

4. Calculate the volume of a box measuring 2.5 m long, 1.5 m wide and 0.5 m high.
OBJECTIVE 4

TO CALCULATE THE VOLUME OF A CYLINDER

A regular circular cylinder is a solid, similar to a regular prism, whose bases are circles.

Remembering that the volume of a prism = area of base \( \times \) height and also that the area of a circle = \( \pi r^2 \), then we have the volume of a cylinder given as:

\[
V (\text{cylinder}) = \pi r^2 h
\]

Example 1

Calculate the volume of a cylindrical tank whose radius is 3 m and height 4 m.

Solution

\[
V (\text{cylinder}) = \pi r^2 h = (3.14 \times 3 \times 3 \times 4) \text{ m}^3 = 113.04 \text{ m}^3
\]
Example 2  Calculate how many cubic metres of wheat will fit into a cylindrical silo with a diameter of 4 m and a height of 5 m.

Solution

\[
\text{Diameter} = 4 \text{ metres} \\
\therefore \quad \text{Radius} = 2 \text{ metres} \\
V (\text{cylinder}) = \pi r^2 H \\
= (3.14 \times 2 \times 2 \times 5) \text{ m}^3 \\
= 62.8 \text{ m}^3
\]

EXERCISE

1. Calculate the volume of a cylinder with radius 8 cm and height 10 cm.
2. Calculate the volume of water necessary to fill a cylindrical tank with a diameter of 10 m to a depth of 4 m.
3. Calculate the volume of a pipe which is 3 m long with a diameter of 0.4 m.
4. Calculate the volume of a rod which has a radius of 0.2 cm and is 50 cm long.
OBJECTIVE 5

TO CALCULATE THE VOLUME OF A PYRAMID OR CONE, GIVEN THE FORMULA \[ V = \frac{1}{3} (\text{Area of Base}) \times \text{Height} \]

A regular pyramid is a solid with a base (which may be any regular shape such as a triangle, square, rectangle, circle, etc.) and sides which meet at a point, called the vertex. Here are some examples.

(i) triangular pyramid
(ii) rectangular pyramid
(iii) circular pyramid (CONE)

In general, the volume of a pyramid is given by:

\[ V = \frac{\text{Area of Base}}{3} \times \text{Height} \]

That is, find the area of the base, divide by 3 and multiply by the vertical height.

The table below shows the specific formulas for certain pyramids.

<table>
<thead>
<tr>
<th>type of pyramid (named by shape of the base)</th>
<th>area of base</th>
<th>volume of pyramid</th>
</tr>
</thead>
<tbody>
<tr>
<td>(i) triangular pyramid</td>
<td>( \frac{B \times h}{2} )</td>
<td>( \frac{(B \times h)}{6} \times H )</td>
</tr>
<tr>
<td>(ii) rectangular pyramid</td>
<td>( B \times L )</td>
<td>( \frac{(B \times L)}{3} \times H )</td>
</tr>
<tr>
<td>(iii) circular pyramid (CONE)</td>
<td>( \pi r^2 )</td>
<td>( \frac{(\pi r^2)}{3} \times H )</td>
</tr>
</tbody>
</table>
**Example 1**
Calculate the volume of a pyramid with a base which measures 30 m by 20 m (rectangular) and whose vertical height is 15 m.

Use \( V = \frac{1}{3} \text{(area of base)} \times \text{vertical height} \).

**Solution**

\[
\text{Area of base} = B \times L \\
= (30 \times 20) \text{ m}^2 \\
= 600 \text{ m}^2 \\
V = \frac{1}{3} \text{(area of base)} \times H \\
= \frac{600}{3} \times \frac{15}{1} \text{ m}^3 \\
= 3000 \text{ m}^3
\]

**Example 2**
Calculate the volume of a cone with a base radius of 5 cm and a vertical height of 20 cm.

Use \( V = \frac{1}{3} \text{(area of base)} \times \text{vertical height} \).

**Solution**

\[
\text{Area of base} = \pi \times r \times r \\
= 3.14 \times 5 \times 5 \text{ cm}^2 \\
= 78.5 \text{ cm}^2 \\
V = \frac{1}{3} \text{(area of base)} \times H \\
= \frac{78.5}{3} \times \frac{20}{1} \text{ cm}^3 \\
= 523.3 \text{ cm}^3
\]

**EXERCISE**

1. Find the volume of a square pyramid if the base has sides of length 3.2 m and the height is 7.3 m.
2. Find the approximate volume of a cone (circular pyramid) if the radius of the base is 3 m and the height is 4 m.
3. Find the volume of a rectangular pyramid if the base measures 8.2 cm by 6.7 cm and its vertical height is 12 cm.
4. Sand is poured onto a circular area of radius 6 m, to a height of 14 m. What is the approximate volume of sand?
OBJECTIVE 6

TO CALCULATE THE VOLUME OF A SPHERE, GIVEN THE FORMULA: \( V = \frac{4}{3} \pi r^3 \)

In Objective 2 of this module, we found the surface area of a sphere. We will now find its volume.

The formula for the volume of a sphere is given by:

\[
V (\text{sphere}) = \frac{4}{3} \pi r^3
\]

If we expand this, it becomes:

\[
V = \frac{4 \times \pi \times r \times r \times r}{3}
\]

That is, we multiply the radius by itself 3 times, then multiply by both \( \pi \) and 4, then finally, we divide by 3.

Example 1 Find the volume of a sphere with a radius of 2 cm.

Solution

\[
V = \frac{4}{3} \pi r^3
\]

\[
= \frac{4 \times 3.14 \times 2 \times 2 \times 2}{3} \text{ m}^3
\]

\[
= 33.493 \text{ m}^3
\]
Example 2 Calculate the volume of a spherical ball whose radius is 0.5 m.
Use $V \text{ (sphere)} = \frac{4}{3} \pi r^3$.

Solution

\[
V = \frac{4}{3} \pi r^3
\]
\[
= \frac{4 \times 3.14 \times 0.5 \times 0.5 \times 0.5}{3} \text{ m}^3
\]
\[
= 0.523 \text{ m}^3
\]

EXERCISE

Use $V \text{ (sphere)} = \frac{4}{3} \pi r^3$, $\pi \approx 3.14$ to

1. Find the volume of a sphere with radius 5 cm.
2. Find the volume of a sphere with radius 15 cm.
3. What is the volume, in cubic centimetres, of a sphere with a radius of 3 cm?
4. The planet Mercury has a radius of approximately 2 600 000 m ($2.6 \times 10^6$). Find its volume in cubic metres.
5. Find the volume of air in a spherical ball of radius 7 cm.
SECTION 6

Ratio and Proportion
A ratio is a way of comparing two numbers. For example, suppose in a family there are 3 boys and 1 girl.

We can compare the number of boys and girls by using the idea of a ratio – the ratio of the number of boys to the number of girls is 3:1. Notice that the order is important. If we wrote 1:3 we would be saying that the first number (the number of boys) is less than the second number (the number of girls), but in our example there are more boys than girls.

Ratios can also be written as fractions. In our example, we could write the ratio of the number of boys to the number of girls as \( \frac{3}{1} \). When ratios are written in this fraction form, if an improper fraction is involved it should not be changed to a mixed number or a whole number. In other words, although we would normally simplify the fraction \( \frac{3}{1} \) to just 3, it does not make sense to write the ratio \( \frac{3}{1} \) as 3.
OBJECTIVE 1

TO FORM A RATIO FROM APPROPRIATE DATA

Example 1  In the diagram below what is the ratio of the number of circles to the number of squares?

Altogether there are 4 circles and 12 squares, so the required ratio is 4:12 or \( \frac{4}{12} \).

Example 2  In 1987, Perth television viewers had a choice of two commercial television channels and one non-commercial channel. Express this information as a ratio.

Solution  The ratio of the number of commercial channels to the number of non-commercial channels is 2:1.

or

The ratio of the number of non-commercial channels to the number of commercial channels is 1:2.

Example 3  A large pad measures 30 cm by 20 cm, while a small pad measures 15 cm \( \times \) 10 cm. Compare the areas using a ratio.

Solution  The area of the large pad is \( 30 \times 20 = 600 \text{ cm}^2 \).

The area of the small pad is \( 15 \times 10 = 150 \text{ cm}^2 \).

Therefore the ratio of the area of the large pad to the area of the small pad is 600:150.

Example 4  Bill walked 100 metres. Ben walked 1 kilometre. Use a ratio to compare the distances walked.

Solution  Before we can compare the distances walked, we must express both distances in the same unit of measure.
So, Bill walked 100 metres
and Ben walked 1 kilometre or 1 000 metres.
Thus, the ratio of Bill’s distance to Ben’s distance is 100:1 000.

**EXERCISE**

1. What is the ratio of 1 hour to 12 minutes?
2. What is the ratio of 4 hours to 2 days?
3. What is the ratio of 2 metres to 3 kilometres?
4. Express \(\frac{2}{6}\) as a ratio.
OBJECTIVE 2

TO EXPRESS A RATIO IN ITS LOWEST TERMS

When ratios are used to compare two numbers, we are usually not interested in the actual numbers themselves, but in their comparative sizes. For example, in the previous section (see Example 1) we had 4 circles and 12 squares, and the ratio (circles to squares) was expressed as 4:12. Since we are mainly interested in relative numbers of circles and squares, we could say that there are 3 times as many squares as circles, or, the ratio of circles to squares is 1:3, ie for every 1 circle there are 3 squares. If we re-arrange the diagram this is more apparent.

Although the ratio of circles to squares is 4:12, a better way to write this is 1:3. This is called ‘expressing the ratio in its lowest terms’. We can see an easy way to do this, if we write the ratios in fraction form. The ratio of circles to squares is 4:12 or $\frac{4}{12}$. Clearly this fraction can be reduced to its lowest terms by dividing the numerator and the denominator by 4. Thus $\frac{4}{12} = \frac{1}{3}$.

So, the ratio of circles to squares is $4:12 = \frac{4}{12} = \frac{1}{3} = 1:3$.

- Write the ratio as a fraction.
- Reduce the fraction to lowest terms.
- Write the reduced fraction as a ratio.

**Example 1**  
A large pad has an area of 600 cm$^2$ while a small pad has an area of 150 cm$^2$. Compare the areas using a ratio in its lowest terms.

Area of large pad = 600 cm$^2$  
Area of small pad = 150 cm$^2$  

same units of measure
\[ \therefore \text{Ratio of areas (large to small) is } \frac{600}{150}. \]
\[ = \frac{600}{150} \]
\[ = \frac{4}{1} \]
\[ = 4:1 \]

Notice that we leave the fraction as \( \frac{4}{1} \). We do not write it as 4.

That is the large pad has an area 4 times that of the small pad.

**Example 2**

Tom has $13.00 and Ted has $5.20. What is the ratio of Ton’s money to Ted’s money?

\[ = \frac{1300}{520} \]
\[ = \frac{5}{2} \]
\[ = 5:2 \]

**(Express both amounts in cents.)**

**EXERCISE**

Write the following comparisons as ratios reduced to lowest terms.

1. 400 homes to 360 bathtubs
2. 200 homes to 600 television sets
3. 45 boys to 30 girls
4. 4 seconds to 2 hours
5. 10 centimetres to 1 metre
OBJECTIVE 3

TO DIVIDE A QUANTITY INTO TWO PARTS GIVEN THE RATIO OF THE TWO PARTS

Example 1  Tom and Bill agree to clean out a neighbour’s garage for $60.00. Tom works all afternoon (4 hours) but Bill goes home after only 2 hours. How should the $60 be divided between Tom and Bill so that each receives a fair share for the amount of work done?

Solution  Tom worked for 4 hours.
Bill worked for 2 hours.
The total number of hours worked is 6, for which the amount paid is $60. So, each hour’s work is worth $10.
Tom worked 4 hours so he should receive $40 and Bill worked 2 hours so he should receive $20.

What we have done in this example is divide $12 into two parts, which have a given ratio, namely the ratio of the hours worked, i.e. 4:2. The answers we obtain must satisfy two conditions:
1. The amount must add to $60 ($40 + $20 = $60).
2. The amount must form the correct ratio 4:2 ($40:$20 = 4:2).

Example 2  Share $72 between A and B so that the ratio of A’s share to B’s share is 1:3.

Solution  To say that the ratio A’s share to B’s share is 1:3 means that for every $1 that A gets, B gets $3. That is, B always gets 3 times as much as A. Because of this, we could think of the $72 as being divided into four parts, with A getting 1 part and B getting 3 parts. How much is in each part?

$72 \div 4 = $18
So, A’s share is $18 and B’s share is $54.

Example 3  Divide $48 between X and Y so that the ratio of X’s share to Y’s share is 7:5.

Solution  This time the $48 is divided into 12 parts (7 + 5 = 12) with 7 of these parts going to X and the remaining 5 parts going to Y. How big is each part?

$48 \div 12 = $4.
So, X’s share is $28 and Y’s share is $20.
Example 4  A piece of wood 45.5 cm long is to be cut into two pieces so that the ratio of the lengths of the pieces is 5:8. How long will each piece be?

Solution  Total length 45.5 cm is divided into $5 + 8 = 13$ parts so each part is $45.5 \div 13 = 3.5$ cm long.
So the first piece is $5 \times 3.5$ cm = 17.5 cm
and the second piece is $8 \times 3.5$ cm = 28.0 cm.

EXERCISE

1. Divide 15 cm in the ratio 2:3.
2. Divide $12$ in the ratio 3:1.
3. Tracy and Beryl agree to distribute pamphlets in letter boxes for $20. Tracy delivers 400, whereas Beryl delivers 600. How should the $20 be divided between Tracy and Beryl so that each receives a fair share for the amount of work done?
5. Divide 14.4 m in the ratio 5:7.
OBJECTIVE 4

TO SOLVE AN EQUATION IN THE FORM OF A PROPORTION

A *proportion* is a statement which expresses the equality of two ratios. For example, the ratios 4:12 and 1:3 are equal, so the statement 4:12 = 1:3 is a proportion.

Example 1  Is 3:2 = 9:6 a proportion?

Solution  The easiest way to check whether the statement is a proportion, i.e. whether the two ratios are equal, is to write the ratios in fraction form. The question then becomes:

Are \( \frac{3}{2} \) and \( \frac{9}{6} \) equivalent fractions?

We can check this by using the method of cross-multiplication.

\[
\begin{align*}
\frac{3}{2} \times \frac{9}{6} &= \frac{3 \times 6}{2 \times 6} \\
&= \frac{18}{12} \\
&= \frac{3}{2}
\end{align*}
\]

\( \therefore \) 3:2 = 9:6  is a proportion, since the ratios are equal.

Example 2  What number does b represent if 3:4 = b:16 is a proportion?

Solution  We want to find the value of b which makes 3:4 equal to b:16.

i.e. \( \frac{3}{4} = \frac{b}{16} \)

By cross-multiplying we have:

\[
\begin{align*}
3 \times 16 &= 4b \\
b \times 4 &= 4b
\end{align*}
\]

and these must be equal since the ratios are to be equal.

\( ie \) 4b = 48  

so  \( b = 12 \)  Divide both sides by 4.
Example 3  Solve the proportion $5:7 = 10:m$.

**Solution**

\[
\begin{align*}
5:7 &= 10:m \\
\text{ie } \frac{5}{7} &= \frac{10}{m} \\
5m &= 70 \\
m &= 14
\end{align*}
\]

Example 4  Find $a$ if the statement $a:42 = 5:35$ is a proportion.

**Solution**

\[
\begin{align*}
a:42 &= 5:35 \\
\text{ie } \frac{a}{42} &= \frac{10}{35} \\
\text{(Cross-multiply.)} \\
a &= \frac{10 \times 42}{35} \\
\therefore a &= 6
\end{align*}
\]

**EXERCISE**

Say whether the following ratios form a proportion. (Use cross-multiplication to help you decide.)

1. \( \frac{8}{12} = \frac{10}{15} \)
2. \( \frac{12}{21} = \frac{10}{15} \)
3. \( 9:32 = 12:48 \)
4. \( \frac{15}{21} = \frac{3}{5} \)

Use cross-multiplication to solve the following proportions.

5. \( \frac{1}{2} = \frac{x}{8} \)
6. \( \frac{7}{5} = \frac{m}{10} \)
7. \( 8:7 = 32:n \)
8. \( 4:10 = 22:y \)
OBJECTIVE 5

TO SOLVE WORD PROBLEMS WHICH INVOLVE PROPORTIONS

**Example 1** 12 bottles of soft drink cost $3.36. What would 5 bottles cost?

**Solution** This type of problem can be solved using proportions, as follows:

Ratio of bottles = ratio of cost

ie 12 : 5 = 3.36 :

\[ \frac{12}{5} = \frac{3.36}{X} \]

\[ 12 \times X = 5 \times 3.36 \]

\[ 12X = 16.80 \]

\[ X = \frac{16.80}{12} \]

\[ X = 1.40 \]

ie 5 bottles will cost $1.40.

However, it is probably easier to find the solution using the so-called 'unitary method' as below:

12 bottles cost $3.36

\[ 1 \text{ bottle cost } \frac{3.36}{12} = 0.28 \]

\[ 5 \text{ bottles cost } 5 \times 0.28 = 1.40 \]

**Example 2** 290 bolts weigh 130 kg. How much would 440 of these bolts weigh?

**Solution**

290 bolts weigh 130 kg

\[ 1 \text{ bolt weighs } \frac{130}{290} \text{ kg} \]

\[ 440 \text{ bolts weigh } 440 \times \frac{130}{290} \text{ kg} \]

\[ = 197.24 \text{ kg} \]
Example 3  
On a map of the Perth Metropolitan area, a scale of 1:10 000 is used. The distance from the Barrack Street Jetty to Perth Oval, on the map, is approximately 20 cm. What is the actual distance between these two places?

Solution  
The scale used is 1:10 000,  
\[ \text{ie } 1 \text{ cm on the map represents } 10 000 \text{ cm in real life. Therefore, } 20 \text{ cm on the map represents } 20 \times 10 000 \text{ cm} \]
\[ = 200 000 \text{ cm} \]
\[ = 2 000 \text{ m} \]
\[ = 2 \text{ km}. \]
\[ \text{ie } \]the distance from the Barrack Street Jetty to Perth Oval is approximately 2 kilometres.

So far we have used ratios to compare numbers, eg numbers of boys and girls, numbers of circles and squares, and quantities expressed in the same unit of measure, eg areas of pads, and distances walked. Sometimes it is useful to compare things which are measured differently. For example, suppose a man walks 10 kilometres in 2 hours, we can use a fraction to compare the numbers of kilometres with the number of hours, ie \( \frac{10}{2} \) kilometres per hour.

\[
\text{We could also use the fraction } \frac{2}{10}, \text{ provided we used the appropriate units; namely } \frac{2}{10} \text{ hours per kilometre.}
\]

Notice that we now have a fraction in which the number at the top is a different sort of number from the number at the bottom:
\[
\frac{10}{2} \quad \leftarrow \quad \text{kilometres}
\]
\[
\frac{2}{10} \quad \leftarrow \quad \text{hours}
\]

This is an example of a special kind of ratio called a rate. The important difference between rates and the other types of ratios is that with a rate, units must be shown, eg kilometres per hour, or hours per kilometre, depending on which way the fraction is written.

Example 4  
$6.02 will buy 7 kilograms of apples. Express this as rate.

Solution  
\[
\frac{\$6.02}{7} \quad \text{kilograms} \quad \leftarrow \quad \text{The fraction } \frac{6.02}{7} \quad \text{reduces to } \frac{0.86}{1}.
\]
\[
= \frac{\$0.86}{1} \quad \text{kilogram} \quad \leftarrow \quad \\text{ie } \$0.86 \text{ per kilogram.}
\]
EXERCISE

1. A pack of 6 hamburger buns costs $2.10. How much would 5 buns cost?
2. Batteries cost $7.50 for 3. How much would 5 cost?
3. Colour film may be bought as 12 exposures for $9.96 or 20 exposures for $14.20. Which represents the best buy, and by how much?
4. Cool drink cans are $18.72 for 12. How much would 7 cost?
SECTION 7

Scientific Notation
Scientists use very large and very small numbers.

The speed of light is 29 900 000 000 centimetres per second, the mass of a proton is 0.000 000 000 000 000 000 000 001 65 grams.

It is customary to express such numbers in a shorter way in scientific notation (or standard notation), as the product of a whole number or decimal between 1 and 10 and an integral power of 10. Thus, the above two numbers would be $2.99 \times 10^{10}$ and $1.65 \times 10^{-24}$ respectively.
OBJECTIVE 1

TO SIMPLIFY A PRODUCT OF THE FORM $10^m \cdot 10^n$ WHERE m AND n ARE COUNTING NUMBERS

$10^m$ is read as the ‘$m$th power of 10’ and is expanded as

$10^m = 10 \times 10 \times 10 \times \ldots \times 10$ m times

ie m factors of 10 are multiplied together.

NOTE: 10 is referred to as the base and m is referred to as the index.

Example 1

$10^3 = 10 \times 10 \times 10 = 1000$

Example 2

$10^5 = 10 \times 10 \times 10 \times 10 \times 10 = 100000$

$10^3 \times 10^5 = (10 \times 10 \times 10) \times (10 \times 10 \times 10 \times 10 \times 10) = 10^8$

ie $10^3 \times 10^5 = 10^{3+5} = 10^8$

In general,

$10^m \times 10^n = 10^{m+n}$.

EXERCISE

Simplify:

1. $10^4 \times 10^3$
2. $10^2 \times 10^5$
3. $10 \times 10^4$
4. $10 \times 10^5$
5. $10^2 \times 10^4 \times 10$
6. $10^3 \times 10^4 \times 10^5$
OBJECTIVE 2

TO SIMPLIFY A QUOTIENT OF THE FORM $10^m \div 10^n$ WHERE $m$ AND $n$ ARE COUNTING NUMBERS

Example 1  Simplify $10^7 \div 10^4$.

Solution

$$10^7 \div 10^4 = \frac{10^7}{10^4} = \frac{10 \times 10 \times 10 \times 10 \times 10 \times 10 \times 10}{10 \times 10 \times 10}$$

(We can ‘cancel’ four factors of 10 from the numerator with four factors of ten in the denominator.)

$$= 1 \times 10 \times 10 \times 10$$

$$= 10^3$$

or $\frac{10^7}{10^4} = 10^{7-4} = 10^3$

Again, consider the quotient $\frac{10^5}{10^2} = \frac{10 \times 10 \times 10 \times 10}{10 \times 10}$

$$= 10 \times 10$$

$$= 10^3$$

or $\frac{10^5}{10^2} = 10^{5-2} = 10^3$

In general,

$$10^m \div 10^n = \frac{10^m}{10^n} = 10^{m-n}$$

EXERCISE

Simplify:
1. $10^4 \div 10^3$
2. $10^7 \div 10^5$
3. $10^6 \div 10^2$
4. $10^3 \div 10$
OBJECTIVE 3

TO SIMPLIFY AN EXPRESSION OF THE FORM $10^m \cdot 10^n$ OR $10^m \div 10^n$ WHERE $m$ AND $n$ ARE ZERO OR COUNTING NUMBERS

Example 1  Simplify $10^3 \div 10^3$.

Solution

\[
\frac{10^3}{10^3} = \frac{10 \times 10 \times 10}{10 \times 10 \times 10} = 1 \quad \text{Obviously!}
\]

Remember also that we have the rule $\frac{10^m}{10^n} = 10^{m-n}$.

So, $\frac{10^3}{10^3} = 10^{3-3} = 10^0 \left( \text{but } \frac{10^3}{10^3} = 1 \right)$.

Therefore, $10^0 = 1$.

Example 2  Simplify $\frac{10^8}{10^0}$

Solution

\[
\frac{10^8}{10^0} = 10^{8-0} = 10^8
\]

EXERCISE

Simplify:

1. $\frac{10^4}{10^4}$  
2. $\frac{10^6}{10^0}$  
3. $\frac{10^8}{10^2}$  
4. $\frac{10^0}{10}$  
5. $\frac{10^3}{10^0}$  
6. $\frac{10^0}{10^0}$
OBJECTIVE 4

TO SIMPLIFY AN EXPRESSION OF THE FORM $10^m \cdot 10^n$ OR $10^m \div 10^n$ WHERE $m$ AND $n$ ARE INTEGERS

Example 1  Simplify $10^2 \div 10^5$.

Solution

\[
\frac{10^2}{10^5} = \frac{10 \times 10}{10 \times 10 \times 10 \times 10 \times 10} = \frac{1}{10^3}
\]

But \( \frac{10^2}{10^5} = 10^{-3} \)

Remember also that we have the rule \( \frac{10^m}{10^n} = 10^{m-n} \)

or another way to consider this is \( \frac{1}{10^3} = \frac{10^9}{10^9} = 10^{9-3} = 10^{-3} \)

In general,

\[
\frac{1}{10^m} = 10^{-m}
\]

Example 2  Simplify $10^3 \times 10^2$.

Solution

\[
10^3 \times 10^2 = 10^{3+2} = 10^{-5}
\]

\[
\therefore 10^{-5} = 10^{-3} \text{ or } \frac{1}{10^5}
\]

Example 3  Simplify $\frac{10^{-4}}{10^{-6}}$.

Solution

\[
\frac{10^{-4}}{10^{-6}} = 10^{-4-(-6)} = 10^{-4+6} = 10^2
\]
**EXERCISE**

Simplify:

1. \( \frac{10^{-3}}{10^{-4}} \)

2. \( 10^{-3} \times 10^{-5} \)

3. \( \frac{10^{-2} \times 10^4}{10^4} \)
OBJECTIVE 5

TO SIMPLIFY EXPRESSIONS OF THE FORM \((10^m)^n\)
WHERE \(m\) AND \(n\) ARE INTEGERS

Example 1  Simplify \((10^3)^2\).

Solution

\[
(10^3)^2 = 10^3 \times 10^3 = (10 \times 10 \times 10) \times (10 \times 10 \times 10) = 10^6
\]

\[\text{i.e. } (10^3)^2 = 10^6 = [10^2 \times 3]
\]

Example 2  Simplify \((10^2)^4\)

Solution

\[
(10^2)^4 = 10^2 \times 10^2 \times 10^2 \times 10^2 = 10^{2+2+2+2} = 10^8
\]

\[\text{i.e. } (10^2)^4 = 10^8 = [10^4 \times 2]
\]

In general,

\[(10^m)^n = 10^{m \times n}\]

and is not to be confused with:

\[10^m \times 10^n = 10^{m+n}\]

Note the difference carefully, and remember it.

EXERCISE

Simplify:

1. \((10^3)^{-1}\)
2. \((10^{-2})^3\)
3. \((10^{-2})^{-3}\)
4. \((10^3)^4\)
5. \((10^{-1})^{-1}\)
6. \((10^4)^{-2}\)
OBJECTIVE 6

SIMPLIFY EXPRESSIONS OF THE FORM \((10^m \cdot 10^n)^p\) OR \((10^m \div 10^n)^p\) WHERE \(m, n\) AND \(p\) ARE INTEGERS

Example 1  Simplify

\[
\frac{10^3 \times 10^4}{10^2}.
\]

Solution

\[
\frac{10^3 \times 10^4}{10^2} = \frac{10^7}{10^2} = 10^5
\]

Example 2  Simplify

\[
\frac{10^2 \times 10^3}{10^{-2}}.
\]

Solution

\[
\frac{10^2 \times 10^3}{10^{-2}} = \frac{10^5}{10^{-2}} = 10^{5-(-2)} = 10^7
\]

Example 3  Simplify \((10^3 \times 10)^5\).

Solution

\[
(10^3 \times 10)^2 = (10^4)^2 = 10^8
\]

Example 4  Simplify \((10^{-2} \times 10^4)^{-3}\).

Solution

\[
(10^{-2} \times 10^4)^{-3} = (10^2)^{-3} = 10^{-6} \text{ or } \frac{1}{10^6}
\]
Example 5  Simplify \((10^{-3} + 10^4)^{-2}\).

Solution

\[
(10^{-3} + 10^4)^{-2} = \left(\frac{10^{-3}}{10^4}\right)^{-2} \\
= (10^{-3-4})^{-2} \\
= (10^{-7})^{-2} \\
= 10^{14}
\]

EXERCISE

Simplify:

1. \((10^2 \times 10^3)^{-1}\)
2. \(\left(\frac{10^{-3}}{10^3}\right)^2\)
3. \((10^{-2} \times 10^{-4})^2\)
4. \((10^4 + 10^{-2})^0\)
5. \((10^{-2} \times 10^{-3})^{-2}\)
OBJECTIVE 7

TO EXPRESS A RATIONAL NUMBER IN SCIENTIFIC NOTATION

In the introduction we stated that to express a number in scientific notation is to express it as the product of a decimal number, between 1 and 10, and an integral power of 10.

Example 1 \[2500000 = 2.5 \times 10^6\]

Example 2 \[0.025 = \frac{2.5}{100} = \frac{2.5}{10^2} = 2.5 \times 10^{-2}\]

The effect of multiplying or dividing a number in the decimal system by 10 is to shift the position of the decimal point. Therefore, changing from one form to the other becomes a matter of counting the number of places you must shift the decimal point. The following have all been expressed in scientific notation.

standard position

\[0.00194 = 1.94 \times 10^{-3}\] ie the decimal point is three positions to the left of the standard position.

\[0.0194 = 1.94 \times 10^{-2}\] ie the decimal point is two positions to the left of the standard position.

\[0.194 = 1.94 \times 10^{-1}\]
\[1.94 = 1.94 \times 10^0\]
\[19.4 = 1.94 \times 10^1\]
\[194.0 = 1.94 \times 10^2\] ie the decimal point is two positions to the right of the standard position.
\[1940 = 1.94 \times 10^3\]

EXERCISE

Express each of the following in scientific notation.

1. 4 600 000
2. 8 700
3. 0.018
4. 0.000 29
5. 7.5
6. 0.09
7. 145.2
8. 713.6
9. 0.000 4
OBJECTIVE 8

TO CONVERT A NUMBER EXPRESSED IN SCIENTIFIC NOTATION INTO ORDINARY DECIMAL NOTATION

Example 1  Express \(2.76 \times 10^{-3}\) in ordinary decimal notation.

Solution  \(2.76 \times 10^{-3} = 0.00276\)

The negative index in \(10^{-3}\) indicates that the decimal point must be shifted 3 times to the left, and zeros added if necessary. This is simply because:

\[2.76 \times 10^{-3} = \frac{2.76}{10^3} = \frac{2.76}{1000}\]

ie a negative index in scientific notation indicates that we have divided by a power of ten. Whereas, a positive index on the other hand indicates that we have multiplied by a power of ten, and hence the decimal point would be shifted to the right.

Example 2  Express \(3.8 \times 10^4\) in ordinary decimal notation.

Solution  \(3.8 \times 10^4 = 3.8 \times 10000\)

\[= 38000\]

Example 3  Express \(4.7 \times 10^{-2}\) in ordinary decimal notation.

Solution  \(4.7 \times 10^{-2} = 0.047\)

Example 4  Express \(3.85 \times 10^5\) in ordinary decimal notation.

Solution  \(3.85 \times 10^5 = 385000\)

Example 5  \(3.26 \times 10^{-4} = .000326\)

Example 6  \(9.8 \times 10^3 = 9800\)
### EXERCISE

Express each of the following in ordinary decimal notation.

1. \(2.5 \times 10^{-3}\)
2. \(8.6 \times 10^{3}\)
3. \(9.1 \times 10^{-2}\)
4. \(7.4 \times 10^{-4}\)
5. \(5.9 \times 10^{4}\)
6. \(6.2 \times 10^{-1}\)
OBJECTIVE 9

TO SIMPLIFY THE PRODUCT OR QUOTIENT OF TWO NUMBERS EXPRESSED IN SCIENTIFIC NOTATION

Example 1  Simplify

\[
\frac{4.8 \times 10^{-3}}{6.0 \times 10^{-4}}.
\]

Solution

\[
\frac{4.8 \times 10^{-3}}{6.0 \times 10^{-4}} = \frac{4.8}{6} \times \frac{10^{-3}}{10^{-4}}
\]

\[
= 0.8 \times 10^{-3-(-4)}
\]

\[
= 0.8 \times 10^{1}
\]

\[
= 8
\]

When dividing, the only helpful thing we can do is to ensure that the divisor (bottom number) is a whole number before carrying out the actual division.

Example 2  Simplify

\[
\frac{3.6 \times 10^{-2}}{1.2 \times 10^{-5}}.
\]

Solution

\[
\frac{3.6 \times 10^{-2}}{1.2 \times 10^{-5}} = \frac{3.6}{1.2} \times \frac{10^{-2}}{10^{-5}}
\]

Remember that when doing objective 10 in decimals we would have multiplied 1.2 by 10 to obtain a whole number – 1.2 \times 10 = 12, and then we must also multiply 3.6 by 10. (whatever we do to the divisor – the 1.2 – we must also do to the dividend – the 3.6.)

\[
= \frac{36}{12} \times 10^{-2+5}
\]

\[
= 3 \times 10^{3} \text{ or } 3 \,000
\]
Example 3  Simplify

\[(9.2 \times 10^3) (4.0 \times 10^{-2})\].

Solution

\[(9.2 \times 10^3) (4.0 \times 10^{-2}) = (9.2 \times 4.0) \times 10^3 \times 10^{-2}\]

\[= 36.8 \times 10^1\]

\[= 368\]

Example 4  Simplify

\[(8.7 \times 10^{-2}) (9.0 \times 10^{-1})\].

Solution

\[(8.7 \times 10^{-2}) (9.0 \times 10^{-1}) = (8.7 \times 9.0) \times 10^{-2} \times 10^{-1}\]

\[= 78.3 \times 10^{-3}\]

\[= 0.0783\]

**EXERCISE**

Simplify:

1. \[\frac{5.5 \times 10^{-2}}{5.0 \times 10^{-2}}\]

2. \[\frac{8.2 \times 10^{3}}{4.1 \times 10^{-2}}\]

3. \[\frac{1.08 \times 10^{-2}}{1.2 \times 10^{-4}}\]

4. \[(6.9 \times 10^{3}) (7.0 \times 10^{4})\]

5. \[(1.1 \times 10^{3}) (8.4 \times 10^{2})\]

6. \[(1.2 \times 10^{-1}) (5.4 \times 10^{3})\]
SECTION 8

Algebra
OBJECTIVE 1

TO TRANSLATE A VERBAL EXPRESSION ABOUT THE SUM, DIFFERENCE PRODUCT OR QUOTIENT OF TWO NUMBERS INTO ALGEBRAIC SYMBOLS

In algebra as well as in arithmetic, there are four fundamental operations:

1. addition (sum)
2. subtraction (difference)
3. multiplication (product)
4. division (quotient)

The answer for each operation is the word in brackets, eg

(i) In the operation ‘2 add 4’ the answer 6 is called the ‘sum of 2 and 4’.
(ii) In the operation ‘3 × 5’ the answer 15 is called the ‘product of 3 and 5’.

In algebra, small letters (x, y, a, b, ...) are used to represent numbers. By using letters and mathematical symbols, short algebraic statements replace lengthy verbal statements. Note how this is done in the following examples.

<table>
<thead>
<tr>
<th>verbal statement</th>
<th>algebraic statement</th>
</tr>
</thead>
<tbody>
<tr>
<td>* 1. seven times a number</td>
<td>7 × a = 7a</td>
</tr>
<tr>
<td>2. twice a number</td>
<td>2 × b = 2b</td>
</tr>
<tr>
<td>3. a number y, plus five</td>
<td>y + 5</td>
</tr>
<tr>
<td>4. a number increased by two</td>
<td>m + 2</td>
</tr>
<tr>
<td>5. four more than a number</td>
<td>x + 4</td>
</tr>
<tr>
<td>6. the sum of two different numbers</td>
<td>x + y</td>
</tr>
<tr>
<td>7. the quotient of a number divided by 2</td>
<td>[ a + 2 = \frac{a}{2} ]</td>
</tr>
<tr>
<td>* 8. one quarter of a number, m</td>
<td>[ \frac{1}{4} \times m = \frac{m}{4} ]</td>
</tr>
<tr>
<td>9. a number, reduced by six</td>
<td>p − 6</td>
</tr>
<tr>
<td>10. five less than a number</td>
<td>r − 5</td>
</tr>
<tr>
<td>11. a number decreased by two</td>
<td>x − 2</td>
</tr>
<tr>
<td>12. the sum of a number and one</td>
<td>m + 1</td>
</tr>
<tr>
<td>* 13. the product of two numbers</td>
<td>a × b = ab</td>
</tr>
</tbody>
</table>
* We notice that when a number and a letter, or when two letters are written next to each other, multiplication is implied. For example, 2x means ‘2 times x’ and xy means ‘x times y’.

**NOTE:** It can be seen from the examples that when a letter is not stated (for the unknown number) then any letter may be used.

**EXERCISE**

Write an algebraic expression for each of the following.

1. a number increased by 15
2. 4 less than a number
3. the product of 7 and a number
4. a number divided by 20
5. a number multiplied by 10
6. 20 divided by a number
7. one fifth of a number
8. the sum of a number and 6
9. a number diminished by 5
10. three quarters of a number
OBJECTIVE 2

TO TRANSLATE A VERBAL EXPRESSION ABOUT THE SQUARE OR CUBE OF A NUMBER INTO ALGEBRAIC SYMBOLS

The square of a number is the product obtained by multiplying the number by itself, eg
3 squared means $3 \times 3$ and can be written $3^2$.
A number $x$, squared, means $x \times x$ and can be written as $x^2$.
5 squared means $5 \times 5$ and can be written as $5^2$.
A number 6, squared, means $6 \times 6$ and can be written as $6^2$.

The cube of a number is the product obtained by multiplying the number by itself, three times, eg
3 cubed means $3 \times 3 \times 3$ and can be written $3^3$.
A number $x$, cubed, means $x \times x \times x$ and can be written $x^3$.
5 cubed means $5 \times 5 \times 5$ and can be written $5^3$.
A number $y$, cubed, means $y \times y \times y$ and can be written as $y^3$.

NOTE: This is sometimes referred to as the ‘third power’ of a number.

EXERCISE

Express each of the following algebraically.

1. the square of 7
2. the cube of 4
3. a number squared
4. a number cubed
5. the product of $m$ times itself
6. the third power of a number
OBJECTIVE 3

TO TRANSLATE A VERBAL EXPRESSION INVOLVING TWO OR MORE ALGEBRAIC OPERATIONS INTO ALGEBRAIC SYMBOLS

When a second operation is performed on a sum or difference, we use brackets ( ) to show the order of the operations, eg

$2 + 3 \times 4$ could mean $(2 + 3) \times 4 = 20$ or it could mean $2 + (3 \times 4) = 14$.

The wording of a question should be studied very carefully to see where the brackets belong. The brackets must be placed around the operation which is to be done first.

Example 1  Express in algebraic symbols ‘twice the sum of $x$ and 4’.
Solution  Clearly, the operation of summing $x$ and 4 must be done first, and is represented as $(x + 4)$ then, secondly, we want twice this sum, i.e. $2 \times (x + 4)$ or simply $2(x + 4)$.

Example 2  Express in algebraic symbols ‘one half the difference of $x$ decreased by 3’.
Solution  Clearly, the operation of finding the difference of $x$ decreased by 3 must be done first, and is represented as $(x - 3)$, then, secondly, we want one half of this difference i.e. $\frac{1}{2} (x - 3)$ or $\frac{(x - 3)}{2}$.

Example 3  Express in algebraic symbols ‘3 less than the product of 2 and $x$’.
Solution  Firstly, the product of 2 and $x$ is $2x$. Secondly, 3 less than this product is $(2x) - 3$ or simply $2x - 3$.

Example 4  Express in algebraic symbols ‘2 times the quotient of $x$ divided by 5’.
Solution  Firstly, the quotient of $x$ divided by 5 is $\left(\frac{x}{5}\right)$. Secondly, twice this quotient is $2\left(\frac{x}{5}\right)$ or $\frac{2x}{5}$.

Example 5  Express in algebraic symbols ‘15 decreased by 3 times the square of $x$’.
Solution  Firstly, the square of $x$ is $x^2$. Secondly, 3 times the square of $x$ is $(3x^2)$ and finally, 15 decreased by the above amount is $15 - (3x^2)$ or simply $15 - 3x^2$. 

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**EXERCISE**

Express each of the following in algebraic symbols.

1. $x$ increased by twice $y$
2. twice the sum of $x$ and $y$
3. 40 decreased by three times $x$
4. 15 minus the product of 3 and $x$
5. half of $x$, increased by the product of 7 and $y$
OBJECTIVE 4

TO EVALUATE AN ALGEBRAIC EXPRESSION IN ONE OR TWO VARIABLES BY SUBSTITUTION

If a variable \( x \) appears in an equation, it means that we can assign a value, or a number of values to the variable in order to make the equation true. If we then write \( x = 6 \) this means that \( x \) now has the numerical value of 6 units for that equation. We can then substitute 6 wherever an \( x \) appears in the equation.

Example 1  Evaluate \( 2x - 4 \) for \( x = 6 \).

Solution

\[
2x - 4 = 2(6) - 4 \\
= 12 - 4 \\
= 8
\]

(Replace \( x \) with 6.)

(Do multiplication first.)

Example 2  Evaluate \( x + 4y \) where \( x = 6 \) and \( y = -2 \).

Solution

\[
x + 4y = 6 + 4(-2) \\
= 6 - 8 \\
= -2
\]

(Replace \( x \) with 6 and \( y \) with \(-2\).)

(Do multiplication first.)

EXERCISE

Evaluate each of the following, using \( x = 2 \), \( y = -3 \) and \( z = 0 \).

1. \( 2x - y \)  
2. \( 3x^2 + 2y \)  
3. \( \frac{x^2y}{3x} \)  
4. \( 3xyz \)  
5. \( \frac{1}{4}(x - 2y) \)
OBJECTIVE 5

TO WRITE THE SUM OF TWO OR THREE LIKE TERMS AS A SINGLE TERM

Consider the situation in which a person has $2 in his hand and is then given a further $3. By using the letter d for dollars, the above could be written algebraically as:

\[ 2d + 3d. \]

It is obvious to us that the person has received $5 altogether, so clearly:

\[ 2d + 3d = 5d \]

Similarly,

\[ 2x + 3x = 5x \]
\[ 3y + 2y + 4y = 9y \]
\[ 7d - 2d = 5d \]
\[ -2x - 3x = -5x \]
\[ 2xy + 4xy = 6xy \]
\[ 3x^2 + 5x^2 = 8x^2 \]
\[ 2x + x = 3x \quad (x = 1x, \text{ie the number 1 need not appear.}) \]
\[ m + m = 2m \]
\[ -3mn - 2mn + mn = -4mn \]

EXERCISE

Find each of the following sums.

1. \[ 5m + 2m \]
2. \[ 6y - 5y \]
3. \[ 3x + 2x + 5x \]
4. \[ 2n + 3n - 6n \]
5. \[ 6y + 3y - 6y \]
6. \[ 6a - 3a - 10a \]
OBJECTIVE 6

TO SIMPLIFY AN EXPRESSION BY COLLECTING LIKE TERMS

In the previous objective we discussed the case in which a person received $2 and then another $3. This was represented as $2d + 3d = 5d$.

Consider now the case of a person who received $2 and then a further 10 cents. Using $d$ for dollars and $c$ for cents, this could be expressed algebraically as:

$$2d + 10c.$$

Note that there is no way of simplifying this algebraic statement any further, i.e. $2d + 10c$ do not equal $12dc$ or any other such combination. It is a well known fact that in terms of money, he will always have simply $2$ plus 10 cents.

What we are saying algebraically is that you cannot combine unlike terms, but you can always combine like terms, e.g.

1. $2d + 3d + 10c$ cannot be simplified any further, i.e. we could combine the like terms $(2d + 3d)$ but not the unlike terms $(5d + 10c)$.
2. $2d + 3d + 10c + 5c = 5d + 15c$ – notice that we have combined only the like terms.
3. $2x - 3y + 4x + 7y = 2x + 4x - 3y + 7y$
   $$= 6x + 4y$$
4. $2x^2 - 5y + x^2 + 8y = 2x^2 + x^2 - 5y + 8y$
   $$= 3x^2 + 3y$$
5. $3x - 5xy - 2x + 7xy = 3x - 2x - 5xy + 7xy$
   $$= x + 2xy$$
6. $-2m - 3n - 4m + 6n + 7m = m + 3n$

In each of the above cases we say that we have ‘simplified’ the algebraic expressions. If it so happens that the expression does not contain any like terms, then we say that it cannot be simplified any further since we cannot add unlike terms.
EXERCISE

Simplify each of the following.

1. $5x + 3y - y + 2x + 2y - x$
2. $5a + 3b - 2a + b - 3a + 4b$
3. $3x^2 - 2x + 4x - 2x^2 + 3x - 5x$
4. $5x^2 - 2xy - xy - x^2$
5. $5a - 4ax - x^2 - 4x - 3a + x^2 - 2a + 4ax$
6. $-3x^2 - 2x + 3 - 4x + 2x^2 - 3x - 2$
7. $2a - 3b + 4c - 7$
OBJECTIVE 7

TO REMOVE BRACKETS FROM A GIVEN EXPRESSION BY USING THE DISTRIBUTIVE PROPERTY

Consider the following.

Example 1 Find the sum \((x + 3) + (x + 3) = x + 3 + x + 3 = 2x + 6\).

However, \((x + 3) + (x + 3) = 2(x + 3)\).

So, clearly \(2(x + 3) = 2x + 6\).

In other words, \(2(x + 3) = (2 \cdot x) + (2 \cdot 3)\).

We are said to have distributed (multiplied) the number 2 over the sum \((x + 3)\), \(ie\) we multiply each term inside the bracket by 2. This is applying the distributive property, which can be summarised as:

\[ a(b + c) = ab + ac \]

or expanding:

\[ a(b + c + d) = ab + ac + ad \]

\(ie\) the expression outside the bracket is multiplied in turn with each expression inside the brackets.

Similarly,

\[ a(b - c) = ab - ac \]

\[ -(b + c) = (-1)(b + c) = -b - c \]

\[ -(b - c) = (-1)(b - c) = -b + c \]

Example 2 Rewrite \(2(x + 5 - 3y)\) without ‘brackets’.

Solution \(2(x + 5 - 3y) = 2x + 10 - 6y\)

Example 3 Expand \(-3(2x - y)\) by removing the ‘brackets’.

Solution \(-3(2x - y) = -6x + 3y\)

Remember that when we multiply like signs, we get a + whereas when we multiply unlike signs, we get a −.

Example 4 Expand \(-2x(3 - 5y)\) by using the distributive property.

Solution \(-2x(3 - 5y) = -6x + 10xy\)
Example 5  Expand $-(4x + 9y - 8)$

Solution  $-(4x + 9y - 8) = (-1) (4x + 9y - 8)$

$= -4x - 9y + 8$

EXERCISE

Write each of the following without ‘brackets’ by applying the distributive property.

1. $3(x - 5)$  
2. $-4(2y - 5)$  
3. $-(2y - 1)$  
4. $-8(x - 2)$  
5. $-3(x + 2y - 1)$  
6. $-(4x + 9y - 6)$  
7. $2x(3x^2 - 2x + 1)$
OBJECTIVE 8

TO SIMPLIFY AN EXPRESSION BY REMOVING BRACKETS AND COLLECTING LIKE TERMS

So far you have learnt two skills:

(a) removing brackets

(b) collecting like terms.

We now want to do problems which involve both of these operations. It is necessary to remove all brackets first (by use of the distributive property) and then to collect any like terms.

Example 1

Simplify $3x + 2(x - 5)$.

Solution

$$3x + 2(x - 5)$$

= $3x + 2x - 10$  \[(Remove\ the\ bracket.\ Collect\ like\ terms.)\]

= $5x - 10$

Example 2

Simplify $3(x - 5) - 2(x - 4)$.

Solution

$$3(x - 5) - 2(x - 4)$$

= $3x - 15 - 2x + 8$  \[(Remove\ brackets\ –\ take\ care\ to\ multiply\ through\ every\ term\ inside\ the\ bracket.)\]

= $x - 7$

Example 3

Simplify $6 - (y + 5 - 2x)$.

Solution

$$6 - (y + 5 - 2x)$$

= $6 - 1(y + 5 - 2x)$

= $6 - y - 5 + 2x$

= $1 - y + 2x$

Example 4

Simplify $3x^2 - (-x^2 - 5x + 6)$.

Solution

$$3x^2 - (-x^2 - 5x + 6)$$

= $3x^2 - 1(-x^2 - 5x + 6)$

= $3x^2 + x^2 - 5x + 6$

= $4x^2 + 5x - 6$
Example 5  
Simplify $2x(3x - 5) - 3(x^2 - 6x)$.

Solution

$$2x(3x - 5) - 3(x^2 - 6x)$$
$$= 6x^2 - 10x - 3x^2 + 18x$$
$$= 3x^2 + 8x$$

EXERCISE

Simplify:

1. $2(x + 3) - (x - 4)$
2. $-3x^2 - (2x - 3x^2)$
3. $4(x + 2x^2) - (3x - 5x^3)$
4. $2x(x - 3) + x(2x - 1)$
5. $8(r - 2s) - 3(2r + s)$
SECTION 9

First Degree Equations
OBJECTIVE 1

TO SOLVE AN EQUATION OF THE FORM $ax = c$ or $p + x = q$

The equation $y – 5 = 3$ cannot be said to make a false statement or a true one. It merely states that $(y – 5)$ and $3$ represent the same number.

The symbol $y$ (or $x$, $a$, $b$, etc) is called a variable, because it can take on a variety of different values.

When we are asked to ‘solve an equation’, or ‘find the truth number of an equation’, or simply ‘find the answer’, we have been asked to find that value for the variable which makes the equation a true statement. For example:

When we choose $y = 10$, the equation becomes $10 – 5 = 3$ (replacing $y$ by $10$), which is false.

Similarly, when we choose $y = –2$, the equation becomes $–2 – 5 = 3$ which is also false.

Perhaps you can already see that if we choose $y = 8$ then the equation becomes $8 – 5 = 3$ which is true.

So, if asked to solve $y – 5 = 3$, we would say $y = 8$ or the truth number $= 8$ or the answer is $8$, etc.

However, it is not always so easy to see the solution to an equation, especially when they become more complicated. So we will now develop a set of rules or steps to be followed in solving an equation.

**RULE 1**

If $ax = c$

then $x = \frac{c}{a}$.

In effect, we have simply divided both sides of the equation by ‘$a$’ This is similar to the process of cross-multiplying which we used in both fraction and ratio-proportion work.

**Example 1**

Solve $2x = 10$.

**Solution**

$2x = 10$

then $x = \frac{10}{2}$  

$\therefore x = 5$
Example 2  Solve \(-6x = 15\).

Solution

\[ -6x = 15 \]

then \( x = \frac{15}{-6} \) \hspace{1cm} Rule 1

\[ \therefore x = -2\frac{1}{2} \]

(since unlike signs give a \(-ve\) value when multiplied or divided)

NOTE:  If \( \frac{x}{a} = c \)

then \( x = ac \)

This is another form of Rule 1. Again you can see that we have simply cross-multiplied. In effect, we have multiplied both sides of the equation by ‘a’.

Example 3  Solve \( \frac{x}{2} = 6 \).

Solution

\[ \frac{x}{2} = 6 \]

\[ \frac{x}{2} = \frac{6}{1} \]

then \( x = 2 \times 6 \) \hspace{1cm} Rule 1

\[ \therefore x = 12 \]

Example 4  Solve \( \frac{1}{3} x = 2 \).

Solution

\[ \frac{1}{3} x = 2 \]

ie \( x = \frac{2}{1} \)

then \( x = 3 \times 2 \) \hspace{1cm} Rule 1

\[ \therefore x = 6 \]
If \( p \times q \) then \( x \equiv q \div p \).

\[
\text{RULE 2}
\]

<table>
<thead>
<tr>
<th>Rule</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>If ( p + x = q ) then ( x = q - p ).</td>
<td></td>
</tr>
</tbody>
</table>

In effect, we have subtracted ‘\( p \)’ from both sides of the equation.

**Example 5** Solve \( 5 + x = 7 \).

**Solution**

\[
5 + x = 7
\]

then \( x = 7 - 5 \) \quad \text{Rule 2}

\[
\therefore \ x = 2
\]

**Example 6** Solve \( y + 3 = 4 \).

**Solution**

\[
y + 3 = 4
\]

then \( y = 4 - 3 \) \quad \text{Rule 2}

\[
\therefore \ y = 1
\]

**Example 7** Solve \( x - 1 = 5 \).

**Solution**

\[
x - 1 = 5
\]

then \( x = 5 - (-1) \) \quad \text{Rule 2} \quad \text{(since two negatives multiplied, ie \(-1\) give a +ve)}

\[
= 5 + 1
\]

\[
\therefore \ x = 6
\]

**Example 8** Solve \( a - 3 = -2 \).

**Solution:**

\[
a - 3 = -2
\]

then \( a = -2 - (-3) \) \quad \text{Rule 2}

\[
= -2 + 3
\]

\[
\therefore \ a = 1
\]

If we look back at the examples and the two rules, we can see that we can summarise them roughly as follows:

*Any number or variable which is moved from one side of an equation to the other will appear there as the opposite operation to which it was previously.*

So, in Example 1, \( \times 2 \) on the left became \( \div 2 \) on the right.

in Example 2, \( \times (-6) \) on the left became \( \div (-6) \) on the right.
in Example 3, \( +2 \) on the left became \( \times 2 \) on the right.
in Example 4, \( +3 \) on the left became \( \times 3 \) on the right.
in Example 5, \( +5 \) on the left became \( -5 \) on the right.
in Example 6, \( +3 \) on the left became \( -3 \) on the right.
in Example 7, \( -1 \) on the left became \( +1 \) on the right.
in Example 8, \( -3 \) on the left became \( +3 \) on the right.

**Example 9**

Solve \( 5x = 30 \).

**Solution**

\[
5x = 30 \\
\text{then } x = \frac{30}{5} \\
\therefore x = 6
\]

**Example 10**

Solve \( x - 8 = 11 \).

**Solution**

\[
x - 8 = 11 \\
\text{then } x = 11 - (-8) \\
\text{So } x = 11 + 8 \\
\text{= 19}
\]

**EXERCISE**

Solve each of the following.

1. \( 2a = 6 \)
2. \( -3b = 12 \)
3. \( \frac{1}{4}y = 5 \)
4. \( \frac{y}{2} = -10 \)
5. \( \frac{1}{-3}y = -7 \)
6. \( 2 + x = 8 \)
7. \( m + 3 = -5 \)
8. \( t - 1 = 6 \)
9. \( x - 10 = -15 \)
10. \( -m = 7 \)
OBJECTIVE 2

TO SOLVE AN EQUATION OF THE FORM \( ax + b = c \)

In studying Objective 1, we found two rules:

**RULE 1**

If \( ax = c \)

then \( x = \frac{c}{a} \).

**RULE 2**

If \( p + x = q \)

then \( x = p - q \),

which we summarised as:

*Any number or variable which is moved from one side of an equation to the other will appear there as the opposite operation to which it was previously.*

We now need to note one further point:

**RULE 3** Where possible, carry out any necessary addition or subtraction before cross-multiplying.

---

**Example 1** Solve \( 2x - 3 = 5 \).

**Solution**

\[
2x - 3 = 5 \\
\text{then } 2x = 5 + 3 \quad \text{Rules 2 and 3} \\
\text{so } 2x = 8 \\
\text{now } x = \frac{8}{2} \\
\therefore x = 4
\]

---

**Example 2** Solve \( 3x + 1 = 16 \).

**Solution**

\[
3x + 1 = 16 \\
\text{then } 3x = 16 - 1 \quad \text{Rules 2 and 3} \\
\text{so } 3x = 15 \\
\text{then } x = \frac{15}{3} \quad \text{Rule 1} \\
\therefore x = 5
\]
Example 3  Solve $5x - 2 = -17$.

Solution

\[
5x - 2 = -17
\]

then  \(5x = -17 + 2\) \quad \text{Rules 2 and 3}

so  \(5x = -15\)

then  \(x = \frac{-15}{5}\) \quad \text{Rule 1}

\[
\therefore \quad x = -3
\]

Example 4  Solve $-2x + 3 = 11$.

Solution

\[
-2x + 3 = -11
\]

then  \(-2x = 11 - 3\) \quad \text{Rules 2 and 3}

so  \(-2x = 8\)

then  \(x = \frac{8}{-2}\) \quad \text{Rule 1} \quad \text{(Remember that $-2x$ means $(-2)$ times $x$.)}

\[
\therefore \quad x = 4
\]

Example 5  Solve $-3x - 2 = -14$.

Solution

\[
-3x - 2 = -14
\]

then  \(-3x = -14 + 2\) \quad \text{Rules 2 and 3}

so  \(-3x = -12\)

then  \(x = \frac{-12}{-3}\) \quad \text{Rule 1}

\[
\therefore \quad x = 4
\]

**EXERCISE**

Solve each of the following.

1.  \(4a + 1 = 9\)
2.  \(3y - 7 = -1\)
3.  \(3x - 2 = 10\)
4.  \(\frac{x}{5} + 4 = -16\)
5.  \(-11m + 7 = 7\)
6.  \(4x - 6 = 6\)
7.  \(\frac{x}{9} - 16 = 11\)
8.  \(2r - 18 = -22\)
OBJECTIVE 3

TO SOLVE AN EQUATION OF THE FORM $px + q = rx + s$

So far we have established three rules for solving equations. We now find that a fourth is necessary.

RULE 4. If the variable (x, y, z, a, etc) appears in more than one term of the equation, then our first step is to bring these terms together by applying our previous rules.

Example 1
Solve $10x + 2 = 8x - 4$.

Solution
Here we have two terms involving the variable $x$ (they are $10x$ and $8x$). So according to our new rule, these terms must be brought together. We usually bring the variables together on the left hand side of the equation.

\[
10x + 2 = 8x - 4
\]

then $10x - 8x + 2 = -4$  \hspace{1cm} Rules 2 and 4

so, $2x + 2 = -4$  \hspace{1cm} \text{(This can now be solved as we did in Objective 2.)}

\[
2x = -4 - 2
\]

then $2x = -6$

so $x = \frac{-6}{2}$  \hspace{1cm} Rule 1

\[
\therefore x = -3
\]

Example 2
Solve $3x + 4 = 20 - x$.

Solution

\[
3x + 4 = 20 - x
\]

then $3x + x + 4 = 20$  \hspace{1cm} Rules 2 and 4

\[
4x + 4 = 20
\]

then $4x = 16$  \hspace{1cm} Rule 1

\[
x = \frac{16}{4}
\]

\[
\therefore x = 4
\]
Example 3  Solve \(5x + 12 = 4x + 22\).

Solution

\[
\begin{align*}
5x + 12 &= 4x + 22 \\
\text{then } & \quad 5x - 4x + 12 = 22 \\
& \quad x + 12 = 22 \\
& \quad x = 22 - 12 \\
& \quad x = 10 \\
\end{align*}
\]

Example 4  Solve \(-y - 2 = -3y + 12\).

Solution

\[
\begin{align*}
-y - 2 &= -3y + 12 \\
\text{then } & \quad -y + 3y - 2 = 12 \\
& \quad 2y - 2 = 12 \\
& \quad 2y = 14 \\
& \quad y = \frac{14}{2} \\
& \quad y = 7 \\
\end{align*}
\]

Example 5  Solve \(7x + 10 = 3x + 18\).

Solution

\[
\begin{align*}
7x + 10 &= 3x + 18 \\
\text{then } & \quad 7x - 3x + 10 = 18 \\
& \quad 4x + 10 = 18 \\
& \quad 4x = 18 - 10 \\
& \quad 4x = 8 \\
& \quad x = \frac{8}{4} \\
& \quad x = 2 \\
\end{align*}
\]
EXERCISE

Calculate the value of the variable for each of the following.

1. \( 7x - 2 = 4x + 10 \)  
2. \( 3 - 4x = 19 - 6x \)  
3. \( -x - 2 = -4 - 2x \)  
4. \( -x - 3 = -12 - 4x \)  
5. \( 11 - 3x = -4x - 59 \)  
6. \( 3m + 2 = m + 12 \)  
7. \( 5y - 3 = -6 + 2y \)  
8. \( 9 - 6p = 11 - p \)
OBJECTIVE 4

TO SOLVE AN EQUATION WHICH FIRST REQUIRES SIMPLIFICATION USING THE DISTRIBUTIVE PROPERTY (i.e. CONTAINING BRACKETS)

In the Algebra Section, Objective 7, the distributive property was summarised as:

\[ a(b + c) = ab + ac \]

eg

\[ 2(x - 3) = 2x - 6 \]

and again,

\[ -3(-2x + 4) = 6x - 12 \]

This brings us to our final rule for solving equations.

**RULE 5** Any brackets in an equation must be removed at the very beginning (by using the distributive property) before any solution can be found. Then we proceed as before.

---

**Example 1** Solve \( 2(x + 3) = -2 \).

**Solution**

\[
2(x + 3) = -2 \\
\text{then} \quad 2x + 6 = -2 \quad \text{Rule 5} \\
\text{then} \quad 2x = -2 - 6 \quad \text{Rules 2 and 3} \\
\text{so,} \quad 2x = -8 \\
\text{then} \quad x = \frac{-8}{2} \quad \text{Rule 1} \\
\therefore \quad x = -4
\]
Example 2  Solve $-3(2m - 4) = 6$.

Solution

$-3(2m - 4) = 6$
then $-6m + 12 = 6$  \hspace{1cm} Rule 5
then $-6m = 6 - 12$  \hspace{1cm} Rules 2 and 3
so, $-6m = -6$
then $m = \frac{-6}{-6}$  \hspace{1cm} Rule 1
\therefore $m = 1$

Example 3  Solve $6(2x + 1) = 3(x + 8)$.

Solution

$6(2x + 1) = 3(x + 8)$
then $12x + 6 = 3x + 24$  \hspace{1cm} Rule 5
then $12x - 3x + 6 = 24$  \hspace{1cm} Rules 2 and 4
so, $9x + 6 = 24$
then $9x = 24 - 6$  \hspace{1cm} Rules 2 and 3
so, $9x = 18$
then $x = \frac{18}{9}$  \hspace{1cm} Rule 1
\therefore $x = 2$

Example 4  Solve $2(x + 4) - 6 = 3(x + 1)$.

Solution

$2(x + 4) - 6 = 3(x + 1)$
then $2x + 8 - 6 = 3x + 3$  \hspace{1cm} Rule 5
so, $2x + 2 = 3x + 3$  \hspace{1cm} (Always simplify like terms whenever possible.)
then $2x - 3x + 2 = 3$  \hspace{1cm} Rules 2 and 4
so, $-x + 2 = 3$
then $-x = 3 - 2$  \hspace{1cm} Rules 2 and 3
so, $-x = 1$
then $x = \frac{1}{-1}$  \hspace{1cm} Rule 1
\therefore $x = -1$
Example 5  Solve $4(x - 2) - 6(x - 4) = 26$.

Solution

$$4(x - 2) - 6(x - 4) = 26$$

then  $4x - 8 - 6x + 24 = 26$  \hspace{1cm} \text{Rule 5}$

so,  $4x - 6x - 8 + 24 = 26$

$\text{ie}$  $-2x + 16 = 22$

then  $-2x = 26 - 16$  \hspace{1cm} \text{Rules 2 and 3}$

so,  $-2x = 10$

then  $x = \frac{10}{-2}$  \hspace{1cm} \text{Rule 1}$

$\therefore$  $x = -5$

EXERCISE

Find the truth number for (solve) each of the following.

1.  $5 + 4x = 5(x - 2) + 8$
2.  $2(m - 3) = 3(13 - m)$
3.  $7y - 5(y + 2) = 6$
4.  $9p - 4(2p - 3) = 12$
5.  $2(y + 4) = 5y - 2(y - 3)$
OBJECTIVE 5

TO TRANSLATE WORD PROBLEMS INTO FIRST DEGREE EQUATIONS

In the Algebra Section, Objectives 1 to 3, you learnt to translate a verbal expression into algebraic symbols. Now, we will translate verbal expressions into equations.

**Example 1**

A fully loaded jet prior to take-off carries 160 000 kg of fuel, luggage and people. Its total weight is 325 000 kg. Write an equation connecting the plane’s empty weight (w), the extra weight of fuel etc. and the total weight.

**Solution**

Plane + extra weight = total weight.
So, \( w + 160\,000 = 325\,000 \) would be our equation where \( w \) = empty weight.

**Example 2**

Bill originally had $54, then he saved $24 per month for \( x \) months, till he had a total of $150. Write an equation which would enable you to find the number of months Bill saved for.

**Solution**

Bill saved $24 for \( x \) months.
\[ \text{ie he saved a total of } 24x \]
Now, original amount + savings = total amount.
So, \( 54 + 24x = 150 \) would be our required equation.

**Example 3**

A rectangle is exactly twice as long as it is wide. Its perimeter is 48 centimetres. Write an equation which could be used to find how wide it is.

**Solution**

Since no variable was given for the unknown (the width) we must first choose one.
Let \( w \) represent the width.

Now, we express the information given in the question using algebraic symbols.

\[ \text{the length } = \text{ twice the width} = 2w \]

Finally, express the above in an equation, using words first, then symbols.

Perimeter = 2(length + width)
So we have:
\[ 48 = 2(2w + w) \]
which would serve as our required equation.
Example 4  A pipe 24 m long is cut into two pieces. One piece of the pipe is twice as long as the other. Write an equation to find the length of each piece.

Solution  Let \( x \) = the length of one piece. 
then \( 2x \) = the length of the other piece. 
Since piece 1 + piece 2 = total length 
then \( x + 2x = 24 \)

is our required equation.

Example 5  We wish to write an equation which can be used to find how much pure acid should be added to 24 L of a 35% solution (a solution which contains 35% pure acid) to obtain a 50% solution.

Solution  Let \( n \) litres be the amount of pure acid added. 
So, \( (24 + n) \) L will be the final amount of 50% solution.

\[
\text{Pure acid in } 24 \text{ L of 35\% solution} + \text{Pure acid added} = \text{Pure acid in final 50\% solution} \\
35\% \text{ of } 24 \text{ litres} + n \text{ litres} = 50\% \text{ of } (24 + n) \text{ litres} \\
\text{So, } .35 \times 24 + n = .5 \times (24 + n)
\]

would be the required equation.

**EXERCISE**

1. A fully loaded car carries 350 kg of fuel, luggage and people. Its total weight is 985 kg. Write an equation connecting the car’s empty weight (\( w \)), the extra weight of fuel etc. and the total weight.

2. Jill originally had $38, then she saved $18 per week for ‘\( n \)’ weeks, till she had a total of $110. Write an equation which would enable you to find the number of weeks Jill saved for.

3. A rectangle is 2 m longer than it is wide. Its perimeter is 44 m. Write an equation which could be used to find how wide it is.

4. A beam 25 m long is cut into two pieces. One piece is \( \frac{2}{3} \) of the length of the other. Write an equation which could be used to find the length, \( \ell \), of the longer piece.

5. Write an equation which can be used to find how many litres, \( n \), of a 20% solution of alcohol should be added to 40 litres of an 80% solution of alcohol to obtain a final 60% solution.
OBJECTIVE 6

TO SOLVE WORD PROBLEMS WHICH TRANSLATE INTO EQUATIONS OF THE TYPES SOLVED IN OBJECTIVES 1, 2 AND 3

The following steps are used to solve word problems.

1. Determine the unknown quantity and choose a variable to represent it.
2. Write an equation that expresses the relation between the numbers in the problem.
3. Solve the equation.
4. Use the solution of the equation to answer the problem.

Example 1
A man walked up a hill at 2 km/h, and then walked back down the hill, at 3 km/h, to where he began. Altogether, he walked for 5 hours. Find how far up the hill he walked.

Solution
Let $d$ represent the distance in km that he walked up the hill.

Also, remember that $speed = \frac{distance}{time}$.

So, cross-multiplying, we have $time = \frac{distance}{speed}$.

Now, time to walk uphill + time to walk downhill = total time.

So, $\frac{d}{2} + \frac{d}{3} = 5$

then $\frac{3d + 2d}{6} = 5$.

So, $\frac{5d}{6} = 5$

$ie$ $5d = 30$. (Cross-multiply.)

So, $d = \frac{30}{5}$ (Cross-multiply.)

$ie$ $d = 6$.

So he walked 6 km up the hill.
Example 2  
Bananas cost 10 cents more than oranges. Three of each costs $3.30 cents altogether. Find the cost of one orange.

Solution  
Let \( x \) be the cost in cents of 1 orange.

Then \((x + 10)\) is the cost in cents of 1 banana.

Cost of 3 bananas + cost of 3 oranges = $3.30.

\[
3(x + 10) + 3x = 330
\]

\[
3x + 30 + 3x = 330
\]

\[
6x + 30 = 330
\]

\[
6x = 300
\]

\[
x = \frac{300}{6}
\]

\[
\therefore x = 50
\]

So, 1 orange costs 50 cents.

Example 3  
I invested \( d \) dollars, for 1 year at 5\% simple interest. If at the end of the year I have $420, find the original amount \( d \) which I invested.

Solution  
Since I invested \( d \) dollars at 5\% and we know that:

original amount + interest = final amount.

Then \( d + \frac{5}{100}d = 420 \) \( \quad \left( \text{Interest is 5\% of } d = \frac{5}{100} \times d. \right) \)

So,

\[
\frac{105}{100}d = 420
\]

Then

\[
d = \frac{420 \times 100}{105}
\]

\[
\therefore d = 400
\]

So, the original amount invested was $400.

Example 4  
The total takings at a local football match were $9 200. It is known that twice as many children as adults came. If the adults paid $2.20 each and children $1.20 each, how many adults attended?

Solution  
Let \( n \) be the number of adults.

Then \( 2n \) would be the number of children.

Total paid by adults + total paid by children = total received.
\[
\begin{align*}
n(2.20) + 2n(1.20) &= 9\,200 \\
\text{So,} & \quad 2.2n + 2.4n = 9\,200 \\
\text{then} & \quad 4.6n = 9\,200 \\
\text{So,} & \quad n = \frac{9\,200}{4.6} \\
\therefore & \quad n = 2\,000
\end{align*}
\]

So, 2 000 adults attended the match.

---

**EXERCISE**

1. A man is 30 years older than his son. If their combined ages are 88, find the actual age of each man.

2. A room is 2 m longer than it is wide. If the perimeter of this room is 16 m, find its length and width.

3. A man drove at 100 km/h to a town, and then he drove back at 75 km/h. Altogether, he drove for \(3\frac{3}{4}\) hours. Find how far away the town was.

4. Cricket balls cost \$1.60 more than tennis balls. Also, 2 cricket balls plus 3 tennis balls cost \$13.20 altogether. Find the cost of each tennis ball.
PART 3

MEM2.13C5A
Perform Mathematical Computations
SECTION 10

Formula Transposition
OBJECTIVE 1

TO EVALUATE A VARIABLE (WHICH IS THE SUBJECT OF A FORMULA) GIVEN THE VALUES OF ALL OTHER VARIABLES

In many practical workshop calculations it is frequently convenient to use symbols. These symbols are usually letters of the alphabet which replace words or numbers, eg consider the rule for finding the area of a rectangle.

Area of rectangle = length units \times breadth units.

This rule may be abbreviated by using the following symbols:

Let
- Area be represented by \( A \)
- Length be represented by \( L \)
- Breadth be represented by \( B \)

Then the rule becomes:

\[
A = L \times B
\]

When the rule is written in this way, it is known as a formula. (It is also called an equation.)

**NOTE:** To save time, \( L \) multiplied by \( B \) can be written as \( LB \) not as \( L \times B \).

If \( L = 4 \text{ m} \) and \( B = 3 \text{ m} \), then the area \( A \) is found by substituting into the formula:

\[
A = LB = 4 \times 3 \text{ m}^2 = 12 \text{ m}^2
\]

Many rules may be written as formulae. It will be evident that to apply a formula to a particular case, the values of all the symbols but one must be known, then the unknown value can easily be calculated.

**Example 1** The area of a circle is given by the formula \( A = \pi r^2 \) where \( A = \text{area}, \ r = \text{radius} \) and \( \pi = \frac{3}{2} \). Find the area of a circle of radius \( 3\frac{1}{2} \text{ metres} \).

**Solution** \( A = \pi r^2 \)

The meaning of \( r^2 \) is \( r \times r \).
Here $\pi = 3\frac{1}{7} = \frac{22}{7}$ \hspace{1cm} \text{(we often use } \pi = 3.14) \\
\therefore r = 3\frac{1}{2} = \frac{7}{2} \text{ m} \hspace{1cm} \text{(or } r = 3.5 \text{ m if using a calculator)} \\
A = \frac{22}{7} \times \frac{7}{2} \times \frac{7}{2} \text{ m}^2 \\
= \frac{77}{4} \text{ m}^2 \\
= 38.5 \text{ m}^2

The area has been found by substituting the arithmetical value of the symbols in the formula.

When particular values are to be substituted for the symbols, the number within each pair of brackets must be worked out first, for the symbols enclosed by a pair of brackets represent a single number.

Example 2 Find the value of $y$ in the formula

$$y = 4a - (b - c) + \frac{d^2}{4}$$

when $a = 5$, $b = 7$, $c = 2$ and $d = 3$.

Solution

$$y = 4a - (b - c) + \frac{d^2}{4}$$

$$= 4 \times 5 - (7 - 2) + \frac{3 \times 3}{4} \hspace{1cm} \text{(Substitute.)}$$

$$= 20 - 5 + \frac{9}{4} \hspace{1cm} \text{(Do brackets first.)}$$

$\therefore y = 17\frac{1}{4} \text{ or } 17.25$

It has been shown that a formula expresses concisely in symbols the connection between related quantities; it is a brief mathematical statement of some law or rule. Such a statement is also called an equation.

From the previous examples, it is evident that to apply a formula, the values of all the symbols but one must be known, then the unknown value can be calculated. It is usual to make the symbol on the left hand side of a formula as the unknown value.
EXERCISE

1. If $V$ is the volume, $r$ is the radius and $H$ the height of a cylinder, its volume is given by the formula $V = \pi r^2 H$. Use this formula to find the volume of a cylinder of:
   (a) radius $1\frac{3}{4}$ m, height 6 m (Use $\pi = 3\frac{1}{7}$.)
   (b) radius 2.4 m, height 7.3 m (Use $\pi = 3.14$.)

2. The amount of power units (expressed in watts) consumed by an electric radiator can be found by the formula:
   \[ P = V \times I \]
   where $P$ is the power in watts, $V$ is the applied voltage and $I$ is the current draw in amps.

   Using the above formula calculate:
   (a) the power consumed by a 250 volt radiator which draws 5 amps.
   (b) the power consumed by a 220 volt radiator which draws 4.5 amps.

3. Given $S = \frac{\pi dN}{12}$ find $S$ when $\pi = 3.14$, $d = 2$ and $N = 154$.

4. If $A = \pi (R + r) (R - r)$ find $A$ when $\pi = 3.142$, $R = 7.5$ and $r = 6.25$.

5. Find $r$ given $r = \frac{A^2 + H^2}{2H}$ and $A = 3.7$, $H = 2.5$. 
OBJECTIVE 2

TO TRANPOSE A FORMULA IN WHICH THE VARIABLES INVOLVE ADDITION OR SUBTRACTION (ie THE VARIABLES ARE CONNECTED BY + OR – SIGNS)

A formula or equation should be thought of as a balanced scale (or seesaw) which must at all times remain balanced (this is what is implied by the use of the equal sign). Remembering this fact is most important when we consider transposing (or rearranging) formula. For example, if we subtract (or add) something on the left hand side of a formula (or equation) then we must subtract (or add) this exact same something from (or to) the right hand side of our formula (only then will our formula remain 'balanced'). So, if we have:

\[ I = E_2 + E_1 \]

then

\[ I - E_1 = E_2 \]

(subtracting \(E_1\) from both sides)

Notice that in effect, the term \(+E_1\) has moved to the opposite side of the equals sign and appears there as \(-E_1\). This can be roughly summarised by the following rule:

A term which is moved from one side of the equals sign to the other, will be written as the opposite operation to which it was originally.

* That is, in the above, the opposite of \(+E_1\) is \(-E_1\). So when we move the term \(+E_1\) to the other side of our formula, then it is written as \(-E_1\). Diagrammatically, we have

\[
\begin{array}{c}
I \\
E_2 + E_1
\end{array}
\]

which becomes

\[
\begin{array}{c}
I - E_1 \\
E_2
\end{array}
\]

Again, if \(R = d - m\)

then \(R + m = d\)

(The opposite operation to \(-m\) is \(+m\).)

\[ d = R + m \]

In a formula, the term which is by itself on the left side of the equals sign is called the subject of the formula.

**So in the above formula, \(R\) was the subject but now \(d\) is.**
Example 1
Make \( c \) the subject of the formula

\[
a = b + c.
\]

Solution

If \( a = b + c \)

then \( a - b = c \)  
\( \therefore c = a - b \)

(The opposite of \( b \) is \( -b \).)

(Simply reverse sides, since it is more common to have the subject on the left hand side of our equation.)

Example 2
Make \( c \) the subject of the formula

\[
y = a - b + c.
\]

Solution

If \( y = a - b + c \)

then \( y - a = -b + c \)  
\( \therefore y - a = -b + c \)

(The opposite of \( a \) is \( -a \).)

(The opposite of \( c \) is \( -c \).)

\( \therefore -a - c = -b \)

\( \therefore ie \ -b = y - a - c \)

Obtaining a negative subject (as we have above) instead of the required positive subject is quite common. In such a case, both sides of the equation should be multiplied by \(-1\). The effect of this is to change the sign of every term throughout the equation.

\( \therefore b = -y + a + c \)

**EXERCISE**

Make \( m \) the subject for each of the following formulae.

1. \( k = m - n \)
2. \( p + m = n \)
3. \( r - m = t \)
4. \( p = m - r + t \)
5. \( s + t = r - m - n \)
6. \( p - m = r - t - s \)
OBJECTIVE 3

TO TRANSPOSE A FORMULA INVOLVING MULTIPLICATION OR DIVISION

In the notes on Objective 2 of this topic, we established a general rule for transposing terms in a formula, ie

A term which is moved from one side of the equals sign to the other, will be written as the opposite operation to which it was originally.

So, if the term was + b then it becomes – b when transposed.
if the term was – b then it becomes + b when transposed.
if the term was × b then it becomes ÷ b when transposed.
if the term was ÷ b then it becomes × b when transposed.

Example 1 Transpose \( M - KV \) to make \( K \) the subject.

Solution

If \( M = KV \) (Remember that \( KV \) is \( K \times V \).)
then \( \frac{M}{V} = K \) (The opposite operation to \( \times V \) is \( \div V \).)
\[ ie \quad K = \frac{M}{V}. \]

Example 2 Transpose \( I = \frac{V}{R} \) to make \( V \) the subject.

Solution

If \( I = \frac{V}{R} \) (Remember that \( \frac{V}{R} \) is \( V \div R \).)
then \( IR = V \) (The opposite operation to \( \div R \) is \( \times R \).)
\[ ie \quad V = IR. \]

Example 3 Transpose the formula \( I = \frac{V}{R} \) to make \( R \) the subject.

Solution

If \( I = \frac{V}{R} \)
then \( = V \)
\[ \therefore \quad R = \frac{V}{I}. \]
NOTE: As shown in this last example, it is important to always keep in the back of our minds exactly which term is to be the new subject. This will help direct our transposing.

EXERCISE

1. Find \( r \) if \( C = 2\pi r \).

2. Transpose the formula \( A = \frac{1}{2}BH \) to make \( H \) the subject.

3. Transpose the formula \( P = \frac{2\pi d T}{60} \) to make \( T \) the subject.

4. The cutting speed of a drill may be calculated from the formula:

\[
s = \frac{\pi d N}{1000}
\]

Transpose this formula to make \( N \) (the drill speed in revolutions per minute) the subject.

5. The power delivered by an electric motor is given by:

\[
P = V \times I \times 0.85
\]

(If its efficiency is 85%).

Transpose this formula to make the current \( I \) the subject.

6. Find \( R \) if \( P = \frac{E^2}{R} \).
OBJECTIVE 4

TO TRANSPOSE A FORMULA INVOLVING ADDITION, SUBTRACTION, MULTIPLICATION OR DIVISION

Often in transposing a formula we will have to use two or more different operations. The order in which we do this is important. The following rules should guide you in this:

1. Find the terms which involves your new subject.
2. Move all other terms which are added to or subtracted from this term to the other side of the equal sign.
3. Isolate the required subject by transposing anything which is multiplied or divided into it.

Example 1 Transpose the formula \( V = u + at \) to make \( t \) the subject.

Solution

If \( V = u + at \) \( \text{(The term 'at' involves our new subject t.)} \)
then \( -u = at \) \( \text{Rule 2} \)
\( ie \quad \frac{V - u}{a} = t \) \( \text{Rule 3} \)
\( ie \quad t = \frac{V - u}{a} \)

Example 2 Transpose the formula \( S = 8.5 (A + L) \) to make \( L \) the subject.

Solution

If \( S = 8.5 (A + L) \) \( \text{(Expand bracket first.)} \)
then \( S = 8.5A + 8.5L \)
\( ie \quad S - 8.5A = 8.5L \) \( \text{Rule 2} \)
\( ie \quad \frac{S - 8.5A}{8.5} = L \) \( \text{Rule 3} \)
\( L = \frac{S - 8.5A}{8.5} \)
<table>
<thead>
<tr>
<th>EXERCISE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Make $B$ the subject of the formula $C = A + 2B$.</td>
</tr>
<tr>
<td>2. Make $b$ the subject of the formula $4 - 2bc = p + q$.</td>
</tr>
<tr>
<td>3. Make $m$ the subject of the formula $y = mx + b$.</td>
</tr>
<tr>
<td>4. Make $c$ the subject of the formula $F = \frac{2}{5}c + 32$.</td>
</tr>
<tr>
<td>5. Make $f$ the subject of the formula $v = u + \frac{ft}{m}$.</td>
</tr>
</tbody>
</table>
OBJECTIVE 5

TO TRANPOSE A FORMULA CONTAINING THE REQUIRED NEW SUBJECT IN A BRACKET

Whenever the new subject is contained within a bracket, it is advisable to isolate this bracket on one side of the equals sign, then proceed as before.

Example 1  The temperature $C\degree$ Centigrade corresponding to a temperature $F\degree$ Fahrenheit is given by:

$$C = \frac{5}{9} (F - 32)$$

Transpose this formula so that $F$ is the subject.

Solution

If $C = \frac{5}{9} (F - 32)$ then $9C = 5 (F - 32)$

$$\text{ie} \quad \frac{9C}{5} = (F - 32)$$

$$\text{ie} \quad \frac{9C}{5} = F - 32$$

$$\text{ie} \quad \frac{9C}{5} + 32 = F$$

$$\text{ie} \quad F = \frac{9C}{5} + 32$$

EXERCISE

1. Transpose the equation $s = \frac{t}{2}(u + v)$ to make $u$ the subject.

2. Make $\theta$ the subject of the formula $L_2 = L_1 (1 + \alpha \theta)$.

3. Make $r$ the subject of the formula $S(1 - r) = A$

4. Make $H$ the subject of the formula $A = 2\pi r (r + H)$

5. Make $x$ the subject of the formula $w = aq(x - t)$
OBJECTIVE 6

TO TRANSPOSE A FORMULA INVOLVING SQUARE ROOTS, CUBE ROOTS OR POWERS

Whenever the proposed new subject is involved within a square root or cube root, then that root should be isolated and both sides of the formula should be squared or cubed.

Example 1  Make b the subject of the formula

\[ M = \frac{\sqrt{a + b}}{y}. \]

Solution

If \[ M = \frac{\sqrt{a + b}}{y} \]

then \[ My = \sqrt{a + b} \]

\[ \therefore My^2 = a + b \]  \hspace{1cm} \text{(Square both sides.)}

ie \[ My^2 = a + b \]

ie \[ M^2y^2 - a = b \]

ie \[ b = M^2y^2 - a \]

Whenever the proposed new subject is raised to the \( n \)th power, that term should be isolated. Then take the \( n \)th root of both sides of the formula.

Example 2  Make \( R \) the subject of the formula

\[ V = \frac{4\pi R^3}{3}. \]

Solution

If \[ V = \frac{4\pi R^3}{3} \]

then \[ 3V = 4\pi R^3 \]

ie \[ \frac{3V}{4\pi} = R^3 \]

ie \[ \sqrt[3]{\frac{3V}{4\pi}} = R \] \hspace{1cm} \text{(Take the cube root of both sides.)}

ie \[ R = \sqrt[3]{\frac{3V}{4\pi}} \]
Example 3  Make \( p \) the subject of the formula

\[ L = \left( \frac{c}{p} \right)^3. \]

Solution

If \( L = \left( \frac{c}{p} \right)^3 \)

then \( \sqrt[3]{L} = \frac{c}{p} \)  

(Take the cube root of both sides.)

\[ ie \quad p \sqrt[3]{L} = c \]

\[ ie \quad p = \frac{c}{\sqrt[3]{L}} \]

EXERCISE

1. Make \( I \) the subject \( P = \dot{I}R \).

2. Make \( V \) the subject of \( E = \frac{WV^2}{2g} \).

3. Make \( V \) the subject of \( D = \frac{0.334 WV^2}{P} \).

4. Make \( m \) the subject of \( d = \frac{\sqrt[5]{12m}}{5B} \).

5. Make \( a \) the subject of \( c = \sqrt{(a^2 - b^2)} \).
OBJECTIVE 7

TO TRANSPOSE A FORMULA IN WHICH THE REQUIRED SUBJECT APPEARS MORE THAN ONCE

If the required subject appears more than once then:

(a) isolate all terms involving this subject on one side of the equals sign
(b) factorise the required subject out of each of these terms – it must be a common factor
(c) isolate the common factor (the required subject) by the usual methods – as applied in earlier objectives.

Example 1
Given \( R = \frac{r_1 r_2}{r_1 + r_2} \) make \( r_1 \) the subject.

Solution

If \( R = \frac{r_1 r_2}{r_1 + r_2} \)

then \( R(r_1 + r_2) = r_1 r_2 \) (Cross multiply.)

\( ie \ Rr_1 + Rr_2 = r_1 r_2 \) (Expand the bracket.)

\( ie \ Rr_1 = r_1 r_2 - Rr_2 \)

\( ie \ Rr_1 - r_1 r_2 = -Rr_2 \) (Isolate the terms involving \( r_1 \).)

\( ie \ r_1(R - r_2) = -Rr_2 \) (Take out the common factor \( A \).)

\( ie \ r_1 = \frac{-Rr_2}{R - r_2} \) (Cross multiply.)

Example 2
Given \( B = \frac{A}{1 - A} \) make \( A \) the subject.

Solution

If \( B = \frac{A}{1 - A} \)

then \( B(1 - A) = A \) (Cross multiply.)

\( ie \ B - AB = A \) (Isolate the terms involving \( A \).)

\( ie \ B = A + AB \) (Take out the common factor \( A \).)

\( ie \ \frac{B}{1 + B} = A \) (Cross multiply.)

\( ie \ A = \frac{B}{1 + B} \)
Example 3

Given the differential pulley formula

\[ 2 RF = W(R - r), \]

make \( R \) the subject.

Solution

If \( 2 RF = W(R - r) \)

then \( 2 RF = WR - Wr \)  \(\text{(Expand the bracket.)}\)

\( ie \) \( 2 RF - WR = -Wr \)  \(\text{(Isolate the terms involving } R \text{.)}\)

\( ie \) \( R(2F - W) = -Wr \)  \(\text{(Factorise.)}\)

\( ie \) \( R = \frac{-Wr}{(2F - W)} \)  \(\text{(Cross - multiply.)}\)

EXERCISE

1. Make \( r \) the subject of the formula \( m = rp - rt \).
2. Make \( W \) the subject of the formula \( PL = \frac{WR}{1 + W} \).
3. Make \( A \) the subject of the formula \( AB = C(A - B) \).
4. Make \( x \) the subject of the formula \( y = \frac{x + 3}{x - 1} \).
5. Make \( t \) the subject of the formula \( m = \frac{t}{t + p} \).
SECTION 11

Geometry 1
OBJECTIVE 1

TO MEASURE ANGLES (USING DEGREE MEASURE ONLY)

If the size of an angle is 60 degrees, we say the size of the angle is 60 times as great as the size of an angle of 1 degree.

An angle is formed when two line segments meet at a common end point.

The above angle is referred to as angle ABC or simply angle B. From this you can see that in naming an angle, the common point (vertex) is always between the other 2 letters. If it is clear which angle you are referring to, it is acceptable to name the vertex only.

Furthermore, angle ABC can be written $\angle ABC$, ie the symbol $\angle$ means angle.

The instrument used to find the size of an angle is a protractor. Use a protractor to find the size of an angle.

Example 1 Use a protractor to find the size of $\angle Y$.

Solution

(i) Place the centre of the protractor at Y, the centre of $\angle Y$.

(ii) Make sure that the base line of the protractor lies on the lower ray $YZ$ (a ray has one end point only – point Y).

(iii) Read off the numeral on the protractor closest to the upper ray $XY$.

(iv) This numeral tells us the size of the angle in degrees (to the nearest whole number).

(v) The size of $= \angle Y = 60^\circ$. 

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The size of angle $Y$ is 60 degrees and can be shown in 2 ways.

(i) By $s \angle Y = 60^\circ$ where ‘$s$’ refers to ‘size’ and

(ii) by a diagram where $60^\circ$ is written near the vertex and between the 2 rays.

**Example 2** Find the size of $\angle DOE$ in the diagram below. Also measure angle $\angle FOD$.

**Solution** First turn the protractor so that the zero line lies on $\overline{OE}$. Then read off the size (20$^\circ$) in the usual way.

Similarly, for $\angle FOD$, we, first turn the protractor so that the zero line lies on $\overline{OD}$. Then read off the size (90$^\circ$) in the usual way.

**EXERCISE**

Use a protractor to find the size of the angles represented below.

(a) $\angle AOB$  (d) $\angle AOE$  (g) $\angle AOH$
(b) $\angle AOC$  (e) $\angle AOF$  (h) $\angle AOI$
(c) $\angle AOD$  (f) $\angle AOG$
OBJECTIVE 2

TO CLASSIFY A GIVEN ANGLE AS A RIGHT ANGLE, A STRAIGHT ANGLE, AN ACUTE ANGLE, OR AN OBTUSE ANGLE

Angles can be classified according to their size.

1. Acute Angles

   An acute angle is an angle whose size is less than 90°.

   Below are some diagrams which represent acute angles. (The word ‘acute’ in everyday language means ‘sharp’).

2. Right Angles

   A right angle (rt ∠) is an angle whose size is 90°.

   Below are some diagrams which represent right angles.

   We sometimes indicate that a right angle is represented in a diagram by marking the diagram as shown below.
3. Obtuse Angles

*An obtuse angle is an angle whose size is greater than 90° and less than 180°.*

The angles represented in the diagrams below are obtuse angles. (The word ‘obtuse’ in everyday language means ‘blunt’).

![Obtuse angles diagram](image)

4. Straight Angles

*A straight angle is an angle whose size is 180°.*

The angle shown in the following diagram is a straight angle.

![Straight angle diagram](image)

An angle of 180° is called a straight angle, because the rays of the angle are opposite rays; and as we have seen, the union of two opposite rays is a straight line.

**Example 1** Without using a protractor, state whether the diagrams below represent or determine:

(i) acute angles  
(ii) right angles  
(iii) obtuse angles  
(iv) straight angles

![Angle diagrams](image)

**Solution**  
(a) Since this angle is greater than 90°, it is obtuse.  
(b) Since this angle is less than 90°, it is acute.  
(c) Since this angle is greater than 90°, it is obtuse.  
(d) Since this angle is 180°, it is straight.
(e) Since this angle is less than 90°, it is acute.
(f) Since this angle equals 90°, it is a right angle.

**EXERCISE**

The diagram below represents a structure for a bridge. State whether the angles named are:

(i) acute
(ii) right
(iii) obtuse

(a) \( \angle A \)  
(b) \( \angle NCD \)  
(c) \( \angle BNC \)  
(d) \( \angle AMN \)  
(e) \( \angle MBC \)  
(f) \( \angle DNC \)  
(g) \( \angle DNB \)  
(h) \( \angle DNM \)
OBJECTIVE 3

TO RECOGNISE, DESCRIBE AND NAME POLYGONS WITH 3, 4, 5, 6, 8 OR 10 SIDES

The simple closed curves drawn above are somewhat special, because they consist of line segments. To describe any figure which is a simple closed curve consisting of line segments only, we use the word polygon.

A polygon is a simple closed curve which is the union of three or more line segments.

When referring to a polygon, we usually call the line segments the sides of the polygon and the end points of the line segments the vertices of the polygon. ('Vertices' is the plural of 'vertex'.) In the diagram below the sides and vertices of a polygon are marked.

To name a polygon, we use the letters naming its vertices. It is usual to select these in either a clockwise or an anticlockwise direction. Polygon QRST, QTSR or RSTQ are all names for this quadrilateral. We would not refer to it as polygon QSRT.

Other names, could be polygon RQTS, SRQT, STQR, TQRS or RSTQ.

We can name a polygon according to the number of sides it has.
However, the polygons we will be concerned with have special names. These special names also indicate the number of sides they have. Some special polygons are listed here.

<table>
<thead>
<tr>
<th>number of sides</th>
<th>diagrams</th>
<th>name</th>
<th>special name</th>
</tr>
</thead>
<tbody>
<tr>
<td>3</td>
<td><img src="image" alt="3-gon" /></td>
<td>3-gon</td>
<td>triangle</td>
</tr>
<tr>
<td>4</td>
<td><img src="image" alt="4-gon" /></td>
<td>4-gon</td>
<td>quadrilateral</td>
</tr>
<tr>
<td>5</td>
<td><img src="image" alt="5-gon" /></td>
<td>5-gon</td>
<td>pentagon</td>
</tr>
<tr>
<td>6</td>
<td><img src="image" alt="6-gon" /></td>
<td>6-gon</td>
<td>hexagon</td>
</tr>
<tr>
<td>8</td>
<td><img src="image" alt="8-gon" /></td>
<td>8-gon</td>
<td>octagon</td>
</tr>
</tbody>
</table>

A 10 sided polygon is called a **decagon**.

Look at the diagrams again. Note that the number of vertices of a polygon and the number of angles determined by a polygon are both equal to the number of sides of that polygon.

So to find the special name for these polygons, we need simply to count the number of sides, or the number of vertices (since these are always the same).

**EXERCISE**

Give the special name for each of the polygons drawn below.

1. ![Polygon 1](image)
2. ![Polygon 2](image)
3. ![Polygon 3](image)
4. ![Polygon 4](image)
5. ![Polygon 5](image)
6. ![Polygon 6](image)
**OBJECTIVE 4**

**TO DEFINE THE TERMS ISOSCELES TRIANGLE, SCALENE TRIANGLE, EQUILATERAL TRIANGLE, RIGHT TRIANGLE AND RECOGNISE EXAMPLES OF EACH**

1. *A triangle which has 3 congruent (equal length) sides is called an equilateral triangle.* The diagrams below represent equilateral triangles and the sides of equal length are shown by placing identical markings on each (usually a stroke, or squiggle, etc).

   ![Equilateral Triangle Diagrams](image)

2. *A triangle which has 2 congruent (equal length) sides is called an isosceles triangle.* In the diagrams below, the sides of equal length have been marked.

   ![Isosceles Triangle Diagrams](image)

   \[ \text{ie } PQ \equiv PR \quad \text{KM} \equiv KL \quad \text{ED} \equiv EF \]

3. *A triangle which has no congruent sides is called a scalene triangle.* The 2 diagrams below represent scalene triangles.

   ![Scalene Triangle Diagrams](image)

4. *A right triangle is a triangle with one right angle.* In a right triangle, the side opposite the right angle is called the hypotenuse.

   ![Right Triangle Diagrams](image)
Example 1  From the diagram below:
(i) Make a list of all the triangles.
(ii) Alongside each, indicate whether it is
   (a) an isosceles
   (b) an scalene
   (c) an equilateral, or
   (d) a right triangle.
(iii) State briefly why each triangle is in this category.

Solution  Triangle ABC is equilateral since all sides are congruent.
           Triangle CBD is right since angle BDC is 90 degrees.
           Triangle CBE is scalene since no sides are congruent.
           Triangle CEF is isosceles since two sides are congruent.
           Triangle CBF is scalene since no sides are congruent.
           Triangle ABF is scalene since no sides are congruent.

EXERCISE

From the diagram below, find:
(a) an isosceles triangle
(b) a right triangle
(c) a scalene triangle
(d) an equilateral triangle

NOTE: If sides are marked as equal then assume that this is true for the question. The diagram is not drawn to scale.
OBJECTIVE 5

TO DEFINE THE TERMS PARALLELOGRAM, RECTANGLE, RHOMBUS, SQUARE AND TRAPEZOID (TRAPEZIUM) AND RECOGNISE EXAMPLES OF EACH

All of the above terms are examples of special types of quadrilaterals (4-sided polygons) and are identified by naming the 4 letters associated with each of the vertices (corners).

Listed below are examples of each of these, together with a list of their properties.

**Definition**

A parallelogram is a quadrilateral with its opposite sides parallel.

**Properties**

1. The opposite sides are congruent.
2. The opposite angles are congruent.
3. The diagonals bisect each other.

**NOTE:**

(i) Parallel sides are shown by using similar arrowheads.

(ii) A diagonal is the line segment drawn from one corner to the opposite one.

**Definition**

A rhombus is a quadrilateral with opposite sides parallel and all sides congruent (but angles not 90°).

**Properties**

1. The opposite sides are congruent.
2. The opposite angles are congruent as for parallelogram.
3. The diagonals bisect each other.
4. The diagonals bisect the angles of the rhombus.
5. The diagonals are perpendicular to each other.
**Definition**  
A rectangle is a right quadrilateral with its opposite sides parallel.

**Properties**  
1. The opposite sides are congruent.  
2. The opposite angles are congruent as for parallelogram.  
3. The diagonals bisect each other.  
4. The diagonals are congruent.

**Definition**  
A square is a right quadrilateral with its opposite sides parallel and all sides congruent.

**Properties**  
1, 2, 3, 4, 5

**Definition**  
A trapezoid (trapezium) is a quadrilateral with one pair of opposite sides parallel.

**Properties**  
1. One pair of opposite sides is parallel only.

**Example 1**
Use the previous diagram to name the type of 4-sided polygon represented by each of the following figures. Briefly state the reason for each of your choices.

(a) ABEF  
(b) ACDE  
(c) BCDE  
(d) ABHG  
(e) AEHG  
(f) EFGH

**Solution**

(a) square ABEF : all sides congruent and all angles equal 90°
(b) rhombus ACDE : all sides congruent and angles are not 90°
(c) trapezium BCDE : one pair of opposite sides only is parallel
(d) trapezium ABHG : one pair of opposite sides only is parallel
(e) parallelogram AEHG : the opposite sides are parallel
(f) trapezium EFGH : one pair of opposite sides only is parallel

**EXERCISE**

Identify each of the following 4-sided polygons as square, rectangle, rhombus, parallelogram or trapezium (trapezoid).

1.  
2.  
3.  
4.  
5.
OBJECTIVE 6

TO USE PYTHAGORAS’ THEOREM TO CALCULATE THE LENGTH OF ANY ONE OF THE SIDES OF A RIGHT ANGLED TRIANGLE GIVEN THE LENGTHS OF THE OTHER 2 SIDES

In the diagram below, \( BC^2 = AC^2 + AB^2 \).

Pythagoras is said to have given the first proof of this relationship between the sides of a right triangle. Since then many mathematicians have produced proofs. Several hundred different proofs are known to be in existence. The traditional proof, using the idea of areas of squares actually constructed on the sides of a triangle, was given by Euclid (about 300 BC). This is illustrated below.

So, in the above case \( 5^2 = 4^2 + 3^2 \), i.e. \( BC^2 = AC^2 + AB^2 \).
We will accept the truth of this property.

**THE PYTHAGOREAN PROPERTY**

*If a triangle is right, then the square of the hypotenuse is equal to the sum of the squares of the other two sides.*

The side opposite the right angle in a right triangle is called the *hypotenuse*. The hypotenuse of a right triangle is the longest side of the triangle.

**NOTE:** In the statement of the Pythagorean Property the words ‘hypotenuse’ and ‘sides’ refer to the ‘length of the hypotenuse’ and ‘the lengths of the sides of the triangle’.

**Example**

![Diagram of a right triangle with hypotenuse labeled XZ, sides XY and YZ, and right angle at Z.]

In the $\triangle XYZ$, if $\angle XYZ = 90^\circ$, $XZ$ is the hypotenuse and $(XZ)^2 = (XY)^2 + (YZ)^2$.

**NOTE:** For convenience, we write this statement as:

$XZ^2 = XY^2 + YZ^2$

Remember that:

$XZ^2 = (XY) \cdot (XZ)$

**ie** $7^2 = 7 \times 7 = 49$

If $XZ^2 = 49$

$XZ = \sqrt{49}$ (taking the square root of both sides)

$= 7$ units

**Example 1** Find the hypotenuse of a right triangle if the other two sides are of lengths 5 cm and 12 cm.
Solution Let the hypotenuse be \( x \) centimetres long.

By the Pythagorean Property,

\[
x^2 = 5^2 + 12^2
\]

\[ie \quad x^2 = 25 + 144\]

\[ie \quad x^2 = 169\]

\[ie \quad x = 13 \quad (The \ square \ root \ of \ 169 \ is \ 13.)\]

Therefore the hypotenuse is 13 centimetres long.

Example 2 Calculate the length of MN in the following.

Solution Let the hypotenuse be \( x \) centimetres long.

By the Pythagorean Property,

\[
x^2 = 18^2 + 24^2
\]

\[x^2 = 324 + 576\]

\[ie \quad x^2 = 900\]

\[\therefore \quad x = \sqrt{900} = 30 \text{ cm}\]

\[\therefore \quad MN = 30 \text{ cm}\]

Example 3 Calculate the length of RS in the following.
Solution Let the length of RS be $x$ centimetres.

By the Pythagorean Property,

$$25^2 = x^2 + 24^2$$

$i.e.$ $625 = x^2 + 576$

$i.e.$ $625 - 576 = x^2$

$i.e.$ $x^2 = 49$

$\therefore x = \sqrt{49} \text{ cm}$

$= 7 \text{ cm}$

$\therefore N = 30 \text{ cm}$

Therefore the length of RS is 7 cm.

Example 4 Calculate the length of AB in the following.

![Diagram](image)

Solution Let the length of AB = $x$ m.

By the Pythagorean Property,

$$ (1.25)^2 = x^2 + (0.75)^2$$

$i.e.$ $1.5625 = x^2 + 0.5625$

$i.e.$ $1.5625 - 0.5625 = x^2$

$i.e.$ $x^2 = 1$

$\therefore x = 1 \text{ m}$

EXERCISE

For triangle ABC, in each case, find the third side.

1. angle $A = 90^\circ$, $AB = 8 \text{ cm}$, $AC = 15 \text{ cm}$
2. angle $A = 90^\circ$, $AB = 6 \text{ m}$, $AC = 2.5 \text{ m}$
3. angle $A = 90^\circ$, $BC = 5 \text{ cm}$, $AB = 4 \text{ cm}$
4. angle $A = 90^\circ$, $BC = 26 \text{ cm}$, $AB = 24 \text{ cm}$
5. angle $A = 90^\circ$, $BC = 15 \text{ cm}$, $AB = 12 \text{ cm}$
SECTION 12

Geometry 2
OBJECTIVE 1

TO RECOGNISE THAT THE SUM OF THE MEASURES OF THE ANGLES OF A TRIANGLE IS 180°, AND USE THIS FACT TO SOLVE SIMPLE NUMERICAL PROBLEMS

Try the following activity.

From a piece of stiff paper, cut out the shape of a triangle and label the corners A, B and C. Now tear off the corners. The straight edges of these pieces represent the angles of the triangle. Place two of the pieces alongside each other as in diagram (ii) below. Now fit the third piece alongside B.

Does the figure suggested by the edge of A and the edge of C appear to be a straight angle? ie an angle whose size is 180°?

This activity suggests that for any ΔABC, s∠A + s∠B + s∠C = 180° where s∠A = size of angle A.

This is in fact true, and was first shown some 2,500 years ago, ie the sizes of three angles of a triangle always add up to 180°.

The above angle is referred to as angle ABC or simply angle B. From this you can see that in naming an angle, the common point (vertex) is always between the other 2 letters. If it is clear which angle you are referring to, it is acceptable to name the vertex only.

Example 1 Use the diagram below to calculate the size of angles ABC, BAD and DAC.

Solution

(i) In triangle ABC,

\[ s\angle A + s\angle B + s\angle C = 180° \]

ie \[ 90° + s\angle B + 32° = 180° \]

ie \[ s\angle B + 122° = 180° \]

ie \[ s\angle B = 180° - 122° \]

\[ \therefore s\angle ABC = 58° \]
(ii) In triangle ABD,
\[ \angle A + \angle B + \angle D = 180^\circ \]
\[ ie \quad \angle A + 58^\circ + 90^\circ = 180^\circ \]
\[ ie \quad \angle A + 148^\circ = 180^\circ \]
\[ ie \quad \angle A = 180^\circ - 148^\circ \]
\[ \therefore \quad \angle BAD = 32^\circ \]

(iii) In triangle ADC,
\[ \angle A + \angle D + \angle C = 180^\circ \]
\[ ie \quad \angle A + 90^\circ + 32^\circ = 180^\circ \]
\[ ie \quad \angle A + 122^\circ = 180^\circ \]
\[ ie \quad \angle A = 180^\circ - 122^\circ \]
\[ \therefore \quad \angle DAC = 58^\circ \]

**EXERCISE**

1. In triangle XYZ, if \( \angle X = 27^\circ \), \( \angle Y = 46^\circ \), find \( \angle Z \).

2. In triangle MNO, if \( \angle M = 52^\circ \) and the \( \angle N = \angle O \), find the \( \angle N \).

3. In triangle ABC, if all the angles are equal, find their size.

4. Use the diagram below to find \( \angle L \) and \( \angle NMO \).
OBJECTIVE 2

TO USE THE PROPERTIES OF ISOSCELES TRIANGLES AND EQUILATERAL TRIANGLES TO SOLVE SIMPLE NUMERICAL PROBLEMS

You should remember from Geometry 1, that

(a) An isosceles triangle is one with two sides of equal length. This implies that the sizes of the angles opposite these two sides are also equal. Therefore, in triangle ABC below, since AB = AC, then $\angle C = \angle B$.

(b) An equilateral triangle is one with all three sides of equal length. This implies that the size of the three angles are also equal. Furthermore, since the three angles of a triangle always add up to 180°, then each of the angles in an equilateral triangle must be equal to 60° – always.

Example 1  In the following isosceles triangle ABC (AB = AC), if angle A is 68°, how big is angle B?

Solution  (i) Always draw a sketch first.

(ii) Since the 3 angles must add up to 180°, then $\angle B + \angle C = 180° - 68° = 112°$.

(iii) Since AB = AC, then $\angle B = \angle C$. 
So, $\angle B = \frac{112}{2} = 56^\circ$.

**Example 2**
In the isosceles triangle LMN (LM = MN), if angle M is 52°, how big is angle L?

**Solution**

![Diagram of triangle LMN with angle M = 52°]

Since the 3 angles must add up to 180°, then $\angle L + \angle N = 180° - 52° = 128°$.

Also, since LM = MN,
then $\angle L = \angle N$

$\angle L = \frac{128}{2} = 64°$

**Example 3**
In an equilateral triangle, how large is each angle?

**Solution**
Since all angles are equal, each must be equal to $\frac{180°}{3} = 60°$.

**Example 4**
In triangle XYZ, if XY = YZ and $\angle X = 46°$, how big is angle Z?

**Solution**

![Diagram of triangle XYZ with angle X = 46°]

Since XY = YZ, then $\angle X = \angle Z$, therefore $\angle Z = 46°$.

**EXERCISE**

1. In triangle RST, if RS = ST and angle S is 98°, find the size of angle R.
2. In triangle LMN, if LM = MN = LN, find the size of angle M.
3. In triangle FGH, if FG = FH and angle G is 31°, find the size of angle H.
4. In triangle XZW, if XZ = XW and angle X is 124°, how big is angle Z?
OBJECTIVE 3

TO RECOGNISE THE RELATIONSHIPS BETWEEN THE SIZES OF ANGLES FORMED WHEN A TRANSVERSAL CUTS A PAIR OF PARALLEL LINES AND USE THESE RELATIONSHIPS TO SOLVE SIMPLE NUMERICAL PROBLEMS

The diagrams below each show a pair of parallel lines cut by a transversal.

![Diagram (i)](image1)

![Diagram (ii)](image2)

![Diagram (iii)](image3)

The parallel lines are shown as such by the use of identical arrowheads. Diagram (i) is the more common of the above three, but it should be realised that the following angles will occur in situations which are often quite different from the above and, hence, may take a while to be recognised.

(a) Corresponding angles

![Diagram (iv)](image4)

It can be seen that all pairs of corresponding angles appear in the identical (or corresponding) position where the transversal cuts each of the parallel lines and as such are always congruent (equal in size).

(b) Alternate angles
It can be seen that all alternate angles appear within the parallel lines and on alternate (opposite) sides of the transversal. As such, they are always congruent (equal in size).

(c) Co-interior angles

Co-interior angles are also within the pair of parallel lines but they are on the same side of the transversal. As such, they are not congruent but they are supplementary, i.e., their two sizes always add up to $180^\circ$, e.g., if one is $60^\circ$, the other will be $120^\circ$.

This table summarises the above properties.

<table>
<thead>
<tr>
<th>ANGLES</th>
<th>DIAGRAM</th>
<th>PROPERTIES</th>
</tr>
</thead>
<tbody>
<tr>
<td>corresponding</td>
<td><img src="image1" alt="Diagram" /></td>
<td>Corresponding angles are congruent.</td>
</tr>
<tr>
<td>alternate</td>
<td><img src="image2" alt="Diagram" /></td>
<td>Alternate angles are congruent.</td>
</tr>
<tr>
<td>co-interior</td>
<td><img src="image3" alt="Diagram" /></td>
<td>Co-interior angles are supplementary.</td>
</tr>
</tbody>
</table>
Example 1 For each of the following diagrams, find the value of the angle marked. (Do not use protractors.)

1. \[ \text{Angles are alternate.} \quad \therefore x = 120^\circ \]
2. \[ \text{Angle y is corresponding and angle x is supplementary to y.} \quad \therefore y = 70^\circ, \quad x = 110^\circ \]
3. \[ \text{Angles are alternate.} \quad \therefore x = 30^\circ \]
4. \[ \text{Angles are co-interior.} \quad \therefore x = 180^\circ - 130^\circ = 50^\circ \]
5. \[ \text{Angles are corresponding.} \quad \therefore x = 50^\circ \]
6. \[ \text{Angle x is co-interior and angle y is corresponding.} \quad \therefore x = 180^\circ - 80^\circ = 100^\circ, \quad y = 80^\circ \]
EXERCISE

Find the value of the angle shown in each case below. Name the angle property used.

1. \[ 120^\circ \]
   \[ x^\circ \]
2. \[ 30^\circ \]
   \[ a^\circ \]
3. \[ b^\circ \]
   \[ 100^\circ \]
4. \[ 25^\circ \]
   \[ x^\circ \]
5. \[ \rho^\circ \]
   \[ 60^\circ \]
6. \[ d^\circ \]
   \[ 70^\circ \]
7. \[ c^\circ \]
   \[ 45^\circ \]
8. \[ y^\circ \]
9. \[ e^\circ \]
   \[ 72^\circ \]
10. \[ 65^\circ \]
    \[ a^\circ \]
11. \[ 65^\circ \]
    \[ x^\circ \]
12. \[ 65^\circ \]
    \[ b^\circ \]
SECTION 13

Trigonometry
OBJECTIVE 1

TO USE A TABLE OF TRIGONOMETRIC RATIOS TO FIND THE VALUE OF THE SINE, COSINE OR TANGENT OF A GIVEN ANGLE

In Objective 1 we defined the terms sine (sin) and cosine (cos) in terms of a unit circle. The accuracy of our result was dependent on the accuracy of our circle work. Apart from the lack of accuracy, the unit circle approach is time-consuming. As a result, tables of trigonometric values (sine, cosine, tangent) have been drawn up which are usually correct to 4 decimal places.

Before you learn how to use the tables, there is something about the measurement of angles that you must know.

In all your work so far in geometry and trigonometry, you have only been required to measure angles to the nearest degree. There are times, however, when more accuracy is needed. For this reason each degree is divided into smaller units called minutes. A minute is one sixtieth of a degree. The symbol used to indicate minutes is a raised stroke (′).

\[
60′ = 1°
\]

In the sections that follow, you will often be required to subtract the size of an angle from the size of another.

Example 1  From 90° subtract 23° 25′.
Solution

\[
\begin{align*}
90° &= 90° 00′ \\
-23° 25′ &= 66° 35′ \\
90° 00′ &= 89° 60′
\end{align*}
\]

Example 2  Subtract 46° 29′ from 73° 12′.
Solution

\[
\begin{align*}
73° 12′ &= 72° 60′ + 12′ \\
-46° 29′ &= 26° 43′ \\
73° 12′ &= 72° 72′
\end{align*}
\]
Now turn to the tables at the end of these notes and have a quick look over them. You should notice the following points.

1. The tables are set out in columns (vertical) and rows (horizontal).

2. The left hand column on each page indicates an angle size in degrees (from 0° to 89°).

3. The row at the top of each page indicates a ‘part angle’ size in minutes. Notice that they progress in multiples of six from 0′ to 54′. You may realise that 6′ is one tenth of a degree.

4. Except for the second part of the tangents’ tables, only the first column of values shows the decimal point. For sines and cosines this presents no problems as all the values are in the range of 0 to 1. You must, however, be careful with the tangent tables and make sure that you have the correct whole number part of the value.

5. With the exception of part of the tangents’ tables, all the values are given correct to three decimal places. For this reasons they are called ‘Three Figure Tables’. You may have seen other sets of tables which give the values to four decimal places, or even to seven places. These are more accurate, but for your work ‘Three Figure Tables’ are accurate enough.

6. The tables are clearly named at the top of each page. Whenever you use them, be sure that you use the right ones.

Now that you have had a look at the tables and discovered some of the general facts concerning them, it is time to get down to the business of using them. You will find that they can be used in two ways, *ie* if you know the size of an angle, you can find its sine, cosine and tangent values. If you know the value of the sine, cosine or tangent, you can find the size of the angle.

(a) *When the size of the angle is given in degrees only*, the first column of values gives the required value for the sine, cosine or tangent of that angle.
Example 3  From the tables, find the value of cos 27°.

Solution  First find the table headed cosine, then find 27°.

<table>
<thead>
<tr>
<th>Angle size in degrees</th>
<th>Value of the cosine</th>
</tr>
</thead>
<tbody>
<tr>
<td>25°</td>
<td>0.906</td>
</tr>
<tr>
<td>26°</td>
<td>0.899</td>
</tr>
<tr>
<td>27°</td>
<td>0.891</td>
</tr>
<tr>
<td>28°</td>
<td>0.883</td>
</tr>
<tr>
<td>29°</td>
<td>0.875</td>
</tr>
</tbody>
</table>

\[ \cos 27° = 0.891 \]

The first column is headed 0′. All values given in the column are for angles whose sizes are an exact number of degrees. What we have found in the example above is the cosine of an angle of 27 degrees and 0 minutes.

\[ \text{ie } 27° 00′ \]

(b) When the size of the angle is given in degrees and minutes:

1. Look down the left hand column until you find the row beginning with that number of degrees.
2. Place a ruler or card under the row.
3. Look across the top row until you find the column headed by the required number of minutes.
4. Run your finger down this column to the row found in Step 1.
5. The numeral at the intersection of the row and the column is the decimal part of your answer.
6. Look back along the row to find the whole number part.

Example 4  Find the value of \( \tan 66° 24′ \).
You will have noticed that the tables have values for angles between 0° and 90° only. For angles larger than 90° we need to calculate the size of the reference angle, which is always less than 90°. (See Example 2 of Objective 1.) Look this value up in our tables; then by considering the unit circle on the next page, decide whether the table value for that particular angle should be positive or negative.
Example 5  Find the sin, cos and tan of $145^\circ 36'$.  

Solution  Firstly, sketching in $145^\circ 36'$ on a unit circle, in standard position, we have:

Secondly, to find the reference angle (which is necessary since our original angle was larger than $90^\circ$) we want to find the size of the angle between our terminal ray and the horizontal axis. In this case, the reference angle is found by subtracting $145^\circ 36'$ from $180^\circ$, ie

$$180^\circ = 180^\circ 00' \quad 179^\circ 60' \quad 180^\circ 00' = 179^\circ + 1^\circ = 179^\circ 60'$$

Finally, use your tables to find the sin, cos and tan of this reference angle, ie

$$\sin 34^\circ 24' = 0.565$$
$$\cos 34^\circ 24' = 0.825$$
$$\tan 34^\circ 24' = 0.685$$

Fourthly, consider the quadrant in which the terminal ray is placed. In our case, the angle ($145^\circ 36'$) has ended in the second quadrant. Hence,
the sine value is positive
the cosine value is negative
and the tangent value is negative.

\[ \sin 145^\circ 36' = 0.565 \]
\[ \cos 145^\circ 36' = -0.825 \]
and \[ \tan 145^\circ 36' = -0.685 \]

**EXERCISE**

Use your tables to complete the following.

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>Sin ( \theta )</th>
<th>Cos ( \theta )</th>
<th>Tan ( \theta )</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) 25°</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(b) 35°</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(c) 60°</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(d) 47°</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(e) 73°</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(f) 23° 12'</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(g) 41° 36'</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(h) 55° 24'</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(i) 126° 12'</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(j) 148° 42'</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
OBJECTIVE 2

TO WRITE A SIZE OF THE ACUTE ANGLE WHICH HAS A GIVEN SINE, COSINE OR TANGENT USING A TABLE OF TRIGONOMETRIC RATIOS

Now that you know how to read the tables for the sine, cosine and tangent of an angle, you will be able to solve trigonometric equations by using the tables in reverse, i.e., if you know the value of the sine, cosine or tangent you will be able to find the size of the angle.

Example 1 If \( \sin \theta = 0.427 \) and \( 0^\circ \leq \theta \leq 90^\circ \), what is the size of the angle?

Solution
1. Look in the correct table to find the value you have been given.
2. Place a ruler or card under the row in which the value occurs.
3. The number of whole degrees in the angle is read from the left hand column of the table – in this case 25°.
4. The number of minutes to be added is read from the top of the column in which the 0.427 appears – in this case 18′.
5. This gives the angle size as 25° 18′.
NOTE: If the trigonometric value for sine, cosine or tangent is not an exact table value, then obtain your answer from the table entry which is closest to the value you were given. For example, if given \( \tan \theta = 0.660 \) to solve for \( \theta \), you would find that the value of 0.660 does not appear in the tangents' tables. The closest value to this is 0.659; so the required answer is \( \theta = 33^\circ \ 24' \).

**Example 2** If the cosine of an acute angle (angle less than 90°) is 0.427, what is the size of the angle? (Use tables of trigonometric ratios.)

**Solution** Let \( \theta \) represent the angle.

Then we have \( \cos \theta = 0.427 \).

From the cosine tables, \( \cos 64^\circ \ 42' = 0.427 \)

\[ \therefore \ \theta = 64^\circ \ 42' \] is the size of the angle.

**EXERCISE**

Solve the following trigonometric equations by using tables, i.e. find the value of the angle \( \theta \).

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>(a)</td>
<td>( \sin \theta = 0.349 )</td>
</tr>
<tr>
<td>(b)</td>
<td>( \sin \theta = 0.863 )</td>
</tr>
<tr>
<td>(c)</td>
<td>( \cos \theta = 0.818 )</td>
</tr>
<tr>
<td>(d)</td>
<td>( \cos \theta = 0.522 )</td>
</tr>
<tr>
<td>(e)</td>
<td>( \tan \theta = 0.396 )</td>
</tr>
<tr>
<td>(f)</td>
<td>( \tan \theta = 1.183 )</td>
</tr>
</tbody>
</table>
OBJECTIVE 3

TO WRITE THE SINE, COSINE OR TANGENT OF EITHER ACUTE ANGLE IN A RIGHT TRIANGLE, GIVEN LENGTHS OF THE THREE SIDES

Consider a right triangle ABC with \( \angle C = 90^\circ \) constructed in such a way that \( \angle ABC \) is in standard position.

If \( AB \) intersects the unit circle in D and \( DE \) is drawn perpendicular to \( BC \), then we have:

1. \[ \therefore \frac{DE}{AC} = \frac{BD}{AB} \]
   but \( DE = \sin B \)
   and \( BD = 1 \) (why?)
   so \( \sin B = \frac{1}{AB} \)
   \( ie \) \( \sin B = \frac{AC}{AB} \)

2. \[ \therefore \frac{BE}{BC} = \frac{BD}{AB} \]
   but \( BE = \cos B \)
   and \( BD = 1 \)
   so \( \cos B = \frac{1}{AB} \)
   \( ie \) \( \cos B = \frac{BC}{AB} \)

This shows how we can calculate the value of the sines and cosines of acute angles by using the lengths of the sides of a right triangle.

* \( BD = 1 \) because it is the radius of a unit circle.
Following on from what we have proved about the sine and cosine of an acute angle in a right triangle, we can also find the value of the tangent of an acute angle from the lengths of the sides of the triangle.

In the right triangle $ABC$

\[
\sin B = \frac{AC}{AB} \quad \text{and} \quad \cos B = \frac{BC}{AB}
\]

\[
\tan B = \frac{\sin B}{\cos B} = \frac{AC}{AB} \left[ \frac{AC}{AB} + \frac{BC}{AB} \right]
\]

\[
= \frac{AC}{AB} \times \frac{AB}{BC}
\]

\[
= \frac{AC}{BC}
\]

Let’s simplify what we have found out about the sine, cosine and tangent of the angle $B$.

$\overline{AB}$ is the HYPOTENUSE of the triangle.

$\overline{AC}$ is the side OPPOSITE the angle $B$.

$\overline{BC}$ is next to, or ADJACENT to the angle $B$.

Using these names for the lengths of the sides of a right triangle we have:

\[
\sin B = \frac{AC}{AB} = \frac{\text{OPPOSITE SIDE}}{\text{HYPOTENUSE}} = \frac{O}{H}
\]

\[
\cos B = \frac{BC}{AB} = \frac{\text{ADJACENT SIDE}}{\text{HYPOTENUSE}} = \frac{A}{H}
\]

\[
\tan B = \frac{AC}{BC} = \frac{\text{OPPOSITE SIDE}}{\text{ADJACENT SIDE}} = \frac{O}{A}
\]

If we want to find the sine, cosine or tangent of the angle $A$, this can be done by re-positioning the triangle so that $A$ is in standard position. See diagrams next page.
AB is still the HYPOTENUSE.

but \( \overline{BC} \) is OPPOSITE to the angle A.

and \( \overline{AC} \) is ADJACENT to the angle A.

\[
\sin A = \frac{BC}{AB}, \quad \cos A = \frac{AC}{AB}, \quad \tan A = \frac{BC}{AC}
\]

Any right angle in which \( \beta \) is an acute angle as shown below.

The values for the sine, cosine and tangent of an angle found by using the lengths of the sides of a right triangle are often called the trigonometric ratios of the angle. In the sections that follow, the trigonometric ratios are very important so make sure you know how to find them. Memorise the table above.

As an aid to memorising these, you may try memorising the word:

SOH-CAH-TOA

which when considered in order, gives you:

\[
\sin \beta = \frac{OPP}{HYP}, \quad \cos \beta = \frac{ADJ}{HYP}, \quad \tan \beta = \frac{OPP}{ADJ}
\]
In the examples that follow, we will find the trigonometric ratios (sine, cosine and tangent) of some acute angles in given right triangles.

**Example 1**  Find the sine, cosine and tangent of the angle $P$ in this right triangle.

![Diagram of a right triangle with sides labeled: QR = 3, PR = 4, PQ = 5.]

**Solution**

(a) $\sin P = \frac{QR}{PQ} = \frac{3}{5}$

(b) $\cos P = \frac{PR}{PQ} = \frac{4}{5}$

(c) $\tan P = \frac{QR}{PR} = \frac{3}{4}$

**Example 2**  Find the trigonometric ratios of $\angle X$ and $\angle Y$ in $\triangle XYZ$.

![Diagram of a right triangle with sides labeled: YZ = 9, XZ = 40, XY = 41.]

**Solution**

1. $\sin X = \frac{YZ}{XY} = \frac{9}{41}$

2. $\sin Y = \frac{XZ}{XY} = \frac{40}{41}$

$\cos X = \frac{XZ}{XY} = \frac{40}{41}$

$\cos Y = \frac{YZ}{XY} = \frac{9}{41}$

$\tan X = \frac{YZ}{XZ} = \frac{9}{40}$

$\tan Y = \frac{XZ}{YZ} = \frac{40}{9}$
EXERCISE

1. KST is the right triangle drawn below. Complete the following.

\[ \sin \theta = \frac{KS}{?}, \quad \cos \theta = \frac{?}{KT}, \quad \tan \theta = \frac{?}{?} \]

2. Find the sine, cosine and tangent of the angle D in the right triangle below.

3. Find the trigonometric ratios (sine, cosine and tangent) of \( \angle M \) and \( \angle N \) in \( \angle LMN \) below.

4. In \( \triangle PQR \), \( \angle Q = 90^\circ \), \( PR = 10 \), \( RQ = 8 \) and \( QP = 6 \). Find the trigonometric ratios (sine, cosine and tangent) of \( \angle P \) and \( \angle R \).
SECTION 13 – TRIGONOMETRY

OBJECTIVE 4

TO USE A TRIGONOMETRIC RATIO TO CALCULATE THE LENGTH OF ANOTHER SIDE GIVEN THE MEASURES OF ONE ACUTE AND ONE SIDE IN A RIGHT TRIANGLE

If the length of only one side is known, the length of another side can be found by using trigonometric ratios. The size of one of the acute angles must be known.

Example 1

In \( \Delta XYZ \), \( \angle Y = 90^\circ \), \( \angle Z = 25^\circ \), \( YZ = 10 \text{ cm} \). Find \( XY \) and \( XZ \).

Solution

\[
\tan Z = \tan 25^\circ
\]

\[
XY \tan Z = YZ
\]

\[
XY \tan 25^\circ = 10
\]

\[
XY = \frac{10}{\tan 25^\circ}
\]

\[
XY = \frac{10}{0.466}
\]

\[
XY \approx 21.57
\]

We could have used

\[
\cos Z = \frac{XZ}{YZ}
\]

but this is harder.

\[
0.906 = \frac{XZ}{10}
\]

\[
XZ = \frac{10 \times 0.906}{10}
\]

\[
XZ = 9.06
\]

\[
XZ \approx 9.06
\]

See below.

In all of your work it will only be necessary to give the answers correct to the first decimal place.

So far we have found the length of the side \( XY \). Now the lengths of two sides are known so the Pythagorean Theorem can be used to find the length of the third side \( (XZ) \).

\( XZ \) is the hypotenuse. \( XZ = XY^2 + YZ^2 \)

\[
XZ = 4.7^2 + 10^2
\]

\[
XZ = 22.09 + 100
\]

\[
XZ = 122
\]

\[
XZ = \sqrt{122}
\]

\[
XZ \approx 11 \text{ cm}
\]
Example 2  In ΔPQR, ∠R = 90°, ∠P = 47°, PQ = 10 cm. Find RQ and PR.

Solution

\[
\frac{RQ}{PQ} = \frac{\text{OPP}}{\text{HYP}} \quad \frac{RQ}{PQ} = \sin P
\]

\[ie \quad \frac{RQ}{12} = \sin 47°\]

\[ie \quad RQ = 12 \sin 47°\]

\[= 12 \times 0.731\]

\[= 8.772\]

\[∴ RQ \approx 8.8 \text{ m}\]

\[
\frac{PR}{PQ} = \frac{\text{ADJ}}{\text{HYP}} \quad \frac{PR}{PQ} = \cos P
\]

\[ie \quad \frac{PR}{12} = \cos 47°\]

\[ie \quad PR = 12 \cos 47°\]

\[= 12 \times 0.682\]

\[= 8.184\]

\[∴ PR \approx 8.2 \text{ m}\]

Always try to find a ratio which has the ‘unknown’ side as the numerator. This has been done in the previous examples because it makes the working out easier.

So, the steps required are:

(i) Draw a diagram (if not already supplied) and place on it all the given information.

(ii) Write down a ratio of the unknown side (the side you have been asked to find) over the given side, ie place the unknown side in the top line (numerator) of your ratio – if possible.

(iii) Consider your diagram and the angle whose size was given to decide the relationship (opposite, adjacent, hypotenuse) of each of the two sides in the above ratio.

(iv) Write the ratio in terms of O, A, H then write an equation involving this ratio and the corresponding trigonometric ratio (sine, cosine, tangent) associated with your given angle.

(v) Look up your tables for the value of your trigonometric ratio then solve the equation by cross-multiplying.
Example 3  
In $\triangle STU$, $\angle TR = 90^\circ$, $SU = 13\, \text{m}$ and $\angle S = 67^\circ\, 24'$. Find $ST$.

Solution  
(i) 

(ii) $\frac{ST}{SU} = \left( \frac{\text{unknown side}}{\text{known side}} \right)$

(iii) $\frac{ST}{SU} = \frac{A}{H}$ (cosine for angle S)

(iv) $\cos \angle S = \frac{ST}{SU}$

$\therefore \cos 67^\circ\, 24' = \frac{ST}{13}$

(v) $0.384 = \frac{ST}{13}$

$\therefore 0.384 \times 13 = ST$

$\therefore ST = 4.992\, \text{m}$

$= 5.0\, \text{m}$ (to one decimal place)

EXERCISE

1. Calculate BC given:

2. Calculate MN given:
3. Calculate RT given:

4. Calculate BC given:

5. In ΔXYZ, \( \measuredangle Y = 90^\circ \), \( \measuredangle X = 54^\circ \) and XZ = 17 cm. What is the length of YZ?

6. In ΔPQR, \( \measuredangle Q = 90^\circ \), \( \measuredangle P = 43^\circ \) and PR = 8 km. Find PQ and RQ.
OBJECTIVE 5

TO USE A TRIGONOMETRIC RATIO TO CALCULATE THE SIZE OF ONE OF THE ACUTE ANGLES GIVEN THE LENGTHS OF TWO SIDES OF A RIGHT TRIANGLE

If only the right angle is known, the size of one of the acute angles can be found from the ratios of the lengths of the sides. The lengths of at least two sides must be known.

Example 1  In ΔXYZ, ∠X = 90°, XY = 4 cm and XZ = 7 cm. Find ∠X and ∠Y.

Solution

Draw the diagram and put in all the given information.

\[
\tan Z = \frac{XY}{XZ} = \frac{4}{7} = 0.571
\]

From tables

\[\therefore \, \angle Z = 29°\,42'\]

\[\angle Y = 90° - \angle Z = 90° - 29°\,42' = 60°\,18'\]

You could have used \tan Y.

\[\text{ie } \tan Y = \frac{7}{4}\]

From Δ Angle Theorem. (3 angles add up to 180°.)

Example 2  In ΔPQR, ∠Q = 90°, PR = 8 cm and PQ = 5 cm. Find ∠P and ∠R.

Solution

\[
\cos \angle P = \frac{PQ}{PR} = \frac{5}{8} = 0.625
\]

\[\therefore \, \angle P = 15°\,18'\]

\[\angle R = 90° - \angle P = 90° - 51°\,18' = 38°\,42'\]

You could have used \sin R.

\[\text{ie } \sin R = \frac{5}{8}\]

\[60' = 1°\]
So, in summary, the basic steps are:

(i) Draw a diagram and put in all the given information.

(ii) Considering the angle you wish to find, decide the relationships to it of the given sides (opposite, hypotenuse, adjacent).

(iii) Write an equation involving the ratio of these two sides and the corresponding trigonometric ratio (sine, cosine, tangent) associated with your unknown angle.

(iv) Convert your ratio to a decimal then use your tables – in reverse, to find the size of the unknown angle.

Example 2  
In \( \triangle ABC \), \( \angle A = 90^\circ \), \( AB = 3 \text{ cm} \) and \( BC = 10 \text{ cm} \). Find \( \angle C \) and hence \( \angle B \).

Solution

(i)

(ii) For angle \( C \), \( AB \) is opposite and \( BC \) is the hypotenuse.

(iii) \[ \sin \angle C = \frac{AB}{BC} = \frac{3}{10} \]

(iv) Therefore \( \sin \angle C = 0.300 \) and from tables \( \sin 17^\circ 24' = 0.299 \) and \( \sin 17^\circ 30' = 0.301 \), therefore the size of angle \( C \) is between \( 17^\circ 24' \) and \( 17^\circ 30' \).

Since it would appear to be halfway between these two, let’s say:

\[ \angle C = 17^\circ 27' \]

Hence \( \angle B = 90^\circ - 17^\circ 27' \)

\[ = 72^\circ 33' \]

Since the 3 angles must add up to \( 180^\circ \) and we already know that \( \angle A = 90^\circ \), therefore, \( \angle C + \angle B = 90^\circ \).
EXERCISE

1. Calculate the size of angle A.

2. Find $\angle Z$ of this triangle.

3. Find $\angle Q$ of this triangle.

4. If $\angle M = 90^\circ$ in $\triangle PNM$, $PM = 6$ cm and $NM = 5$ cm, find $\angle N$ and $\angle P$.

5. In $\triangle KST$, $\angle T = 90^\circ$, $SK = 11$ cm and $ST = 4$ cm. Find $\angle S$ and $\angle K$.

6. In $\triangle STV$, $\angle T = 90^\circ$, $ST = 11$ cm and $VT = 8$ cm. Find $\angle V$ and $\angle S$. 
TABLES OF TRIGONOMETRIC RATIOS

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TABLES OF TRIGONOMETRIC RATIOS

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Answers to Exercises
ANSWERS TO EXERCISES

Section 1 – Decimals

Objective 1

60, 8, \( \frac{5}{10}, \frac{7}{100} \)

Objective 3

0.12, 0.7, 0.15, 0.056, 0.45, 0.375, 0.8, 0.875

Objective 4

9\( \frac{1}{8} \), 3\( \frac{1}{4} \), 2\( \frac{1}{4} \), 6\( \frac{2}{5} \), 7\( \frac{1}{8} \), 1\( \frac{1}{8} \)

Objective 5

Exercise 1

1. 10.21
2. 24.25
3. 9.374
4. 10.92
5. 10.678

Exercise 2

1. 985
2. 870
3. 276.3
4. 880 000

Objective 6

1. 15.64
2. 6.499 8
3. 8.200 8
4. 34.102
5. 0.096 324

Objective 7

1. 174.2
2. 43 610
3. 1.65
4. 0.816
5. 249.3
6. 0.642
7. 0.3397
8. 0.00723
9. 0.0003
10. 0.01003

Objective 8
1. 0.25
2. 0.007
3. 4
4. 0.025

Section 2 – Fractions

Objective 1
1. no
2. no
3. yes
4. yes

Objective 2
\[
\frac{4}{6} = \frac{8}{12} = \frac{10}{15} = \frac{6}{9} = \frac{18}{27} = \frac{54}{81}
\]

Objective 3
1. \(\frac{3}{4}\)
2. \(\frac{9}{7}\)
3. \(\frac{3}{4}\)
4. \(\frac{2}{3}\)
5. \(\frac{2}{3}\)

Objective 4
1. \(\frac{7}{3}\)
2. \( \frac{25}{4} \)
3. \( \frac{15}{8} \)
4. \( \frac{5}{3} \)
5. \( \frac{15}{2} \)

**Objective 5**
1. \( 3 \frac{1}{2} \)
2. \( 2 \frac{1}{5} \)
3. \( 7 \frac{2}{3} \)
4. \( 4 \frac{2}{5} = 4 \frac{1}{2} \)
5. \( 2 \frac{4}{5} = 2 \frac{3}{5} \)
6. \( 2 \frac{3}{5} \)
7. \( 12 \frac{1}{2} \)
8. \( 13 \frac{1}{4} \)

**Objective 6**
1. \( 1 \frac{1}{5} \)
2. \( \frac{2}{7} \)
3. \( 1 \frac{3}{17} \)
4. \( \frac{1}{21} \)
5. \( 1 \frac{1}{21} \)
6. \( \frac{11}{24} \)

**Objective 7**
1. \( 8 \frac{5}{17} \)
2. \( 7 \frac{2}{21} \)
3. \( 8 \frac{2}{15} \)
4. \( 4 \frac{1}{4} \)
5. $\frac{7}{8}$
6. $\frac{44}{27}$

**Objective 8**
1. 4
2. $7\frac{1}{4}$
3. $5\frac{1}{3}$
4. 10
5. 8
6. $5\frac{5}{8}$

**Objective 9**
1. $\frac{5}{4}$
2. $\frac{3}{7}$
3. $\frac{2}{5}$
4. $\frac{3}{5}$

**Objective 10**
1. 28
2. 6
3. $11\frac{1}{4}$
4. $2\frac{5}{8}$
5. $4\frac{2}{3}$
6. $\frac{9}{10}$

**Section 3 – Per cent**

**Objective 1**
1. 24%
2. 30%
3. 35%
4. 48%

Objective 2
1. 53%
2. 954%
3. 8%
4. 382%
5. 0.7%
6. 66%
7. 1860%
8. 1500%

Objective 3
1. $\frac{1}{4}$
2. $\frac{63}{100}$
3. $1\frac{31}{30}$
4. $\frac{13}{200}$
5. $\frac{53}{1000}$
6. $\frac{4}{5}$

Objective 4
1. 0.85
2. 1.35
3. 0.085
4. 0.61
5. 0.385
6. 0.0025

Objective 5
1. 9
2. 11.36
3. 58.76
4. 255
5. $1380$
Objective 6
1. 60%
2. 52%
3. 37 1/2%
4. 130%

Objective 7
1. 16
2. 1 300
3. 250
4. 0.08
5. $465

Section 4 – Measurement 1
The S.I. System
28 900 mm
3 790 mm
80 mm
50 000 mm
397 mm

8.014 km
0.748 943 km
0.907 km
0.070 5 km

49.034 m
384 m
7 049 m
3.12 m
7.162 m

34 000 g
34 000 g
894 g

0.307 kg
481 kg
3.97 kg

47.348 t
4.093 648 t
0.347 t
Objective 1
1. 16 mm
2. 12 cm
3. 480 cm
4. 10.8 mm
5. 412 cm

Objective 2
1. 31.4 cm
2. 43.96 cm
3. 157 m
4. 15.7 cm
5. 879.2 mm
6. 47.1 m

Objective 3
1. 7 cm
2. 90 cm
3. 153 m
4. 40 mm (neither was mm)
5. 0.3 cm
6. 12.25 m

Objective 4
1. 132 mm
2. 2.576 m
3. 208 cm
4. 294 mm
5. 27.03 m

Objective 5
1. 936 mm
2. 418 cm
3. 560 mm
4. 48 cm

Objective 6
1. 153.86 cm
2. 1256 cm
3. 3.14 m
4. 55.389 6 m
5. 1 384.74 m²
6. 31 400 km²
7. 9.616 25 m²

Objective 7
1. 27.5 m²
2. 15 cm²
3. 72 mm²
4. 50 km²
5. 84 m²

Objective 8
1. 27 cm²
2. 14 m²
3. 189.25 m²
4. 83.44 m²
5. 21.98 m²

Objective 9
1. 94 cm²
2. 15 400 mm²
3. 392 cm²
4. 7 m²

Objective 10
1. 9 420 cm²
2. 36.31 m²
3. 628 cm²
4. 83.586 8 m²

Section 5 – Measurement 2

Objective 1
1. 1 695.6 mm²
2. 489.84 m²
3. 471 m²
4. 320.28 cm²

Objective 2
1. 615.44 m²
2. 113.04 cm²
3. 804.84 cm²
4. 200.96 cm²

**Objective 3**
1. 24 m³
2. 8 cm³
3. 3 m³
4. 1.875 m³

**Objective 4**
1. 2009.6 cm³
2. 314 m³
3. 376.8 m³
4. 6.28 cm³

**Objective 5**
1. 24.917 m³
2. 37.68 m³
3. 219.76 cm³
4. 528 m³

**Objective 6**
1. 523.33 cm³
2. 14130 cm³
3. 113.04 cm³
4. $7.358 \times 10^9$ cm³
5. 1436 cm³

**Section 6 – Ratio and proportion**

**Objective 1**
1. 60:12 or 5:1
2. 4:48 or 1:12
3. 2:3 000 or 1:1 500
4. 2:6 or 1:3

**Objective 2**
1. 10:9
2. 1:3
3. 3:2
4. 1:1 800
5. 1:10
Objective 3
1. 6 cm and 9 cm
2. $9 and $3
3. $8 and $12
4. $2.5 and $10
5. 6 m and 8.4 m

Objective 4
1. yes
2. no
3. no
4. no
5. 4
6. 14
7. 28
8. 55

Objective 5
1. $1.75
2. $12.50
3. 20 exposures (12¢ each)
4. $10.92

Section 7 – Scientific notation

Objective 1
1. $10^7$
2. $10^7$
3. $10^5$
4. $10^3$
5. $10^6$
6. $10^9$

Objective 2
1. 10
2. $10^7$
3. $10^9$
4. $10^7$
Objective 3
1. 1
2. $10^6$
3. $10^6$
4. $\frac{1}{10}$ or $10^{-1}$
5. $10^1$
6. 1

Objective 4
1. 10
2. $10^{-8}$
3. 1 (or $10^0$)

Objective 5
1. $10^{-4}$ or $\frac{1}{10^4}$
2. $10^{-6}$ or $\frac{1}{10^6}$
3. $10^6$
4. $10^8$
5. 10
6. $10^{-8}$ or $\frac{1}{10^8}$

Objective 6
1. $10^{-9}$ or $\frac{1}{10^9}$
2. $10^4$
3. $10^{-12}$ or $\frac{1}{10^{12}}$
4. 1
5. $10^{10}$

Objective 7
1. $4.6 \times 10^6$
2. $8.7 \times 10^3$
3. $1.8 \times 10^{-2}$
4. $2.9 \times 10^{-4}$
5. $7.5 \times 10^6$
6. $9 \times 10^{-2}$
ANSWERS TO EXERCISES

7. \(1.452 \times 10^2\)
8. \(7.136 \times 10^2\)
9. \(4 \times 10^{-4}\)

Objective 8
1. 0.0025
2. 860000
3. 0.091
4. 0.00074
5. 59000
6. 0.62

Objective 9
1. \(1.1 \times 10^{-4}\)
2. \(0.9 \times 10^2\)
3. \(9.24 \times 10^5\)
4. \(2 \times 10^5\)
5. \(48.3 \times 10^7\)
6. \(6.48 \times 10^2\)

Section 8 – Algebra

Objective 1
(You may have used different letters.)

1. \(x + 15\)
2. \(x - 4\)
3. \(7x\)
4. \(\frac{x}{20}\)
5. \(10x\)
6. \(\frac{20}{x}\)
7. \(\frac{x}{5}\)
8. \(x + 6\)
9. \(x - 5\)
10. \(\frac{3x}{4}\)
Objective 2
1. $7^2$
2. $4^3$
3. $x^2$
4. $x^3$
5. $m^3$
6. $x^3$

Objective 3
1. $x + 2y$
2. $2(x + y)$
3. $40 - 3x$
4. $15 - 3x$
5. $\frac{x}{2} = 7y$

Objective 4
1. 7
2. 6
3. −2
4. 0
5. 4

Objective 5
1. $7m$
2. $y$
3. $10x$
4. $-n$
5. $3y$
6. $-7a$

Objective 6
1. $6x + 4y$
2. $8b$
3. $x^2$
4. $4x^2 - 3xy$
5. $-4x$
6. $-x^2 - 9x + 1$
7. This cannot be simplified.
Objective 7
1. 3x – 15
2. –8y + 20
3. –2y + 1x
4. –8x + 16
5. –3x – 6y + 3
6. –4x – 9y + 6
7. 6x^3 – 4x^2 + 2

Objective 8
1. x + 10
2. –2x
3. 13x^2 + x
4. 4x^2 – 7x
5. 2r – 19s

Section 9 – First degree equations
Objective 1
1. 3
2. –4
3. 20
4. –20
5. 21
6. 6
7. –8
8. 7
9. –5
10. –7

Objective 2
1. 2
2. 2
3. 4
4. –100
5. 0
6. 3
7. 243
8. –2
Objective 3
1. 4
2. 8
3. –2
4. –3
5. –70
6. 5
7. –1
8. $\frac{-2}{5}$

Objective 4
1. 7
2. 9
3. 8
4. 0
5. 2

Objective 5
1. $w + 350 = 985$
2. $38 + 18n = 110$
3. $2(w + (w + 2)) = 44$
4. $l + \frac{2}{5}l = 25$
5. $.2n + .8(40) = .6(40 + n)$

Objective 6
1. 29 years and 59 years
2. 5 m and 3 m
3. 150 km
4. $2.00$

Section 10 – Formula transposition

Objective 1
1. (a) 57.75 m$^3$ (b) 132.03 m$^3$
2. (a) 1 250 watts (b) 990 watts
3. 80.593
4. 54.003
5. 3.988
Objective 2
1. \( m = k + n \)
2. \( m = n - p \)
3. \( m = r - t \quad \text{or} \quad m = -t + r \)  
   (Obviously these are the same. The order of \( r \) and \( t \) is simply reversed.)
4. \( m = p + r - t \)
5. \( m = r - n - s - t \)  
   (Again the order may be different but the signs in front of each term must be the same.)
6. \( m = p - r + t + s \)

Objective 3
1. \( r = \frac{c}{2\pi} \)
2. \( H = \frac{2A}{B} \)
3. \( T = \frac{60P}{2\pi n} = \frac{30P}{\pi n} \)
4. \( N = \frac{1\,000\,s}{\pi d} \)
5. \( I = \frac{P}{0.85\,V} \)
6. \( R = \frac{E^2}{P} \)

Objective 4
1. \( B = \frac{C - A}{2} \)
2. \( b = \frac{4 - P - q}{2c} \)
3. \( m = \frac{y - b}{x} \)
4. \( c = -\frac{5(F - 32)}{9} \)
5. \( f = \frac{m(v - u)}{t} \)

Objective 5
1. \( u = \frac{2s}{t} - v \)
2. \( \theta = \frac{1}{\alpha} \left( \frac{L_2}{L_1} - 1 \right) \)
3. \( r = 1 - \frac{A}{S} \)

4. \( H = -\frac{A}{2\pi r} - r \)

5. \( x = \frac{w}{aq} - t \)

**Objective 6**

1. \( I = \sqrt{\frac{P}{R}} \)

2. \( V = \sqrt{\frac{2gE}{W}} \)

3. \( V = \sqrt{\frac{DP}{0.334W}} \)

4. \( m = \frac{5Bd^3}{12} \)

5. \( a = \sqrt{c^2 + b^2} \)

**Objective 7**

1. \( r = \frac{m}{p - t} \)

2. \( W = \frac{PL}{R - PL} \)

3. \( A = \frac{-BC}{B - C} \) or \( A = \frac{BC}{-B + C} \)

4. \( x = \frac{3 + y}{y - 1} \)

5. \( t = \frac{mp}{1 - m} \)

**Section 11 – Geometry 1**

**Objective 1**

(a) 10°

(b) 30°

(c) 80°

(d) 90°
(e) $120^\circ$
(f) $135^\circ$
(g) $150^\circ$
(h) $180^\circ$

**Objective 2**
(a) acute
(b) right
(c) acute
(d) obtuse
(e) right
(f) acute
(g) obtuse
(h) obtuse

**Objective 3**
1. triangle
2. pentagon
3. quadrilateral (parallelogram)
4. hexagon
5. quadrilateral (trapezium)
6. octagon

**Objective 4**
(a) Triangle PMX is isosceles.
(b) Triangle XYM and XMZ are right.
(c) Triangle MPZ is scalene.
(d) Triangle XYZ is equilateral.

**Objective 5**
1. trapezium (trapezoid)
2. parallelogram
3. square
4. rhombus
5. rectangle

**Objective 6**
1. 17 cm
2. 6.5 m
3. 3 cm
4. 10 cm
5.  9 cm

Section 12 – Geometry 2

Objective 1
1.  107°
2.  64°
3.  60°
4.  62° and 62°

Objective 2
1.  41°
2.  60°
3.  31°
4.  28°

Objective 3
1.  x = 120°  corresponding angles
2.  a = 30°  alternate angles
3.  b = 80°  co-interior angles
4.  x = 25°  alternate angles
5.  p = 120°  co-interior angles
6.  d = 70°  corresponding angles
7.  c = 45°  corresponding angles
8.  y = 90°  co-interior angles
9.  e = 72°  alternate angles
10.  a = 65°  corresponding angles
11.  x = 115°  co-interior angles
12.  b = 65°  alternate angles

Section 13 – Trigonometry

Objective 1
(a)  0.423,  0.906,  0.466
(b)  0.574,  0.819,  0.700
(c)  0.866,  0.500,  1.732
(d)  0.731,  0.682,  1.072
(e)  0.956,  0.292,  3.271
(f)  0.394,  0.919,  0.429
(g)  0.664,  0.748,  0.888
(h) 0.823, 0.568, 1.450
(i) 0.807, –0.591, –1.366
(j) 0.520, –0.854, –0.608

**Objective 2**
(a) 20° 24’
(b) 59° 36’ or 59° 42’
(c) 35° 6’
(d) 58° 30’
(e) 21° 36’
(f) 49° 48’
(g) 7° 36’
(h) 71° 12’
(i) 40° 24’
(j) 27° 54’
(k) 24° 48’
(l) 41° 6’

**Objective 3**
1. \( \sin \theta = \frac{KS}{KT} \), \( \cos \theta = \frac{ST}{KT} \), \( \tan \theta = \frac{KS}{ST} \)
2. \( \sin D = \frac{12}{13} \), \( \cos D = \frac{5}{13} \), \( \tan D = \frac{12}{5} \)
3. \( \sin M = \frac{24}{25} \), \( \cos M = \frac{7}{25} \), \( \tan M = \frac{24}{7} \)
   \( \sin N = \frac{7}{25} \), \( \cos N = \frac{24}{25} \), \( \tan N = \frac{7}{24} \)
4. \( \sin P = \frac{8}{10} \), \( \cos P = \frac{6}{10} \), \( \tan P = \frac{8}{6} \)
   \( \sin R = \frac{6}{10} \), \( \cos R = \frac{8}{10} \), \( \tan R = \frac{6}{8} \)

**Objective 4**
1. 11.82 m
2. 37.086 cm
3. 7.5 mm
4. 12.852 cm
5. 13.8 cm
6. 5.5 km and 5.8 km
Objective 5
1. 41° 48’
2. 48° 38’
3. 44° 24’
4. 50° 12’ and 39° 48’
5. 68° 42’ and 21° 18’
6. 54° and 36°
TRADE CALCULATIONS FOR FABRICATORS

MEM2.7C10A Perform Computations - Basic
MEM2.8C10A Perform Computations
MEM2.13C5A Perform Mathematical Computations

DESCRIPTION
This resource is a detailed guide to understanding and performing a wide range of calculations and other mathematical functions with practical, problem-solving applications in the workshop. Tasks covered involve decimals, fractions, percentages, measurements, ratio and proportion, scientific notation, algebra, first degree equations, formula transposition, geometry and trigonometry.

CATEGORY
Engineering

TRAINING PACKAGE
• (HLT02) Health
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